

Pre-Contractual Information Acquisition

C. Manuel Willington*

ILADES - Universidad Alberto Hurtado

September, 2004

Abstract

We consider a principal-agent model in which the agent may acquire costly information about his effort costs before he accepts a contract. The model departs from the literature in two ways: (1) the information is 'hard' in the sense that it can be credibly communicated, and (2) the parties are unable to commit to not renegotiate their contract. When the cost of acquiring information is low, the optimal contract induces information acquisition. In this case the contract is renegotiated and leaves the agent no rent. When the information cost is higher, the optimal contract induces the agent not to acquire information. In this case, if the cost is not too high, quantities are inefficient and the agent may receive rent. If the cost is yet higher, the contract again yields efficient quantities and leaves the agent no rent. These results hold also if parties can commit themselves not to renegotiate.

KEYWORDS: Information Acquisition, Hard Information, Renegotiation.

JEL NUMBERS: D82, D83

CORRESPONDENT: C. Manuel Willington, ILADES, Erasmo Escala 1835, Santiago, Chile. Email: mwilling@uahurtado.cl.

*I thank Steve Matthews and Nicola Persico for useful comments. All remaining errors are mine.

1. Introduction

In many agency problems the information structure rather than being exogenous (as it is widely assumed) is affected by the interaction of the players and the incentives they face and, therefore, it is endogenous. The principal and/or the agent may have the opportunity of acquiring information about costs, revenues, etc. *before* entering in a contractual relationship.

We consider a procurement model where the principal (she) hires the agent (he) and, at the time the contract is offered, none of them knows with certainty the production costs. The agent has then the opportunity of costly acquiring information about his own costs. We assume that the acquired information can be credibly communicated -it is hard-, but can not be observed by a court of law -it is non-verifiable- and, therefore, cannot be contracted upon.

The acquired information can be credibly communicated when it is the report of a reputed consultant firm, or the results of some lab tests, or a statistic report, etc. In these circumstances the agent's acquired information can be credibly communicated to the principal. The assumption of non-verifiability makes sense when only the other party (but not the court) can properly assess the validity of the information, or when it is too costly to generate evidence that meets the standards required by a court of law.

To illustrate the assumption consider an example where the principal wants the agent to clean and level a tract of land in a remote place (assume she has been there before). She wants it leveled at a certain height to build on it afterwards. The agent may go and take a look at the land and say "it is really bad, it will be very costly to clean and level it," and the principal has no reason to believe him (the information is soft). Or the agent may go and take some pictures, that the principal would recognize are from his land, and the principal could then assess the validity of the agent's claim.

To generate evidence that can be used in court may require to certify with the local authority that the pictures are really from the principal's land, to have a third party to measure the height at different points, to estimate how much extra soil is needed, etc.

Previous literature in pre-contractual information acquisition (Crémer & Khalil, 1992, Crémer, Khalil & Rochet, 1998a, Crémer, Khalil & Rochet, 1998b, and Kessler, 1998) focuses basically in two dimensions of the problem: the timing of the information acquisition, whether it occurs before or after the contract offer is made, and the information acquisition being productive or strategic (if the agent were to learn for free his production costs before deciding the production level, then the costly acquisition of information before signing the contract is said to be strategic and has no social value). This literature assumes that the acquired information is soft and that parties can commit to avoid renegotiation.

We relax the commitment assumption and analyze the case where the acquired information can be credibly communicated to the other party but it is non-verifiable (moreover, we assume the acquisition of information itself is non-verifiable). We assume the agent can acquire the information after the contract has been offered, the information acquisition is strategic, and the agent can choose to disclose it or not when the parties are about to renegotiate. If the agent decides not to gather information, he will learn his type at no cost when deciding the production level. Principal and agent will be able to renegotiate after the agent learns his type, but the information acquired at this stage is assumed to be soft and therefore can not be credibly communicated at the renegotiation stage.

We may think of this situation as a process where the agent, after incurring some fixed costs or producing a minimal quantity, learns his marginal costs. This information is assumed to be soft and we rule out the possibility of generating hard evidence at this stage.¹

¹All we need to assume is that the cost of generating hard evidence at this stage is larger than the cost of generating it before signing the contract divided by the probability of the agent being high cost type. This will be the case whenever generating hard information or evaluating it takes some time and it is too costly to delay the production process.

In terms of our example, if the principal needs the land clean and even to start building at a certain date it might not be possible to delay the cleaning to take the pictures and hire a third party to estimate the extra soil needed, send the report to the principal and go through the renegotiation process. Alternatively, it could be too costly for the agent to delay the cleaning and leveling process simply because he has all his workers and machinery at the place.

Results

We derive the optimal contract the principal would offer as a function of the cost of acquiring the information. In this model, the agent may choose to gather pre-contractual information basically for two reasons: to learn his type and reject principal's offer if it gives him a negative payoff; and to improve his situation at the renegotiation stage.

We find that for values of the information below a cutoff level, the principal will induce the agent to acquire it. In such cases, renegotiation will take place after the agent shows the information to the principal, and the final production levels will be the efficient ones. The principal will choose an initial contract such that the agent acquires information and obtains zero expected rent. Since production is going to be efficient after the renegotiating with hard information, the principal will appropriate all the expected surplus of the relationship minus the cost of acquiring the information.

For large enough values of the cost of acquiring pre-contractual information, the situation is equivalent to the one in which the agent simply can not acquire pre-contractual information at all. In this case, the principal will offer the agent a contract involving efficient production levels and no expected rent for the agent.

For intermediate values of this cost, the optimal contract will induce a production level below the efficient one for the high cost agent. Depending on the parameters of the model, agent's expected payoff is going to be equal to his reservation utility for any value of the cost of acquiring information, or he can get some positive expected payoff for some intermediate values of the information acquisition cost.

This result contrasts with previous results in the literature, where the agent always finds beneficial to have a lower cost of acquiring pre-contractual information. As will be clear later, this (counterintuitive?) result is directly related to the assumptions that the costly information is hard and the agent decides to acquire it or not only after receiving the contract offer.

Related Literature

Cr mer and Khalil (1992) assume the agent can spend an amount γ to learn his marginal

cost after the contract is offered and before accepting or rejecting it. This information has no productive value since he can learn his type at no cost after accepting the contract and before deciding the production level. The optimal contract will induce no information acquisition for any positive γ : any contract inducing pre-contractual information gathering would be strictly Pareto dominated by one that includes the option for the agent of producing zero and paying to the principal a fraction of γ .² Principal's profit is an increasing function in γ (and for large enough values he is able to extract the total surplus). Therefore, the principal would have an incentive to increase the information acquisition costs and/or to introduce competition between agents.

Cr mer *et al.* (1998a) assume that the information acquisition decision by the agent is prior to the contract offer (the principal can not observe this decision) and this information acquisition is only strategic. For small values of γ the agent always gets the information and the contract offered by the principal is the one derived in Baron & Myerson (1982). For large enough values he never acquires information and the optimal contract involves efficient production levels and zero ex-ante rent for the agent. For intermediate values the agent follows a mixed strategy and the principal designs two different contracts: one for the informed agent and one for the uninformed. Depending on the specific parameter values, the optimal contracts designed for the informed and uninformed more inefficient types may or may not coincide. For the most efficient types the optimal contracts for informed and uninformed agents always differ.

In Cr mer *et al.* (1998b), unlike the previous ones, the pre-contractual information gathering has a productive value since it is assumed that the agent will not learn his type until the end of the game unless he spends γ . It is assumed that the agent can acquire the information about his type after being offered the contract and before accepting or rejecting it. Therefore, the information can help to adjust the production depending on the marginal cost. Optimally, the principal will offer a contract that induces information gathering only when the cost of acquiring is smaller than a critical value. For a small enough cost of

²If the agent spends γ is because he may reject the contract depending on the information.

acquiring the information, the contract offered is the same as if there were no costs at all (the one derived in Baron & Myerson, 1982). For large enough values of γ the contract is ex-ante efficient and gives the agent no rent. For smaller values (but above the critical value) the optimal contract is distorted away from the ex-ante efficient and the agent may have a positive rent.

Kessler (1998) assumes that before a contract is offered the agent can spend some resources to learn his type with a probability that depends on the expenditure level, and the principal can observe this expenditure level but not if the agent learns his type or not. As in Crémer *et al.* (1998b), it is assumed that if she doesn't learn her type here she is not going to learn it until payoffs are realized, therefore the information has a productive value. The main finding is that, in a two-type setting, no matter how cheap is for the agent to learn her type, she will never spend enough money to learn her type for sure. That is, she chooses an expenditure level such that she is going to be ignorant about his type with a positive probability. The asymmetry of information about being or not informed gives the agent a positive rent.

Outline

In the next section the notation and the model are presented. In Section 3 the extreme cases when the cost of acquiring information is zero and infinite are presented. The solution for the general case is presented in Section 4 and the robustness of the model is discussed in Section 5. Section 6 discusses the main results and possible extensions. Proofs missing from the text are in Appendix A.

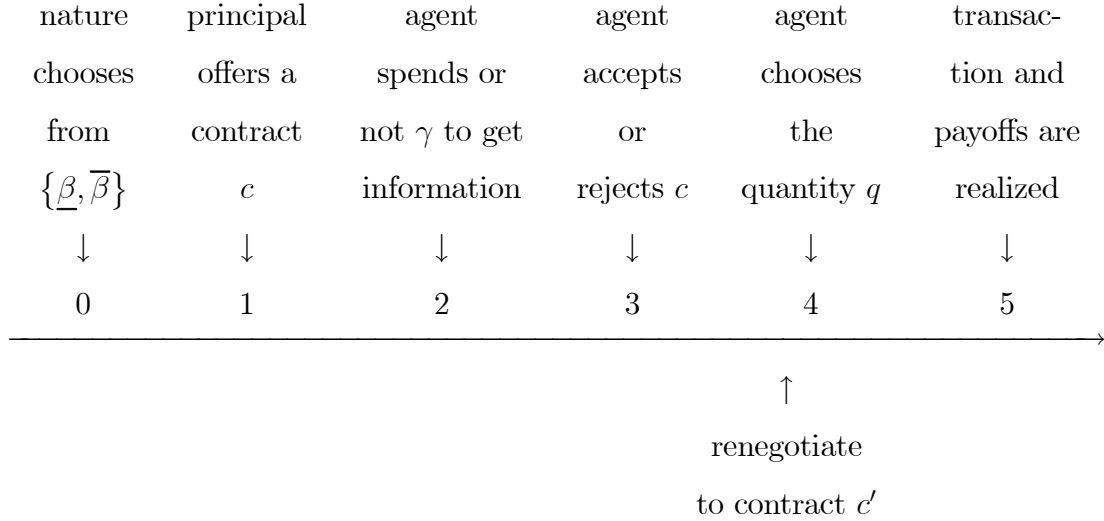
2. The Model

The principal wants the agent to produce a certain good that she values $V(q)$, where q is a contractible variable (quantity or quality) and $V(\cdot)$ is an increasing and strictly concave function, with $V(0) = 0$, $V'(0) = \infty$, $V'(\infty) = 0$.

The production cost is either $C(q) = \underline{\beta}q$ or $C(q) = \bar{\beta}q$, ($\bar{\beta} > \underline{\beta} > 0$) with probabilities $\pi \in (0, 1)$ and $(1 - \pi)$ respectively. We denote by \underline{q}^* and \bar{q}^* the efficient production levels

for each type: $V'(\underline{q}^*) = \underline{\beta}$ and $V'(\bar{q}^*) = \bar{\beta}$.

The timing of the game is as follows:



As mentioned, it is assumed that there is no asymmetric information at the beginning of the game (neither the agent nor the principal knows β), but there is an asymmetry in the possibility of acquiring information: after the principal offered a contract, the agent can learn his type at a cost $\gamma \geq 0$ before accepting or rejecting the contract; while the principal can not acquire this information.³

Once the contract has been signed, we assume parties can renegotiate the terms of the original contract. For simplicity we assume the agent makes a take it or leave it offer at this stage. If the agreed contract is a menu of quantity and transfers, the agent chooses the production level at 4 and payoffs are realized at 5: the principal gets $V(q) - t$ (where t is the payment the principal makes for the q units), and the agent gets either $t - C(q)$ or $t - C(q) - \gamma$ if he acquired the hard information.

A contract

Given a certain contract, the agent will choose to acquire information or not depending on the expected payoffs. To find the optimal contract the principal would offer, our approach is to solve two different problems. First we assume the principal, independently of the cost

³This assumption can be relaxed. It is enough to assume that the principal's cost of acquiring pre-contractual information is at least as large as the agent's cost (γ). See Section 5.

γ , wants to induce the agent to acquire the information. We then solve the case where the principal induces no information acquisition (for any γ). By comparing the principal's expected profits in both cases we determine which contract is optimal for each possible value of γ .

We assume that the principal can not observe if the agent acquires information or not, unless the agent acquires the hard information and decides to share it with the principal. In any case, a court is unable to verify neither the information nor the fact that the agent acquired it.

To solve the problem when the principal induces no information acquisition we can restrict attention to contracts specifying two pairs (t, q) .⁴ That is, a feasible contract is $c_0 = (t_0, q_0, t^0, q^0) \in \tilde{C} = \mathfrak{R} \times \mathfrak{R}_+ \times \mathfrak{R} \times \mathfrak{R}_+$. Without loss of generality, we restrict attention to contracts satisfying:

$$\begin{aligned} t_0 - q_0 \underline{\beta} &\geq t^0 - q^0 \underline{\beta} \\ t^0 - q^0 \bar{\beta} &\geq t_0 - \bar{\beta} q_0. \end{aligned}$$

Let $C \subset \tilde{C}$ denote the subset of contracts satisfying the above conditions.⁵

Renegotiation

Renegotiation can take place in two different scenarios depending on whether the agent spent γ or not. In the first case he would have hard information about his own costs while in the second case his information is soft (cannot be credibly communicated).

We denote by $c_h(c_0) = (t_h(c_0), q_h(c_0))$ and $c^h(c_0) = (t^h(c_0), q^h(c_0))$ the renegotiation

⁴As will become clear in Section 4.2, when the principal induces information acquisition we can also restrict attention to contracts that specify only two pairs (t, q) .

⁵The first pair (t, q) is weakly preferred by the low cost type and the second one by the high cost type. If that is not the case, the contract could be reinterpreted: if both types prefer the same pair (t, q) -i.e. (t_0, q_0) - then the *real* contract is (t_0, q_0, t_0, q_0) ; if type $\bar{\beta}$ prefers (t_0, q_0) and type $\underline{\beta}$ prefers (t^0, q^0) then the reinterpreted contract is (t^0, q^0, t_0, q_0) .

To avoid any ambiguity, attention is restricted to contracts that satisfy the usual incentive compatibility constraints.

offer the agent would make when the original contract is c_0 , he has hard information, and his type is $\underline{\beta}$ and $\bar{\beta}$ respectively.

When he has only soft information, his renegotiation offers are denoted by

$$\begin{aligned} c_{s,\underline{\beta}}(c_0) &= \left(t_{s,\underline{\beta}}(c_0), q_{s,\underline{\beta}}(c_0), t^{s,\underline{\beta}}(c_0), q^{s,\underline{\beta}}(c_0) \right) \\ c_{s,\bar{\beta}}(c_0) &= \left(t_{s,\bar{\beta}}(c_0), q_{s,\bar{\beta}}(c_0), t^{s,\bar{\beta}}(c_0), q^{s,\bar{\beta}}(c_0) \right) \end{aligned}$$

when his type is $\underline{\beta}$ and $\bar{\beta}$ respectively.

3. Two Extreme Cases, $\gamma = \infty$ and $\gamma = 0$

In this section we illustrate the model by informally discussing the two extreme cases in which the cost of acquiring information (γ) is infinite and zero. The principal chooses an initial contract $c_0 = (t_0, q_0, t^0, q^0)$ that will be later renegotiated.

Consider first the situation when $\gamma = \infty$. In this case the agent simply can not acquire pre-contractual information, so principal's problem is:

$$\max_{c_0} \pi \left[V \left(q_{s,\underline{\beta}}(c_0) \right) - t_{s,\underline{\beta}}(c_0) \right] + (1 - \pi) \left[V \left(q^{s,\bar{\beta}}(c_0) \right) - t^{s,\bar{\beta}}(c_0) \right].$$

There is a continuum of contracts involving efficient quantities ($q_0 = \underline{q}^*$ and $q^0 = \bar{q}^*$), and payments t_0 and t^0 such that the incentive compatibility constraints are satisfied, and the agent gets his reservation utility.⁶

It is straightforward to show that these contracts are renegotiation proof (they specify efficient quantities). Since the agent gets no rent and the quantities specified are efficient, it is immediate to conclude that any of these contracts is a solution to the principal's problem. Figure C.1 illustrates the set of optimal contracts in this case.

Consider now the case of $\gamma = 0$ and let $c_0 = (\underline{q}^*\underline{\beta}, \underline{q}^*, q^0\underline{\beta}, q^0)$, where q^0 satisfies $V(q^0) - q^0\underline{\beta} = V(\bar{q}^*) - \bar{q}^*\bar{\beta}$ (illustrated in Figure C.2). If the agent chooses not to acquire the

⁶There is a continuum of equilibria from (t^-, \tilde{t}^-) to (t^+, \tilde{t}^+) where (t^-, \tilde{t}^-) satisfies $t^- - \underline{q}^*\underline{\beta} = \tilde{t}^- - \bar{q}^*\underline{\beta}$ and (t^+, \tilde{t}^+) satisfies $\tilde{t}^+ - \bar{q}^*\bar{\beta} = t^+ - \underline{q}^*\bar{\beta}$ and both pairs gives zero ex-ante rent to the agent.

information, then he would get an expected negative payoff since $c_{s,\underline{\beta}}(c_0) = c_{s,\overline{\beta}}(c_0) = c_0$ (see Section 4.1.2). He would therefore reject the offered contract.

Instead, if he acquires the information and shares it at the renegotiation stage, his expected payoff becomes 0 since $c_{h,\underline{\beta}}(c_0) = (\underline{q}^*\underline{\beta}, \underline{q}^*)$ and $c_{h,\overline{\beta}}(c_0) = (\overline{q}^*\overline{\beta}, \overline{q}^*)$ (see section 4.2.1 and Figure C.2). Note also that the low cost type has no incentives to pretend he did not acquire information. By showing he is low type his payoff is determined by $c_{h,\underline{\beta}}(c_0)$, and he gets $\underline{q}^*\underline{\beta} - \underline{q}^*\underline{\beta} = 0$; while by pretending he is uninformed his payoff would be determined by $c_{s,\underline{\beta}}(c_0)$ and his payoff would also be zero (see Section 4.1.2)

Since the renegotiation of c_0 involves efficient quantities and the agent gets no rent, then it has to be that c_0 is a solution to the principal's problem.

Note that in both extreme cases the production level is always efficient and the principal is able to appropriate all the ex-ante surplus of the relationship, leaving the agent with no ex-ante rent. These results will not hold in general for intermediate values of γ .

4. The General Case, $\gamma \in (0, \infty)$

4.1. Inducing no information acquisition

In this subsection we assume the principal will choose the initial contract to maximize his utility, but constrained to induce the agent not to acquire information. The principal is going to choose a contract $c_0 = \{t_0, q_0, q^0, q^0\}$ that will be renegotiated to $c_{s,\underline{\beta}}(c_0)$ or $c_{s,\overline{\beta}}(c_0)$.⁷

We will solve principal's maximization problem when she wants to induce the agent not

⁷To simplify notation, let (t_s, q_s, t^s, q^s) be the relevant renegotiated contract: when the equilibrium of the renegotiation game is pooling, then

$$(t_s, q_s, t^s, q^s) = (t_{s,\beta}(c_0), q_{s,\beta}(c_0), t^{s,\beta}(c_0), q^{s,\beta}(c_0))$$

for both possible types. If the equilibrium is a separating one, then

$$(t_s, q_s) = (t_{s,\underline{\beta}}(c_0), q_{s,\underline{\beta}}(c_0)) \quad \text{and} \quad (t^s, q^s) = (t^{s,\overline{\beta}}(c_0), q^{s,\overline{\beta}}(c_0)).$$

to acquire information in three steps: first, we will assume that the original contract, when it induces the agent not to acquire information, cannot be renegotiated (subsection 4.1.1); second, we will solve the renegotiation problem when the agent is uninformed (subsection 4.1.2); and finally, we will show that the optimal contract found in the first step is renegotiation proof.

4.1.1. The optimal allocation that induces the agent not to acquire information

To characterize the optimal allocation (for the principal) such that the agent does not acquire information, we solve principal's problem assuming that, if the agent does not acquire pre-contractual information, there is no possibility of renegotiation. We will see that this optimal contract is renegotiation proof (satisfies the conditions of Corollary 1) and, therefore, it is the solution to principal's problem when inducing no pre-contractual information gathering.

The principal chooses $c_0 = \{t_0, q_0, t^0, q^0\}$ to solve

$$\max_{c_0} \pi(V(q_0) - t_0) + (1 - \pi)(V(q^0) - t^0) \quad (P_U)$$

subject to the usual incentive compatibility and participation constraints

$$t_0 - q_0\underline{\beta} \geq t^0 - q^0\underline{\beta} \quad (\text{IC1})$$

$$t^0 - q^0\overline{\beta} \geq t_0 - q_0\overline{\beta} \quad (\text{IC2})$$

$$\pi(t_0 - q_0\underline{\beta}) + (1 - \pi)(t^0 - q^0\overline{\beta}) \geq 0; \quad (\text{IR})$$

and an incentive compatibility constraint between acquiring vs. not acquiring information (IC_U). To derive this constraint notice that if the contract is renegotiated after the agent acquired hard information an efficient outcome will be achieved (see Subsection 4.2.1). Since the agent is assumed to have all the bargaining power at the renegotiation stage, his payoff when he acquires pre-contractual information is going to be

$$\begin{aligned} & \max\{V(\underline{q}^*) - \underline{q}^*\underline{\beta} - [V(q_0) - t_0], 0\} \text{ if } \beta = \underline{\beta}, \text{ and} \\ & \max\{V(\overline{q}^*) - \overline{q}^*\overline{\beta} - [V(q^0) - t^0], 0\} \text{ if } \beta = \overline{\beta}. \end{aligned}$$

It is trivial to show that (IC1), (IC2) and (IR) imply $V(\underline{q}^*) - \underline{q}^*\underline{\beta} - [V(q_0) - t_0] \geq 0$, therefore, the constraint to prevent information acquisition can be written as:

$$\begin{aligned} \pi(t_0 - q_0\underline{\beta}) + (1 - \pi)(t^0 - q^0\bar{\beta}) &\geq \pi[V(\underline{q}^*) - \underline{q}^*\underline{\beta} - (V(q_0) - t_0)] + \\ &(1 - \pi) \max\{V(\bar{q}^*) - \bar{q}^*\bar{\beta} - (V(q^0) - t^0), 0\} - \gamma. \quad (\text{IC}_U) \end{aligned}$$

Which can be rewritten as:

$$\gamma \geq \pi[V(\underline{q}^*) - V(q_0) - \underline{\beta}(\underline{q}^* - q_0)] + (1 - \pi)[V(\bar{q}^*) - V(q^0) - \bar{\beta}(\bar{q}^* - q^0)] \quad (\text{IC}_{U1})$$

$$\gamma \geq \pi[V(\underline{q}^*) - V(q_0) - \underline{\beta}(\underline{q}^* - q_0)] - (1 - \pi)(t^0 - q^0\bar{\beta}). \quad (\text{IC}_{U2})$$

Before presenting the solution to this problem (Propositions (1) and (2)) it is useful to define some critical values:

$$\underline{q}^0 \equiv V'^{-1}\left(\frac{\bar{\beta} - \pi\underline{\beta}}{1 - \pi}\right),$$

$$\tilde{q}^0 : = \left\{ q : q < \bar{q}^* \text{ and } V(q) - q\tilde{\beta} = V(\bar{q}^*) - \bar{q}^*\bar{\beta} \right\}, \text{ where } \tilde{\beta} = (1 - \pi)\bar{\beta} + \pi\underline{\beta},$$

$$\gamma_B \equiv (1 - \pi)[V(\bar{q}^*) - V(\underline{q}^0) - \bar{\beta}(\bar{q}^* - \underline{q}^0)],$$

$$\gamma_A \equiv (1 - \pi)\pi\underline{q}^0(\bar{\beta} - \underline{\beta}),$$

$$\gamma_L \equiv (1 - \pi)\pi\bar{q}^*(\bar{\beta} - \underline{\beta}),$$

$$\gamma_C \equiv \tilde{q}^0(\bar{\beta} - \underline{\beta})\pi(1 - \pi).$$

Proposition 1. *If $\underline{q}^0 > \tilde{q}^0$ (Case 1),⁸ the solution to (P_U) is characterized by:*

$$\begin{aligned} t_0 &= \underline{q}^*\underline{\beta} + q^0(\bar{\beta} - \underline{\beta}) - \frac{\min\{\gamma, \gamma_L\}}{1 - \pi}, \\ q_0 &= \underline{q}^* \\ t^0 &= q^0\bar{\beta} - \frac{\min\{\gamma, \gamma_L\}}{1 - \pi}, \\ q^0 &= \begin{cases} \bar{q}^* & \text{if } \gamma_L \leq \gamma \\ \frac{\gamma}{(1 - \pi)\pi(\bar{\beta} - \underline{\beta})} & \text{if } \gamma_A < \gamma < \gamma_L \\ \underline{q}^0 & \text{if } \gamma_B \leq \gamma \leq \gamma_A \\ \{q : q < \bar{q}^* \wedge V(q) - q\bar{\beta} = V(\bar{q}^*) - \bar{q}^*\bar{\beta} - \frac{\gamma}{1 - \pi}\} & \text{if } \gamma < \gamma_B, \end{cases} \end{aligned}$$

⁸Note that $\underline{q}^0 > \tilde{q}^0$ is equivalent to assume $\gamma_A < \gamma_B$.

Proof: see appendix A.

Proposition 2. *If $q^0 \leq \bar{q}^0$ (Case 2), the solution to (P_U) is characterized by:*

$$\begin{aligned}
t_0 &= \underline{q}^* \underline{\beta} + q^0 (\bar{\beta} - \underline{\beta}) - \frac{\min\{\gamma, \gamma_L\}}{1 - \pi}, \\
q_0 &= \underline{q}^* \\
t^0 &= q^0 \bar{\beta} - \frac{\min\{\gamma, \gamma_L\}}{1 - \pi}, \\
q^0 &= \begin{cases} \bar{q}^* & \text{if } \gamma_L \leq \gamma \\ \frac{\gamma}{(1-\pi)\pi(\bar{\beta}-\underline{\beta})} & \text{if } \gamma_C < \gamma < \gamma_L \\ \{q : q < \bar{q}^* \wedge V(q) - q\bar{\beta} = V(\bar{q}^*) - \bar{q}^*\bar{\beta} - \frac{\gamma}{1-\pi}\} & \text{if } \gamma \leq \gamma_C \end{cases}
\end{aligned}$$

*Proof: see appendix A. Figures C.3 and C.4 summarize the results for q^0 .*⁹

4.1.2. The renegotiation stage when the agent cannot credibly show his type

The initial contract $c_0 \in C$ the principal offers defines a renegotiation game $G(c_0)$ where the agent sends a message (renegotiation offer) $c_s = \{t_s, q_s, t^s, q^s\} \in C$, then the principal either accepts or rejects the offer, and finally the agent decides the quantity to produce and payoffs are realized according to c_0 if the principal rejected the offer and to c_s if she accepted.

As usual in signalling games, many outcomes can be supported in equilibrium if no restrictions are imposed on beliefs off the equilibrium path. In this case, by imposing the *Intuitive Criterion* proposed by Cho and Kreps (1987), it turns out that there is a unique outcome that can be supported in equilibrium.

Proposition 3. *For any initial contract $c_0 \in C$ that satisfies $V(q_0) - t_0 \geq V(q^0) - t^0$,¹⁰ the unique *Intuitive Criterion* equilibrium outcome of $G(c_0)$ satisfies: the low cost type $\underline{\beta}$ produces the efficient quantity \underline{q}^* and receives a transfer $t_s \equiv t_0 + V(\underline{q}^*) - V(q_0)$; and the high*

⁹To draw Figures C.3, C.4, C.6, C.7, C.10, and C.11 we assumed that $V(q) = 100\sqrt{q}$, $\bar{\beta} = 50$, $\underline{\beta} = 25$. π is 0.3 in Case 1 and 0.7 in Case 2.

¹⁰Restricting attention to contracts $c_0 \in C$ and satisfy $V(q_0) - t_0 \geq V(q^0) - t^0$ is with no loss of generality (note that the contracts proposed in Propositions 1 and 2 satisfy these restrictions).

cost type $\bar{\beta}$ receives $t^s(c^0) \equiv t^0 + V(q^s(c_0)) - V(q^0)$ and produces $q^s(c_0) = \min\{\bar{q}^*, q^*(c_0)\}$, where $q^*(c_0) \equiv \{q : t_s - \underline{\beta}(q^* - q) = V(q) - [V(q^0) - t^0]\}$.¹¹

Proof: see Appendix A. Figure C.5 illustrates the definitions of $t_*(c_0)$, $t^{**}(c_0)$, and $q^{**}(c_0)$ when $\bar{q}^* > q^*(c_0)$.

Corollary 1. *If the initial contract $c_0 = \{t_0, q_0, t^0, q^0\} \in C$ satisfies $V(q_0) - t_0 \geq V(q^0) - t^0$, $q_0 = \underline{q}^*$, $q^0 \leq \bar{q}^*$, and $t^0 = t_0 - \underline{\beta}(q^* - q^0)$; then the unique equilibrium outcome satisfying the intuitive criterion of the renegotiation game $G(c_0)$ is $(t_s, q_s) = (t_0, \underline{q}^0)$ if $\beta = \underline{\beta}$ and $(t^s, q^s) = (t^0, q^0)$ if $\beta = \bar{\beta}$. The initial contract c_0 is renegotiation proof.*

Proof: Note that if $q_0 = \underline{q}^*$ then, by definition, $t_0 = t_s(c_0)$. When $t^0 = t_0 - \underline{\beta}(q^* - q^0)$, then $q^*(c_0)$ has to satisfy $V(q^*(c_0)) = V(q^0)$. Then, by definition, $t^s = t^0$. ■

Combining Propositions 1 and 2, the optimal for the principal when she wants to deter information acquisition is characterized by:¹²

$$\begin{aligned} t_0 &= \underline{q}^* \underline{\beta} + q^0(\bar{\beta} - \underline{\beta}) - \frac{\min\{\gamma, \gamma_L\}}{1 - \pi}, \\ q_0 &= \underline{q}^*, \\ t^0 &= q^0 \bar{\beta} - \frac{\min\{\gamma, \gamma_L\}}{1 - \pi}, \\ q^0 &= \begin{cases} \bar{q}^*, & \text{if } \gamma_L < \gamma \\ \frac{\gamma}{(\bar{\beta} - \underline{\beta})\pi(1 - \pi)}, & \text{if } \max\{\gamma_A, \gamma_C\} \leq \gamma \leq \gamma_L \\ q^0 = \underline{q}^0, & \text{if } \gamma_B \leq \gamma \leq \gamma_A \\ \{q : q < \bar{q}^* \wedge V(q) - \bar{\beta}q = V(\bar{q}^*) - \bar{q}^* \bar{\beta} - \frac{\gamma}{1 - \pi}\}, & \text{if } \gamma \leq \min\{\gamma_B, \gamma_C\}. \end{cases} \end{aligned}$$

Remark 1. *The above contract (t_0, q_0, t^0, q^0) satisfies the conditions of Corollary 1 and, therefore, is renegotiation proof.*

Then, the above equations characterize the solution to Principal's problem when he wants the agent not to acquire information.

¹¹ Figure C.5 illustrates the definitions of $t_*(c_0)$, $t^{**}(c_0)$, and $q^{**}(c_0)$ when $\bar{q}^* > q^*(c_0)$.

¹² Note that $\gamma_C > \gamma_A \Leftrightarrow \gamma_C < \gamma_B$.

As intuition suggests, for large values of γ the principal finds optimal to induce efficient levels of production and is able to extract all the surplus of the relationship (the problem is identical to one with an uninformed agent).

For values of γ between γ_L and γ_A (γ_C in Case 2) the principal finds optimal to prevent the information acquisition by distorting the quantity the bad type produces, this gives the agent zero ex-ante rent.

For smaller values of γ (between γ_A and γ_B in Case 1) it is too costly for the principal to keep reducing the quantity the high cost type is supposed to produce. The agent then gets some rent.

For even lower values of γ (smaller than γ_B or γ_C), if the principal were to continue reducing q^0 in Case 2 or maintaining \underline{q} in Case 1, the agent would find that q^0 is too small and he would choose to acquire information and then renegotiate. To prevent this, the principal needs to increase q^0 . In the limit, when $\gamma = 0$, the agent would always spend γ and renegotiate if $q^0 < \bar{q}^*$, therefore the principal has to choose $q^0 = \bar{q}^*$.

As expected, principal's profit when he induces the agent not to acquire information is increasing in γ . The larger is the cost of obtaining information, the easier it is to deter its acquisition. Figures C.6 and C.7 illustrate her expected profits as a function of γ .

4.2. Inducing information acquisition

4.2.1. The renegotiation stage when the agent can credibly show his type

When the agent spends γ to acquire the information and he accepts the contract, he has the opportunity to credibly communicate his type to the principal. Therefore, the renegotiation offer does not have to satisfy the usual incentive compatibility constraints.

If both types of agent choose to share the acquired information, they would solve

$$\begin{aligned} \underset{t_h, q_h}{Max} (t_h - q_h \underline{\beta}) \quad \text{s. to } V(q_h) - t_h &\geq V(q_0) - t_0 \\ \underset{t^h, q^h}{Max} (t^h - q^h \bar{\beta}) \quad \text{s. to } V(q^h) - t^h &\geq V(q^0) - t^0. \end{aligned}$$

It is immediate to show that the solution to these problems are

$$\begin{aligned} t_h &= t_0 + V(\underline{q}^*) - V(q^0), & q_h &= \underline{q}^* \\ t^h &= t^0 + V(\bar{q}^*) - V(q^0), & q^h &= \bar{q}^*. \end{aligned}$$

Figure C.8 illustrates the results for an initial contract $c_0 = \{t_0, q_0, t^0, q^0\}$.¹³

4.2.2. The optimal contract

Note that principal's profit when she induces information acquisition can be at most

$$\pi[V(\underline{q}^*) - \underline{q}^*\underline{\beta}] + (1 - \pi)[V(\bar{q}^*) - \bar{q}^*\bar{\beta}] - \gamma;$$

otherwise the agent's incentive rationality constraint

$$\pi\{V(\underline{q}^*) - \underline{q}^*\underline{\beta} - [V(q_0) - t_0]\} + (1 - \pi)\{V(\bar{q}^*) - \bar{q}^*\bar{\beta} - [V(q^0) - t^0]\} \geq \gamma \quad (\text{IR})$$

would be violated.

On the other hand, for the agent to acquire the information, it has to be that his expected payoff from acquiring information is at least what he would get if he does not acquire the information and accepts the contract. That is,

$$\begin{aligned} \pi\{V(\underline{q}^*) - \underline{q}^*\underline{\beta} - [V(q_0) - t_0]\} + (1 - \pi)\{V(\bar{q}^*) - \bar{q}^*\bar{\beta} - [V(q^0) - t^0]\} - \gamma \geq \\ \pi(t_s(c_0) - \underline{\beta}q_s(c_0)) + (1 - \pi)(t^s(c_0) - \bar{\beta}q^s(c_0)) \quad (\text{IC}_I) \end{aligned}$$

has to be satisfied.

¹³Note that neither an agent type $\underline{\beta}$ nor a type $\bar{\beta}$ can do better by pretending to be uninformed:

$$t_h - q_h\underline{\beta} = V(\underline{q}^*) - \underline{q}^*\underline{\beta} - V(q_0) + t_0 \geq t_0 + V(\underline{q}^*) - V(q_0) - \underline{q}^*\underline{\beta} = t_s - q_s\underline{\beta}$$

and

$$\begin{aligned} t^h - q^h\bar{\beta} &= V(\bar{q}^*) - \bar{q}^*\bar{\beta} - V(q^0) - t^0 \geq t^0 + V(q^{**}(c_0)) - V(q^0) - \bar{q}^*q^{**}(c_0) = t^s - q^s\bar{\beta} \\ &\Leftrightarrow V(\bar{q}^*) - \bar{q}^*\bar{\beta} \geq V(q^{**}(c_0)) - \bar{q}^*q^{**}(c_0) \end{aligned}$$

since \bar{q}^* is the efficient quantity when $\beta = \bar{\beta}$.

Moreover, if both types of agents are expected to accept the contract after acquiring the information, then

$$V(\underline{q}^*) - \underline{q}^* \underline{\beta} - [V(q_0) - t_0] \geq 0 \quad (\text{IR}_{\underline{\beta}})$$

$$V(\bar{q}^*) - \bar{q}^* \bar{\beta} - [V(q^0) - t^0] \geq 0 \quad (\text{IR}_{\bar{\beta}})$$

should be satisfied.¹⁴

Proposition 4 below shows that there is a contract c_0 such that the principal extracts all the surplus (minus γ), and the above constraints (IR , IC_I , $IR_{\underline{\beta}}$ and $IR_{\bar{\beta}}$), plus the usual incentive compatibility ones, are satisfied. Such a contract is therefore optimal for the principal given that she wants to induce information acquisition.

Proposition 4. *For any values of $\gamma \leq (1 - \pi)[V(\bar{q}^*) - \bar{q}^* \bar{\beta}]$,¹⁵ the contract*

$$\begin{aligned} t_0 &= q_0 \underline{\beta} \\ t^0 &= q^0 \underline{\beta} \\ q_0 &= \underline{q}^* \\ q^0 &= \{q : V(\bar{q}^*) - V(q) + \underline{\beta}q - \bar{q}^* \bar{\beta} = \gamma / (1 - \pi)\} \end{aligned}$$

satisfies (IR), (IC_I), ($IR_{\underline{\beta}}$), ($IR_{\bar{\beta}}$), ($IC1$), and ($IC2$) and principal's payoff is

$$\pi[V(\underline{q}^*) - \underline{q}^* \underline{\beta}] + (1 - \pi)[V(\bar{q}^*) - \bar{q}^* \bar{\beta}] - \gamma.$$

Proof: see appendix A.

Figure C.9 illustrates the optimal contracts offered for the two extreme cases when γ is zero and when it is equal to $(1 - \pi)[V(\bar{q}^*) - \bar{q}^* \bar{\beta}]$.

¹⁴The above constraints are the relevant ones since for both types of agent it is optimal, given a contract c_0 , to share the acquired information. See footnote 13.

¹⁵It is enough for us to characterize the solution to this problem for values of $\gamma \leq (1 - \pi)[V(\bar{q}^*) - \bar{q}^* \bar{\beta}]$. Proposition 5 in the next section shows that for values of γ larger than $(1 - \pi)[V(\bar{q}^*) - \bar{q}^* \bar{\beta}]$ the principal will always prefer to induce the agent not to get information.

4.3. Results

In the next Proposition we put the results of previous sections together to find the critical value of the cost of acquiring information (γ) that determines whether the principal is going to induce the agent to acquire information or not.

Proposition 5. *Define*

$$\gamma^* \equiv \frac{1 - \pi}{2 - \pi} \{ (1 - \pi) [V(\bar{q}^*) - \bar{q}^* \bar{\beta} - V(\underline{q}^0) + \underline{q}^0 \bar{\beta}] + \pi \underline{q}^0 (\bar{\beta} - \underline{\beta}) \}.$$

Case 1 - $\underline{q}^0 > \tilde{q}^0$: If $\gamma \leq \gamma^$ then the optimal contract is the one characterized in Proposition 4; and if $\gamma > \gamma^*$, then it is the one characterized by Proposition 1.*

Case 2 - $\underline{q}^0 \leq \tilde{q}^0$: If $\gamma \leq \gamma_C = \tilde{q}^0 (\bar{\beta} - \underline{\beta}) \pi (1 - \pi)$, then the optimal contract is the one characterized in Proposition 4; if $\gamma > \gamma_C$, then it is the one characterized by Proposition 2.

Proof: see appendix A. Figures C.10 and C.11 illustrates the results for both cases.

The principal appropriates all the surplus minus γ when the agent acquires information. Note also that the production is distorted away from the optimal level only when the agent does not acquire information and he is bad type.

It is easy to check that in Case 2 agent's expected payoff is zero for any value of the cost of acquiring of information (γ). On the other hand, in Case 1 agent's expected payoff is positive and decreasing in γ in the interval (γ^*, γ_A) : when $\gamma \in [\gamma_A, \gamma_L)$ it is optimal for the principal to induce no pre-contractual information gathering by distorting the quantity the high cost type will produce and giving the agent zero ex-ante rent. When $\gamma < \gamma_A$ it is too costly for the principal to deter information acquisition by further decreasing the quantity the high cost type is going to produce, and she finds optimal to increase the transfer to the high cost type (as γ decreases) while maintaining the quantity he produces (\underline{q}^0). This will continue up to the point γ^* , where the principal finds that is better for her to induce the agent to acquire pre-contractual information, inducing (after the renegotiation) efficient production levels and giving no ex-ante rent to the agent.

5. Robustness of the Model

The robustness of the model to certain assumptions is discussed in this section. Several assumptions were made to simplify the analysis, but can be somewhat relaxed.

* First of all, we have assumed that the principal can not learn the agent's type at any cost. This is a maintained assumption in the papers discussed in the Introduction. It may be natural for many problems, but certainly not for those in which the uncertainty is related to the project itself rather than to the agent (i.e., the example of cleaning a land mentioned in the Introduction). For our results to hold (in terms of payoffs and production levels as a function of (γ)), it is enough to assume that the principal can learn the type, either before or after offering the contract, at a cost (δ) greater than or equal to the agent's cost (γ) .

The proof is trivial: simply note that principal's expected payoff is greater than or equal to the total expected surplus minus the cost γ , for any value of γ . Therefore, the principal would never be better off spending $\delta \geq \gamma$.

* We also assumed that both the pre-contractual information and the fact of acquiring it were observable but non-verifiable. If we maintain the assumption that the acquisition of information is non-verifiable, we could assume that the acquired information, should the agent disclose it, is verifiable. In this case the results of the paper in terms of payoffs and production levels would still hold.

For values of γ below the critical value (γ^* or γ_C), the contract offered would involve efficient production levels and the principal getting the total surplus of the relationship minus the cost γ .

In those cases where the principal induces the agent not to get information (γ above the critical value), the only difference that this assumption could make is by easing the constraint 4.1, but this constraint is never binding when γ is above the critical value.

Moreover, if we maintain the assumption that the contract can be renegotiated, the constraint 4.1 would not change at all: the constraint 4.1 could be eased only if the principal is able to threat the agent about bringing verifiable information. However, if there is no commitment not to renegotiate, after accepting the contract the agent could approach the

principal and make a renegotiation offer contingent on showing the appropriate information.

* An assumption of the paper is that the principal and the agent cannot commit to avoid renegotiation. If they were able to commit and we restrict attention to contracts of the form $c_0 = \{t_0, q_0, t^0, q^0\}$, the results would certainly be different. In this case, the only purpose for the agent of spending γ would be to eventually reject those contracts that give him a negative payoff.¹⁶ However, if we do not allow for renegotiation and we maintain the assumption of the pre-contractual information being hard, there is no justification to restrict attention to contracts like $c_0 = \{t_0, q_0, t^0, q^0\}$.

More complex contracts (like message contingent contracts) can induce efficient outcomes. In the context of this paper, the principal may use this contracts to induce the agent to acquire hard information and produce the efficient quantities (see Appendix B for an example of such a contract).

When would the principal use this message contingent contracts? Since production would be efficient (and the agent will be left with no ex-ante rent), principal's payoff would be the total expected surplus minus the cost of acquiring the pre-contractual information. Exactly what she was able to get when renegotiation was allowed and she induced the agent to acquire information. On the other hand, if the principal wants to induce the agent not to acquire information, his problem is the one we solved in Section 4.1.1. Therefore, the principal would use a message contingent contract only if the cost of acquiring the information is below the critical value we found in Section 4.3 (γ^* in Case 1 or γ_C in Case 2).

* We assumed that when the agent does not acquire pre-contractual information and accepts the contract, he learns his type at no cost but he can not generate hard information at this stage. (On the example mentioned in the Introduction, to delay the cleaning process once started to generate hard information would be prohibitively costly). If we want to assume that at this stage the agent can still generate hard information at a cost γ' , our results would hold as long as $\gamma' \geq \gamma/(1 - \pi)$ holds.

* Finally, we assumed that the agent makes the renegotiation offer *after* accepting the

¹⁶In such a case the solution will coincide with the one in Cremer and Khalil (1992).

contract. In the case that he does not get pre-contractual information this is a natural assumption to do, since he learns his type only after accepting the contract.

However, when he does get pre-contractual information he could, in principle, make a renegotiation offer before accepting the contract. The underlying assumption in the model is that the principal has the bargaining power before the contract is signed and the agent has it once he has accepted the contract. In this situation, the agent would never choose to reveal the information before accepting the contract.¹⁷

How do we justify this assumption that the bargaining power switches from the principal to the agent? First of all, note that the principal is always willing to give the agent all the bargaining power at the renegotiation stage: the optimal contract that induces no information acquisition (derived in Section 4.1.1) coincides with the one the principal would offer if there were no possibility of renegotiation (that is, renegotiation does not hurt the principal when inducing no pre-contractual information acquisition); and when the principal induces the agent to get pre-contractual information and the agent has all the bargaining power, principal's profit is the maximum possible (total surplus minus γ).

Therefore, it would be enough to find a way in which the principal can commit to give the agent the bargaining power. Following Aghion, Dewatripont & Rey (1994), we could have assumed that the renegotiation game is an infinite-bargaining process with alternating offers (the agent makes the first one) where each party can enforce the initial contract at any period after receiving the offer from the other party, and the principal has to pay a large enough fine to the agent if there is no agreement after two rounds of bargaining.

Under these assumptions, if the agent acquired pre-contractual information and he shows this information to the principal, the unique outcome of the bargaining game is ex-post efficient and gives the principal her reservation utility (given the initial contract). This outcome is exactly the same we obtained by assuming the agent has all the bargaining power at the renegotiation stage.

¹⁷The principal could reject his counteroffer and then make him a new offer for the efficient quantity and the exact cost of it ($(\underline{q}^*, \underline{q}^*)$ if $\beta = \underline{\beta}$ and (\bar{q}^*, \bar{q}^*) if $\beta = \bar{\beta}$), giving the agent a payoff of $-\gamma$.

Since these kind of mechanisms are not observed very often in reality, it would be worth exploring how are our results affected when we consider more general renegotiation games.

6. Final Discussion

Previous agency models with pre-contractual information gathering focus on two dimensions of the problem: the information acquisition being only strategic or productive and the timing of the information acquisition. Here, we assume the agent gathers the pre-contractual information after receiving the offer from the principal and, if he decides not to gather it, he learns his type before deciding the production level (so the information acquisition is for strategic purposes only).

We take into account new dimensions of the problem by assuming the costly acquired information to be hard (but not verifiable) and by allowing for renegotiation. We consider this a plausible case in many situations where the agent can generate evidence (i.e., contracting a consultant firm) about his type that may convince the principal but can not convince a court (the information is observable but not verifiable).

We find that when the cost of acquiring the pre-contractual information (γ) is below a critical level, the agent will acquire it, he will (credibly) communicate it to the principal, and there will be renegotiation. The principal is in this case able to extract all the surplus of the relationship minus the cost γ by choosing the appropriate status quo point with the original contract. Obviously the principal's payoff function is decreasing in γ for values below the critical one.

When γ is above the critical value the optimal contract induces no pre-contractual information acquisition and the contract offered by the principal is renegotiation proof. Principal's expected profits increase as a function of γ in this case and, for large enough values of γ (greater than γ_L), the principal extracts all the expected surplus of the relationship.

The non-monotonicity of the principal's payoff function implies that the principal would not necessarily benefit from an increase in the agent's cost of acquiring pre-contractual information or from facing more than one potential agent in competition for the job (as she

does when the acquired information is soft and there is no renegotiation). She would benefit for sure if γ is above its critical value (that is when the optimal contract with one agent induces no information acquisition), and this is so because when there is competition the potential benefit for an agent of acquiring information is smaller. Therefore the incentive compatibility constraint to deter information acquisition would be eased and principal's benefit would be larger.¹⁸

When $\tilde{q}^0 < \underline{q}^0$ (Case 1) and $\gamma \in (\gamma^*, \gamma_A)$ agent's expected payoff is positive and decreasing in γ and it is zero if γ is smaller than γ^* . These non-monotonocities in the payoff functions are in contrast with the results obtained in Crémer and Khalil (1992) (where agent's expected payoff is decreasing in the cost of acquiring information and principal's is increasing), and their explanation is directly related to the observability of the information gathered and the possibility of renegotiation.

As mentioned before, the situations that fit the environment of this paper include those where the agent is able to convince the principal with non-verifiable information. When γ is below the critical value, there will always be a renegotiation of the initial contract. For these cases, the model provides one (of many) plausible explanation for cost overruns in procurement problems (i.e., defense contracts): principal and agent sign a contract (that they know they will renegotiate) whose main purpose is to set conditions for the renegotiation

¹⁸This point is made in Crémer and Khalil (1992), where the principal's payoff is a non-decreasing function of γ . The optimal contract when there are many agents is not derived in Crémer and Khalil (1992), but they prove that the principal can design a mechanism inducing no information acquisition in which he does better than when there is only one agent.

In our framework, if there are several agents and the principal decides to deal with only one of them, the principal would do as well as in the case with only one agent. But, if the principal is forced by law to offer to all the potential agents the same contract, then he could be worse-off (for some values of γ below the critical one).

and induce the agent to acquire pre-contractual information.^{19,20}

The situation addressed in this paper is one of many one could imagine in the context of principal-agent relationships with pre-contractual information acquisition. While maintaining the assumption of hard information, we could consider cases where the agent does not learn his type until the end of the game unless he spends γ and/or he gets the information before receiving the offer from the principal. Other extensions could consider cases with more than one agent and/or cases where the uncertainty is also about the principal's valuation of the object.

¹⁹Even though the counteroffer by the agent is made after accepting the contract, it is essential in the model that the agent is able to acquire the information before accepting the offer. Otherwise, the principal would solve the problem by offering an incentive compatible contract that induces efficient production levels for every states (and therefore will not be renegotiated) and gives the agent no ex-ante rent.

²⁰Another possible explanation for cost overruns makes use of incomplete contracts: ex-ante, the quality of the good can not be described, therefore a contract for a good with standard characteristics (and low price) is written down. As production takes place the agent learns he can provide a higher quality (and can effectively communicate this to the principal), then the contract is renegotiated (Tirole, 1986).

A. Appendix: Proofs Missing from the Text

A.1. Preliminaries for the Proofs of Propositions 1 and 2

Assuming *IC2* is satisfied (it can be verified afterwards), it is immediate from the first order conditions that $q_0 = \underline{q}^*$. Principal's problem can then be written as

$$\begin{aligned} \underset{t_0, t^0, q^0}{Max} & \pi(V(\underline{q}^*) - t_0) + (1 - \pi)(V(q^0) - t^0) + \mu\{\pi(t_0 - \underline{q}^*\underline{\beta}) + (1 - \pi)(t^0 - q^0\bar{\beta})\} \\ & + \lambda\{-(1 - \pi)[V(\bar{q}^*) - V(q^0) - \bar{\beta}(\bar{q}^* - q^0)] + \gamma\} \\ & + \phi\{(1 - \pi)(t^0 - q^0\bar{\beta}) + \gamma\} + \psi\{t_0 - \underline{q}^*\underline{\beta} - t^0 + q^0\bar{\beta}\} \quad (P'_U) \end{aligned}$$

The first order conditions of (P'_U) are:

$$-(1 - \pi) + \mu(1 - \pi) - \psi + \phi(1 - \pi) = 0 \quad (A.1)$$

$$-(1 - \pi) + \mu(1 - \pi) - \psi + \phi(1 - \pi) = 0 \quad (A.2)$$

$$(1 - \pi)V'(q^0) + \lambda(1 - \pi)(V'(q^0) - \bar{\beta}) - \mu(1 - \pi)\bar{\beta} + \psi\underline{\beta} - \phi(1 - \pi)\bar{\beta} = 0 \quad (A.3)$$

$$\pi(t_0 - \underline{q}^*\underline{\beta}) + (1 - \pi)(t^0 - q^0\bar{\beta}) \geq 0, \quad = 0 \quad \text{if } \mu > 0, \quad \mu \geq 0 \quad (A.4)$$

$$t_0 - \underline{q}^*\underline{\beta} - t^0 + q^0\bar{\beta} \geq 0, \quad = 0 \quad \text{if } \psi > 0, \quad \psi \geq 0 \quad (A.5)$$

$$-(1 - \pi)[V(\bar{q}^*) - V(q^0) - \bar{\beta}(\bar{q}^* - q^0)] + \gamma \geq 0, \quad = 0 \quad \text{if } \lambda > 0, \quad \lambda \geq 0 \quad (A.6)$$

$$(1 - \pi)(t^0 - q^0\bar{\beta}) + \gamma \geq 0, \quad = 0 \quad \text{if } \phi > 0, \quad \phi \geq 0 \quad (A.7)$$

Lemma A1, Corollary 2 and Lemma A2 are preliminary results used in the proofs of Propositions 1 and 2

Lemma A1. *If $\gamma < \gamma_L$ then $\psi > 0$ and $\phi > 0$.*

Proof of Lemma A1.

* Suppose $\psi = 0$, then (A.1) implies $\mu = 1$ and (A.2) implies $\phi = 0$. Then, equation (A.3) and condition (A.4) simplify to:

$$(1 - \pi)(V'(q^0) - \bar{\beta})(1 + \lambda) = 0 \Rightarrow q^0 = \bar{q}^* \quad (A.8)$$

$$t_0 = \frac{1 - \pi}{\pi}(\bar{q}^*\bar{\beta} - t^0) + \underline{q}^*\underline{\beta} \quad (A.9)$$

(A.8), (A.9) and (A.5) imply then:

$$t^0 \leq \bar{q}^*[(1 - \pi)\bar{\beta} + \pi\underline{\beta}] \quad (\text{A.10})$$

Then condition (A.7) and (A.10) give the desired contradiction:

$$\begin{aligned} \gamma &\geq (1 - \pi)\bar{q}^*\bar{\beta} - (1 - \pi)\bar{q}^*[(1 - \pi)\bar{\beta} + \pi\underline{\beta}] \\ \gamma &= \bar{q}^*\pi(1 - \pi)(\bar{\beta} - \underline{\beta}) = \gamma_L \Rightarrow \psi > 0. \end{aligned}$$

* From (A.1) $\psi = \pi(1 - \mu)$, then it must be that $\mu < 1$. Then, combining (A.1) and (A.2) we get $\phi = \frac{1-\mu}{1-\pi} > 0$. ■

Corollary 2. *If $\gamma < \gamma_L$ then $t_0 = \underline{q}^*\underline{\beta} + q^0(\bar{\beta} - \underline{\beta}) - \frac{\gamma}{1-\pi}$ and $t^0 = q^0\bar{\beta} - \frac{\gamma}{1-\pi}$*

Proof of Corollary 2. Immediate from conditions (A.5), (A.7) and Lemma A1. ■

Lemma A2. *For any γ , the solution to (P'_U) satisfies $\max\{q^0, \tilde{q}^0\} \leq q^0 \leq \bar{q}^*$.*

Proof of Lemma A2.

* From equations (A.1) and (A.2) we obtain $\psi = (1 - \mu)\pi$ (then $\mu \leq 1$) and $\phi = \frac{1-\mu}{1-\pi}$.

Combining these results with equation (A.3) we get

$$V'(q^0) = \frac{\bar{\beta}[1 + \lambda(1 - \pi)] - \pi\underline{\beta} - \mu\pi(\bar{\beta} - \underline{\beta})}{(1 - \pi)(1 + \lambda)},$$

therefore

$$\bar{\beta} \leq V'(q^0) \leq \frac{\bar{\beta}[1 + \lambda(1 - \pi)] - \pi\underline{\beta}}{(1 - \pi)(1 + \lambda)} \equiv F(\lambda),$$

where the first inequality follows from $\mu \leq 1$ and the second from $\mu \geq 0$.

* The first inequality implies $q^0 \leq V'^{-1}(\bar{\beta}) = \bar{q}^*$.

* Note also

$$F'(\lambda) = \frac{\bar{\beta}(1 - \pi)}{(1 - \pi)(1 + \lambda)} - \frac{\bar{\beta}[1 + \lambda(1 - \pi)] - \pi\underline{\beta}}{(1 - \pi)(1 + \lambda)^2} = \frac{\pi(\underline{\beta} - \bar{\beta}) - \bar{\beta}(\pi\lambda + 1)}{(1 - \pi)(1 + \lambda)^2} < 0,$$

therefore

$$V'(q^0) \leq \frac{\bar{\beta}[1 + \lambda(1 - \pi)] - \pi\underline{\beta}}{(1 - \pi)(1 + \lambda)}$$

must hold for $\lambda = 0$:

$$V'(q^0) \leq \frac{\bar{\beta} - \pi \underline{\beta}}{(1 - \pi)} = V'(\underline{q}^0) \Rightarrow q^0 \geq \underline{q}^0.$$

* Suppose $\gamma \geq \gamma_C$. Using Corollary 2 condition (A.4) can be written as

$$\begin{aligned} \pi q^0 (\bar{\beta} - \underline{\beta}) &\geq \frac{\gamma}{1 - \pi} \geq \frac{\gamma_C}{1 - \pi} = \pi \tilde{q}^0 (\bar{\beta} - \underline{\beta}) \\ &\Rightarrow q^0 \geq \tilde{q}^0. \end{aligned}$$

* Suppose $\gamma < \gamma_C$. Condition (A.6) implies

$$V(\bar{q}^*) - V(q^0) - \bar{\beta}(\bar{q}^* - q^0) \leq \frac{\gamma}{1 - \pi} < \pi \tilde{q}^0 (\bar{\beta} - \underline{\beta});$$

adding $-V(\tilde{q}^0) + \tilde{q}^0 \bar{\beta}$ on both sides and rearranging terms

$$V(\bar{q}^*) - \bar{q}^* \bar{\beta} - V(\tilde{q}^0) + \tilde{q}^0 \bar{\beta} - \pi \tilde{q}^0 (\bar{\beta} - \underline{\beta}) - V(q^0) + q^0 \bar{\beta} < -V(\tilde{q}^0) + \tilde{q}^0 \bar{\beta},$$

and using the definition of \tilde{q}^0

$$-V(q^0) + q^0 \bar{\beta} < -V(\tilde{q}^0) + \tilde{q}^0 \bar{\beta} \Rightarrow q^0 > \tilde{q}^0.$$

■

Proof of Proposition 1.

1) ($\gamma \geq \gamma_L$):

* Principal's expected profits are equal to the total expected surplus:

$$\begin{aligned} \pi(V(\underline{q}^*) - t_0) + (1 - \pi)(V(\bar{q}^*) - t^0) &= \frac{\gamma_L}{1 - \pi} + \pi V(\underline{q}^*) + (1 - \pi)V(\bar{q}^*) - \\ &\quad \pi(\underline{q}^* \underline{\beta} + \bar{q}^* (\bar{\beta} - \underline{\beta})) - (1 - \pi)\bar{q}^* \bar{\beta} \\ &= \frac{\gamma_L}{1 - \pi} + \pi(V(\underline{q}^*) - \underline{q}^* \underline{\beta}) + \\ &\quad (1 - \pi)(V(\bar{q}^*) - \bar{q}^* \bar{\beta}) - \pi \bar{q}^* (\bar{\beta} - \underline{\beta}) \\ &= \pi(V(\underline{q}^*) - \underline{q}^* \underline{\beta}) + (1 - \pi)(V(\bar{q}^*) - \bar{q}^* \bar{\beta}). \end{aligned}$$

* And all the constraints are satisfied:

- with the expressions for t^0 and t_0 (IC') and (IR') are satisfied for any values of q^0 ,

- (IC'_{UI1}) reduces to $\gamma \geq 0$, and

- (IC'_{UI2}) simplifies to $\gamma \geq \gamma_L$.

2) ($\gamma_A \leq \gamma < \gamma_L$):

* By Lemma A1 we know $\psi, \phi > 0$.

* Assume $\lambda > 0$, condition (A.6) can be written as

$$\begin{aligned} V(q^0) - q^0 \bar{\beta} + \frac{\gamma}{1-\pi} &= V(\bar{q}^*) - \bar{q}^* \bar{\beta} \\ V(q^0) - q^0 \bar{\beta} + \frac{\gamma}{1-\pi} &= V(\tilde{q}^0) - \tilde{q}^0 \tilde{\beta} \\ V(q^0) - q^0 \bar{\beta} + \frac{\gamma}{1-\pi} &= V(\tilde{q}^0) - \tilde{q}^0 \bar{\beta} + \pi \tilde{q}^0 (\bar{\beta} - \underline{\beta}) \end{aligned}$$

where the second and third equalities follow from the definition of \tilde{q}^0 and $\tilde{\beta}$ respectively.

$\gamma \geq \gamma_A$ implies

$$V(q^0) - q^0 \bar{\beta} + \frac{\gamma_A}{1-\pi} \leq V(\tilde{q}^0) - \tilde{q}^0 \bar{\beta} + \pi \tilde{q}^0 (\bar{\beta} - \underline{\beta}),$$

and using the definition of γ_A we get

$$V(q^0) - q^0 \bar{\beta} + \pi \underline{q}^0 (\bar{\beta} - \underline{\beta}) \leq V(\tilde{q}^0) - \tilde{q}^0 \bar{\beta} + \pi \tilde{q}^0 (\bar{\beta} - \underline{\beta}).$$

Since $\underline{q}^0 > \tilde{q}^0$ this inequality implies $q^0 < \tilde{q}^0 < \underline{q}^0$ which contradicts Lemma A2. Therefore it has to be that $\lambda = 0$.

* Assume now $\mu = 0$. If $\lambda = \mu = 0$ then equation (A.3) implies

$$q^0 = V'^{-1} \left(\frac{\bar{\beta} - \pi \underline{\beta}}{1-\pi} \right) = \underline{q}^0.$$

Using Corollary (2) and the above result, condition (A.4) can be rewritten as

$$\begin{aligned} \pi(\underline{q}^0(\bar{\beta} - \underline{\beta}) - \frac{\gamma}{1-\pi}) + (1-\pi)(-\frac{\gamma}{1-\pi}) &\geq 0 \\ \Leftrightarrow \gamma &\leq \pi \underline{q}^0 (\bar{\beta} - \underline{\beta})(1-\pi) = \gamma_A, \end{aligned}$$

which contradicts the assumption that $\gamma_A < \gamma < \gamma_L$. Therefore $\mu > 0$.

* Since $\mu > 0$, then $\pi(t_0 - \underline{q}^* \underline{\beta}) + (1-\pi)(t^0 - q^0 \bar{\beta}) = 0$. Substituting t^0 and t_0 with the result

of Corollary 2 we obtain the desired result:

$$q^0 = \frac{\gamma}{(1-\pi)\pi(\bar{\beta} - \underline{\beta})}.$$

3) ($\gamma = \gamma_A$):

* Following the first two steps of the previous case ($\gamma_A \leq \gamma < \gamma_L$) we conclude $\psi, \phi > 0$ and $\lambda = 0$.

* Assume $\mu = 0$. If $\lambda = \mu = 0$ then equation (A.3) implies

$$q^0 = V'^{-1} \left(\frac{\bar{\beta} - \pi \underline{\beta}}{1 - \pi} \right) = \underline{q}^0.$$

* Assume $\mu > 0$. Then $\pi(t_0 - \underline{q}^* \underline{\beta}) + (1 - \pi)(t^0 - q^0 \bar{\beta}) = 0$. Substituting t^0 and t_0 (recall Corollary 2) we obtain

$$q^0 = \frac{\gamma_A}{(1-\pi)\pi(\bar{\beta} - \underline{\beta})},$$

and, by definition of γ_A , $q^0 = \underline{q}^0$.

4) ($\gamma_B < \gamma < \gamma_A$):

* Assume $\lambda > 0$, condition (A.6) can be written as:

$$V(q^0) - q^0 \bar{\beta} = V(\bar{q}^*) - \bar{q}^* \bar{\beta} - \frac{\gamma}{1-\pi} < V(\bar{q}^*) - \bar{q}^* \bar{\beta} - \frac{\gamma_B}{1-\pi} = V(\underline{q}^0) - \underline{q}^0 \bar{\beta},$$

where the last equality follows from the definition of γ_B . Condition

$$V(q^0) - q^0 \bar{\beta} < V(\underline{q}^0) - \underline{q}^0 \bar{\beta}$$

implies $q^0 < \underline{q}^0$, which contradicts Lemma A2. Therefore $\lambda = 0$.

* Assume $\mu > 0$. Condition (A.4) and the results of Corollary 2 imply:

$$q^0 = \frac{\gamma}{(1-\pi)\pi(\bar{\beta} - \underline{\beta})} < \underline{q}^0 \text{ for } \gamma < \gamma_A,$$

which contradicts Lemma A2. Therefore $\mu = 0$.

* If $\mu = \lambda = 0$ equation (A.3) implies

$$(1-\pi)V'(q^0) + \pi \underline{\beta} - \bar{\beta} = 0 \Rightarrow q^0 = V'^{-1} \left(\frac{\bar{\beta} - \pi \underline{\beta}}{1 - \pi} \right) = \underline{q}^0.$$

5) ($\gamma < \gamma_B$):

* Assume $\mu > 0$. Then condition (A.4) and the results of Corollary 2 imply:

$$q^0 = \frac{\gamma}{(1-\pi)\pi(\bar{\beta} - \underline{\beta})} < \underline{q}^0 \text{ if } \gamma < \gamma_A,$$

which contradicts Lemma A2. Therefore $\mu = 0$.

* If $\lambda = \mu = 0$, then equation (A.3) implies $q^0 = \underline{q}^0$, and condition (A.6) implies

$$\gamma \geq [V(\bar{q}^*) - \bar{q}^*\bar{\beta} - V(\underline{q}^0) + \underline{q}^0\bar{\beta}](1-\pi) = \gamma_B,$$

therefore $\lambda > 0$.

*Now $\lambda > 0$ implies [using condition (A.6)] that q^0 satisfies

$$V(q^0) - q^0\bar{\beta} = V(\bar{q}^*) - \bar{q}^*\bar{\beta} - \frac{\gamma}{1-\pi}.$$

6) ($\gamma = \gamma_B$):

* The same argument used in Step 4 implies $\mu = 0$.

* If $\lambda = \mu = 0$ then equation (3) implies:

$$(1-\pi)V'(q^0) + \pi\underline{\beta} - \bar{\beta} = 0 \Rightarrow q^0 = V'^{-1}\left(\frac{\bar{\beta} - \pi\underline{\beta}}{1-\pi}\right) = \underline{q}^0$$

If $\lambda > 0$, then condition (A.6) implies that q^0 satisfies:

$$V(q^0) - q^0\bar{\beta} = V(\bar{q}^*) - \bar{q}^*\bar{\beta} - \frac{\gamma_B}{1-\pi}$$

$$V(q^0) - q^0\bar{\beta} = V(\underline{q}^0) - \underline{q}^0\bar{\beta},$$

therefore $q^0 = \underline{q}^0$. ■

Proof of Proposition 2.

1) ($\gamma \geq \gamma_L$): Idem Step 1 Proposition 1.

2) ($\gamma_C < \gamma < \gamma_L$):

* Condition (A.4) together with Corollary 2 imply:

$$\pi q^0(\bar{\beta} - \underline{\beta}) \geq \frac{\gamma}{1-\pi} > \pi \tilde{q}^0(\bar{\beta} - \underline{\beta}) \Rightarrow q^0 > \tilde{q}^0,$$

where the strict inequality follows from replacing γ by γ_C .

* Suppose $\lambda > 0$. Condition (A.6) and $\gamma > \gamma_C$ imply

$$V(\bar{q}^*) - \bar{q}^* \bar{\beta} - V(q^0) + q^0 \bar{\beta} = \frac{\gamma}{1 - \pi} > \pi \tilde{q}^0 (\bar{\beta} - \underline{\beta}),$$

adding $-V(\tilde{q}^0) + \tilde{q}^0 \bar{\beta} - \pi \tilde{q}^0 (\bar{\beta} - \underline{\beta})$ on both sides gives

$$V(\bar{q}^*) - \bar{q}^* \bar{\beta} - V(\tilde{q}^0) + \tilde{q}^0 \bar{\beta} - \pi \tilde{q}^0 (\bar{\beta} - \underline{\beta}) - V(q^0) + q^0 \bar{\beta} > -V(\tilde{q}^0) + \tilde{q}^0 \bar{\beta}.$$

Then the definition of \tilde{q}^0 implies:

$$-V(q^0) + q^0 \bar{\beta} > -V(\tilde{q}^0) + \tilde{q}^0 \bar{\beta} \Rightarrow q^0 < \tilde{q}^0,$$

contradicting condition (A.4). Therefore $\lambda = 0$.

* Suppose $\mu = \lambda = 0$. Equation (A.3) implies $q^0 = \underline{q}^0 \leq \tilde{q}^0$, but condition (A.4) requires $q^0 > \tilde{q}^0$, therefore $\mu > 0$.

* If $\mu > 0$, condition (A.4) implies

$$q^0 = \frac{\gamma}{(1 - \pi)\pi(\bar{\beta} - \underline{\beta})}.$$

3) ($\gamma < \gamma_C$):

* Note that the last part of the proof of Lemma A2 implies $q^0 > \tilde{q}^0$ when $\gamma < \gamma_C$.

* If $\mu > 0$, then condition (A.4) implies:

$$q^0 = \frac{\gamma}{(1 - \pi)\pi(\bar{\beta} - \underline{\beta})} < \frac{\gamma_C}{(1 - \pi)\pi(\bar{\beta} - \underline{\beta})} = \tilde{q}^0,$$

therefore $\mu = 0$.

* If $\lambda = \mu = 0$, then equation (A.3) implies

$$V'(q^0) = \frac{\bar{\beta} - \pi \underline{\beta}}{1 - \pi} \Rightarrow q^0 = \underline{q}^0 \leq \tilde{q}^0,$$

therefore $\lambda > 0$.

* $\lambda > 0$ and Lemma A2 imply

$$q^0 = \{q : q < \bar{q}^* \wedge V(q) - q \bar{\beta} = V(\bar{q}^*) - \bar{q}^* \bar{\beta} - \frac{\gamma}{1 - \pi}\}.$$

4) ($\gamma = \gamma_C$):

* Note that $q^0 = \tilde{q}^0$ satisfies $V(q^0) - q^0\bar{\beta} = V(\bar{q}^*) - \bar{q}^*\bar{\beta} - \frac{\gamma_C}{1-\pi}$.

* If $\lambda = \mu = 0$, then equation (A.3) implies $q^0 = \underline{q}^0$.

- If $\underline{q}^0 = \tilde{q}^0$ we have the desired result.

- If $\underline{q}^0 < \tilde{q}^0$, then $q^0 = \underline{q}^0$ contradicts Lemma A2, therefore either λ or μ (or both) are greater than zero.

* If $\lambda > 0$, condition (A.6) implies:

$$V(\bar{q}^*) - \bar{q}^*\bar{\beta} - V(q^0) + q^0\bar{\beta} = \frac{\gamma_C}{1-\pi} = \tilde{q}^0(\bar{\beta} - \underline{\beta})\pi,$$

using the definition of \tilde{q}^0 (and a similar argument to the second part of Step 2) we have $q^0 = \tilde{q}^0$.

* If $\mu > 0$, condition (A.4) implies:

$$q^0 = \frac{\gamma_C}{(1-\pi)\pi(\bar{\beta} - \underline{\beta})} = \tilde{q}^0.$$

■

Proof of Proposition 3. Drop the argument (c_0) in the functions $t_s(c_0)$, $t^s(c_0)$, $q^*(c_0)$ and $q^s(c_0)$ to simplify notation and let the equilibrium outcome be (t_s, q_s) when $\beta = \underline{\beta}$ and (t^s, q^s) when $\beta = \bar{\beta}$.²¹

* Note that for any beliefs the principal may have, he would always accept a renegotiation offer $(t_0 + V(\underline{q}^*) - V(q_0), \underline{q}^*)$,²² therefore the low cost type can guarantee for himself a utility equal to $t_0 + V(\underline{q}^*) - V(q_0) - \underline{q}^*\underline{\beta}$. Then, in equilibrium $t_s - q_s\underline{\beta} \geq t_0 + V(\underline{q}^*) - V(q_0) - \underline{q}^*\underline{\beta}$ must hold.

* Suppose now that $t_s - q_s\underline{\beta} > t_0 + V(\underline{q}^*) - V(q_0) - \underline{q}^*\underline{\beta}$. For the principal to accept the renegotiation, it must be that $V(q^s) - t^s > V(q^0) - t^0$ (otherwise $\lambda[V(q_s) - t_s] + (1-\lambda)[V(q^s) - t^s] < \lambda[V(q_0) - t_0] + (1-\lambda)[V(q^0) - t^0]$ for any beliefs $\lambda \in (0, 1)$ the principal may have, and

²¹The analysis that follows is valid both for a pooling equilibrium -both types send the message (t_s, q_s, t^s, q^s) - and for a separating one where type $\underline{\beta}$ sends (t_s, q_s, \cdot, \cdot) and type $\bar{\beta}$ sends (\cdot, \cdot, t^s, q^s) .

²²Recall we are restricting attention to contracts such that $V(q_0) - t_0 \geq V(q^0) - t^0$.

therefore she would reject the offer). But then the high cost type would have a profitable deviation (point **Z** in Figure C.12).

Formally, assume $V(q^s) - t^s > V(q^0) - t^0$ and (t^s, q^s) is the equilibrium outcome when $\beta = \bar{\beta}$. Let:

$$\begin{aligned} q' &\equiv \{q : t^s - \underline{\beta}(q^s - q) = V(q) - [V(q^s) - t^s]\} \\ t' &= V(q') - [V(q^s) - t^s] = t' - \underline{\beta}(q^s - q') \end{aligned}$$

Consider the deviation for type $\bar{\beta}$ $(\hat{t}, \hat{q}) = (t' - \eta, q')$,²³ where $\eta = (\bar{\beta} - \underline{\beta})\frac{q^s - q'}{2}$.

- Note that type $\bar{\beta}$ strictly prefers (\hat{t}, \hat{q}) to (t^s, q^s) :

$$\begin{aligned} (\bar{\beta} - \underline{\beta})\frac{q^s - q'}{2} > 0 &\Leftrightarrow -\underline{\beta}\frac{q^s - q'}{2} > -\bar{\beta}(q^s - q') + \bar{\beta}\frac{q^s - q'}{2} \\ \Leftrightarrow t^s - \underline{\beta}(q^s - q') + \underline{\beta}\frac{q^s - q'}{2} - \bar{\beta}\frac{q^s - q'}{2} &> t^s - \bar{\beta}q^s + \bar{\beta}q' \\ \Leftrightarrow t' - (\bar{\beta} - \underline{\beta})\frac{q^s - q'}{2} - \bar{\beta}q' &> t^s - \bar{\beta}q^s. \end{aligned}$$

- Note that type $\underline{\beta}$ strictly prefers (t_s, q_s) to (\hat{t}, \hat{q}) :

$$\begin{aligned} 0 > -(\bar{\beta} - \underline{\beta})\frac{q^s - q'}{2} &\Leftrightarrow t^s - \underline{\beta}q^s + \underline{\beta}q' > t' - (\bar{\beta} - \underline{\beta})\frac{q^s - q'}{2} \\ t^s - \underline{\beta}q^s > t' - (\bar{\beta} - \underline{\beta})\frac{q^s - q'}{2} - \underline{\beta}q' &= \hat{t} - \underline{\beta}\hat{q} \end{aligned}$$

and then the incentive compatibility constraint implies

$$t_s - \underline{\beta}q_s \geq t^s - \underline{\beta}q^s > \hat{t} - \underline{\beta}\hat{q}.$$

- Since (\hat{t}, \hat{q}) is strictly preferred to the proposed equilibrium outcome by type $\bar{\beta}$ and the equilibrium outcome (t_s, q_s) is strictly preferred to (\hat{t}, \hat{q}) by the low cost type $(\hat{t} - \underline{\beta}\hat{q} < t_s - \underline{\beta}q_s)$, then the Intuitive Criterion tells that if the principal receives the message $(\hat{t}, \hat{q}, \hat{t}, \hat{q})$ she has to believe that the message was sent by a type $\bar{\beta}$ with probability one.

- Finally, note that:

$$V(\hat{q}) - \hat{t} = V(q') - t' + \eta = V(q^s) - t^s + \eta = V(q^0) - t^0 + \eta > V(q^0) - t^0,$$

²³For brevity we may refer to (t, q) as a message instead of (t, q, t, q) .

and the message $(\widehat{t}, \widehat{q}, \widehat{t}, \widehat{q})$ would be accepted by the principal.

- Therefore $t_s - q_s \underline{\beta} > t_0 + V(\underline{q}^*) - V(q_0) - \underline{q}^* \underline{\beta}$ is not possible in equilibrium. Then it has to be

$$t_s - q_s \underline{\beta} = t_0 + V(\underline{q}^*) - V(q_0) - \underline{q}^* \underline{\beta}. \quad (+)$$

- We also have shown that $V(q^s) - t^s > V(q^0) - t^0$ is not possible in equilibrium.

* Note that the principal accepting the renegotiation requires:

$$\pi[V(q_s) - t_s] + (1 - \pi)[V(q^s) - t^s] \geq \pi[V(q_0) - t_0] + (1 - \pi)[V(q^0) - t^0], \quad (\text{IR}_P)$$

then $V(q^s) - t^s \leq V(q^0) - t^0$ implies

$$V(q_s) - t_s \geq V(q_0) - t_0. \quad (++)$$

* Note that (+) and (++) together imply $t_s = t_0 + V(\underline{q}^*) - V(q_0)$ and $q_s = \underline{q}^*$. To see that replace t_s in (++) with its equal from (+) to get

$$V(q_s) - q_s \underline{\beta} \geq V(\underline{q}^*) - \underline{q}^* \underline{\beta},$$

which can be satisfied only if $q_s = \underline{q}^*$. Then (+) implies $t_s = t_0 + V(\underline{q}^*) - V(q_0)$.

* Now, (IR_P) , $q_s = \underline{q}^*$, and $t_s = t_0 + V(\underline{q}^*) - V(q_0)$ together imply

$$V(q^s) - t^s \geq V(q^0) - t^0.$$

Since $V(q^s) - t^s > V(q^0) - t^0$ cannot hold in equilibrium, then $V(q^s) - t^s = V(q^0) - t^0$ must hold.

* Finally, suppose $V(q^s) - t^s = V(q^0) - t^0$ and $(t^s, q^s) \neq (t^0 + V(q^s) - V(q^0), \min\{\bar{q}^*, q^*\})$.

(Recall $q^*(c_0) \equiv \{q : t_s - \underline{\beta}(q^* - q) = V(q) - [V(q^0) - t^0]\}$).

Consider three possible cases:

1- $t^s - \bar{\beta}q^s = t^0 + V(q^s) - V(q^0) - \bar{\beta} \min\{\bar{q}^*, q^*\}$:

$$V(q^s) - t^s = V(q^0) - t^0$$

simplifies the above expression to $\bar{\beta}q^s = \bar{\beta} \min \{\bar{q}^*, q^*\}$ therefore $q^s = \min \{\bar{q}^*, q^*\}$. Given this,

$$t^s = t^0 + V(q^s) - V(q^0)$$

follows immediately.

2- $t^s - \bar{\beta}q^s > t^0 + V(q^s) - V(q^0) - \bar{\beta} \min \{\bar{q}^*, q^*\}$:

$$V(q^s) - t^s = V(q^0) - t^0$$

implies $q^s < \min \{\bar{q}^*, q^*\}$. Then type $\bar{\beta}$ has a profitable deviation for (accepted by the principal) by sending the message $(t^s + \varepsilon, \min \{\bar{q}^*, q^*\})$, where $\varepsilon = V(\min \{\bar{q}^*, q^*\}) - V(q^s) > 0$. The principal would be indifferent (and therefore she would accept) and the agent increases his payoff:

$$t^s - q^s \bar{\beta} < t^s + V(\min \{\bar{q}^*, q^*\}) - V(q^s) - \bar{\beta} \min \{\bar{q}^*, q^*\}$$

since, by assumption, $q^s < \bar{q}^*$.

3- $t^s - \bar{\beta}q^s < t^0 + V(q^s) - V(q^0) - \bar{\beta} \min \{\bar{q}^*, q^*\}$:

$$V(q^s) - t^s = V(q^0) - t^0$$

implies $q^s > \min \{\bar{q}^*, q^*\}$. Assume also $q^s \leq \underline{q}^*$.²⁴

- Assume $q^s > q^*$. Note:

$$q^s \in (q^*, \underline{q}^*] \Rightarrow V(q^s) - V(\min \{\bar{q}^*, q^*\}) > \underline{\beta}(q^s - \min \{\bar{q}^*, q^*\}). \quad (\#) \quad (\text{A.11})$$

The definition of q^* and the fact that $V(q^s) - t^s = V(q^0) - t^0$ imply

$$t_s - \underline{q}^* \underline{\beta} = t^s - q^* \underline{\beta} - [V(q^s) - V(q^*)].$$

²⁴The case of $q^s > \underline{q}^*$ can be easily ruled out as a possible equilibrium. Since $q_s = \underline{q}^*$ the incentive compatibility constraint for low cost type can be written as

$$t^s \leq t_s + (q^s - \underline{q}^*) \underline{\beta}.$$

But the incentive compatibility constraint for the high cost type requires

$$t^s \geq t_s + (q^s - \underline{q}^*) \bar{\beta}.$$

Using $q_s = \underline{q}^*$, the inequality (#) implies $t_s - q_s \underline{\beta} < t^s - q^s \underline{\beta}$, which violates the incentive compatibility constraint and, therefore, (t_s, q_s, t^s, q^s) cannot be the equilibrium outcome.

- Assume $q^s > \bar{q}^*$. Consider the alternative message $(t', q') = (t^0 + V(\bar{q}^*) - V(q^0), \bar{q}^*)$. The fact that $V(q^s) - t^s = V(q^0) - t^0$ implies

$$t' - q' \bar{\beta} > t^s - q^s \bar{\beta}.$$

Now $t_s - q_s \underline{\beta} \geq t^s - q^s \underline{\beta}$ and $q^s < \underline{q}^*$ imply

$$t_s - q_s \underline{\beta} > t' - q' \underline{\beta},$$

and, therefore, the principal will assign a probability 1 to the agent sending the message (t', q', t', q') being type $\bar{\beta}$. She would therefore accept it since

$$V(q^0) - t^0 = V(q^s) - t^s = V(q') - t'.$$

■

Proof of Proposition 4.

* Note that in order to satisfy (IR), principal's expected profit can be at most

$$\pi[V(\underline{q}^*) - \underline{q}^* \underline{\beta}] + (1 - \pi)[V(\bar{q}^*) - \bar{q}^* \bar{\beta}] - \gamma.$$

* With the proposed solution principal's expected profit is the maximum possible:

$$\begin{aligned} \pi(V(q_0) - t_0) + (1 - \pi)(V(q^0) - t^0) &= \pi(V(\underline{q}^*) - \underline{q}^* \underline{\beta}) + (1 - \pi)(V(q^0) - q^0 \underline{\beta}) \\ &= \pi[V(\underline{q}^*) - \underline{q}^* \underline{\beta}] + (1 - \pi)[V(\bar{q}^*) - \bar{q}^* \bar{\beta}] - \gamma \end{aligned}$$

* Constraints (IC1),(IC2),(IR $_{\underline{\beta}}$), (IR $_{\bar{\beta}}$),(IR) and (IC $_I$) are all satisfied:

$$t_0 - q_0 \underline{\beta} \geq t^0 - q^0 \underline{\beta} \Leftrightarrow 0 \geq 0 \tag{IC1}$$

$$t^0 - q^0 \bar{\beta} \geq t_0 - q_0 \bar{\beta} \Leftrightarrow q^0(\underline{\beta} - \bar{\beta}) \geq q_0(\underline{\beta} - \bar{\beta}) \Leftrightarrow q^0 \leq q_0 \tag{IC2}$$

$$V(\underline{q}^*) - \underline{q}^* \underline{\beta} - [V(q_0) - t_0] \geq 0 \quad \Leftrightarrow \quad 0 \geq 0 \quad (\text{IR}_{\underline{\beta}})$$

$$V(\bar{q}^*) - \bar{q}^* \bar{\beta} - [V(q^0) - t^0] \geq 0 \quad (\text{IR}_{\bar{\beta}})$$

$$\Leftrightarrow V(\bar{q}^*) - \bar{q}^* \bar{\beta} - [V(\bar{q}^*) - \bar{q}^* \bar{\beta} - \gamma/(1 - \pi)] \geq 0 \quad \Leftrightarrow \quad \gamma \geq 0$$

$$\begin{aligned} \gamma &\leq \pi[V(\underline{q}^*) - \underline{q}^* \underline{\beta} - (V(q_0) - t_0)] + (1 - \pi)[V(\bar{q}^*) - \bar{q}^* \bar{\beta} - (V(q^0) - t^0)] \\ &\Leftrightarrow \quad \gamma \leq (1 - \pi)[\gamma/(1 - \pi)]. \end{aligned} \quad (\text{IR})$$

Finally, note that the proposed contract satisfies the conditions of Corollary 1 and therefore it is not renegotiated. Then, (IC_I) reduces to:

$$\begin{aligned} \pi(t_0 - q_0 \underline{\beta}) + (1 - \pi)(t^0 - q^0 \bar{\beta}) &\leq -\gamma + \pi\{V(\underline{q}^*) - \underline{q}^* \underline{\beta} - [V(q_0) - t_0]\} + \\ &\quad (1 - \pi)\{V(\bar{q}^*) - \bar{q}^* \bar{\beta} - [V(q^0) - t^0]\} \quad (\text{IC}_I) \end{aligned}$$

$$\begin{aligned} \Leftrightarrow (1 - \pi)(q^0 \underline{\beta} - q^0 \bar{\beta}) &\leq -\gamma + (1 - \pi)\{V(\bar{q}^*) - \bar{q}^* \bar{\beta} - [V(q^0) - q^0 \underline{\beta}]\} \\ \Leftrightarrow (1 - \pi)q^0(\underline{\beta} - \bar{\beta}) &\leq -\gamma + (1 - \pi)\gamma/(1 - \pi) = 0. \end{aligned}$$

■

Proof of Proposition 5. Since the profit function is increasing (decreasing) in γ when the principal induces the agent to (not to) acquire information, all we need to prove is that profits are identical when the cost is equal to the critical value (γ^* in Case 1 and γ_C in Case 2) and that these critical values are smaller than the expected surplus generated by the high cost type (since Proposition 4 characterizes the optimal contract only in this situation).

Let $PR_h(\gamma)$ be principal's profit as a function of γ when the contract is the one described by Proposition 4 (the agent has hard information), and $PR_s^1(\gamma)$ and $PR_s^2(\gamma)$ principal's profit when contracts are characterized by Propositions 1 and 2 respectively (the agent only has soft information).

Case 1 ($\tilde{q}^0 < \underline{q}^0$):

* $PR_h(\gamma^*) = PR_s^1(\gamma^*) :$

$$\begin{aligned} \pi[V(\underline{q}^*) - \underline{q}^*\underline{\beta}] + (1 - \pi)[V(\bar{q}^*) - \bar{q}^*\bar{\beta}] - \gamma^* &= \pi \left[V(\underline{q}^*) - \underline{q}^*\underline{\beta} - \underline{q}^0(\bar{\beta} - \underline{\beta}) + \frac{\gamma^*}{1 - \pi} \right] \\ &\quad + (1 - \pi) \left[V(\underline{q}^0) - \bar{\beta}\underline{q}^0 + \frac{\gamma^*}{1 - \pi} \right] \end{aligned}$$

\Leftrightarrow

$$(1 - \pi)[V(\bar{q}^*) - \bar{q}^*\bar{\beta} - V(\underline{q}^0) + \bar{\beta}\underline{q}^0] + \pi\underline{q}^0(\bar{\beta} - \underline{\beta}) = \frac{2 - \pi}{1 - \pi}\gamma^*$$

* $\gamma^* < (1 - \pi)[V(\bar{q}^*) - \bar{q}^*\bar{\beta}] :$

$$\frac{1 - \pi}{2 - \pi} \{ (1 - \pi)[V(\bar{q}^*) - \bar{q}^*\bar{\beta} - V(\underline{q}^0) + \underline{q}^0\bar{\beta}] + \pi\underline{q}^0(\bar{\beta} - \underline{\beta}) \} < (1 - \pi)[V(\bar{q}^*) - \bar{q}^*\bar{\beta}]$$

\Leftrightarrow

$$(1 - \pi)[-V(\underline{q}^0) + \underline{q}^0\bar{\beta}] + \pi\underline{q}^0(\bar{\beta} - \underline{\beta}) < [V(\bar{q}^*) - \bar{q}^*\bar{\beta}]$$

\Leftrightarrow

$$\underline{q}^0 \frac{\bar{\beta} - \pi\underline{\beta}}{1 - \pi} - V(\underline{q}^0) < \frac{V(\bar{q}^*) - \bar{q}^*\bar{\beta}}{1 - \pi},$$

which is satisfied since the right hand side is positive and the left hand side is negative (note that $V'(\underline{q}^0) = \frac{\bar{\beta} - \pi\underline{\beta}}{1 - \pi}$ and $V''(q) < 0$).

Case 2 ($\tilde{q}^0 \geq \underline{q}^0$):

* $PR_I(\gamma_C) = PR_U^2(\gamma_C) :$

$$\begin{aligned} \pi[V(\underline{q}^*) - \underline{q}^*\underline{\beta}] + (1 - \pi)[V(\bar{q}^*) - \bar{q}^*\bar{\beta}] - \gamma_C &= \pi \left[V(\underline{q}^*) - \underline{q}^*\underline{\beta} - \underline{q}^0(\bar{\beta} - \underline{\beta}) + \frac{\gamma_C}{1 - \pi} \right] \\ &\quad + (1 - \pi) \left[V(\underline{q}^0) - \bar{\beta}\underline{q}^0 + \frac{\gamma_C}{1 - \pi} \right] \end{aligned}$$

\Leftrightarrow

$$\begin{aligned} (1 - \pi)[V(\bar{q}^*) - \bar{q}^*\bar{\beta}] + \pi\tilde{q}^0(\bar{\beta} - \underline{\beta}) - (1 - \pi)[V(\tilde{q}^0) - \tilde{q}^0\bar{\beta}] &= \frac{2 - \pi}{1 - \pi}\gamma_C \\ &= (2 - \pi)\tilde{q}^0(\bar{\beta} - \underline{\beta})\pi \end{aligned}$$

\Leftrightarrow

$$(1 - \pi)[V(\bar{q}^*) - \bar{q}^*\bar{\beta} - V(\tilde{q}^0)] + (1 - \pi)\tilde{q}^0\bar{\beta} - (1 - \pi)\tilde{q}^0(\bar{\beta} - \underline{\beta})\pi = 0$$

⇔

$$V(\bar{q}^*) - \bar{q}^* \bar{\beta} - V(\hat{q}^0) + \hat{q}^0 [\bar{\beta}(1 - \pi) + \underline{\beta}\pi] = 0,$$

where the last equality holds by the definition of \hat{q}^0 .

$$* \gamma_C < (1 - \pi)[V(\bar{q}^*) - \bar{q}^* \bar{\beta}] :$$

$$\hat{q}^0 (\bar{\beta} - \underline{\beta}) \pi (1 - \pi) < (1 - \pi)[V(\bar{q}^*) - \bar{q}^* \bar{\beta}]$$

⇔

$$\hat{q}^0 (\bar{\beta} - \underline{\beta}) \pi < V(\hat{q}^0) - \hat{q}^0 \tilde{\beta} \Leftrightarrow 0 < V(\hat{q}^0) - \hat{q}^0 \bar{\beta}.$$

■

B. Appendix: A Message Contingent Contract

Message contingent contracts a-la-Maskin-Tirole can induce efficient levels of production in a full commitment case. For any level of γ , the principal can extract all the expected surplus ($[V(\underline{q}^*) - \underline{q}^* \underline{\beta}] \pi + [V(\bar{q}^*) - \bar{q}^* \bar{\beta}] (1 - \pi)$) minus the cost γ (so the payoff for the agent would be equal to his reservation utility) by offering the following message contingent contract:

- 1- The agent reports to the court his cost $\{l, h\}$,
- 2- The principal accepts or challenges agent's report $\{A, C\}$,
- 3- If the principal challenges the report then the agent has to accept the challenge or reject it $\{a, r\}$.

As a function of the messages, production levels, money transfers and fines are as follow (let f and F be the fines the agent and the principal pay to the court):

$$* t(l, A) = \underline{q}^* \underline{\beta} + \gamma ; q(l, A) = \underline{q}^* ; f(l, A) = 0 ; F(l, A) = 0$$

$$* t(h, A) = \bar{q}^* \bar{\beta} + \gamma ; q(h, A) = \bar{q}^* ; f(h, A) = 0 ; F(h, A) = 0$$

$$* t(l, C, a) = \bar{q}^* \bar{\beta} + \gamma ; q(l, C, a) = \bar{q}^* ; f(l, C, a) = P' ; F(l, C, a) = 0$$

$$* t(h, C, a) = \underline{q}^* \underline{\beta} + \gamma ; q(h, C, a) = \underline{q}^* ; f(h, C, a) = P ; F(h, C, a) = 0$$

$$* t(l, C, r) = \underline{q}^* \underline{\beta} + \gamma ; q(l, C, r) = \underline{q}^* ; f(l, C, r) = P''' ; F(l, C, r) = Q$$

$$* t(h, C, r) = \bar{q}^* \bar{\beta} + \gamma ; q(h, C, r) = \bar{q}^* ; f(h, C, r) = P'' ; F(h, C, r) = 0;$$

where P, P', P'', P''' and Q satisfy:

$$(\bar{\beta} - \underline{\beta}) \underline{q}^* \geq P'' \geq P + (\bar{\beta} - \underline{\beta}) \bar{q}^*,$$

$$P > 0,$$

$$P' - (\bar{\beta} - \underline{\beta}) \bar{q}^* \geq P''' \geq P' - (\bar{\beta} - \underline{\beta}) \underline{q}^*,$$

$$Q > 0.^{25}$$

When the principal offers the above contract a game is defined where the agent has to decide to acquire or not pre-contractual information $\{\gamma, \neg\gamma\}$, to accept or not the contract $\{a', r'\}$, to share or not the information with the principal (if he acquired it and accepted the contract) $\{Sh, \neg Sh\}$, then to make a report to the court $\{h, l\}$ and finally, if the principal challenges the report, to accept or reject the challenge $\{a, r\}$. Principal's only participation in this game is to accept or challenge agent's report $\{A, C\}$.

The following (informally described) strategies with any beliefs the principal may have constitute a Perfect Bayesian Equilibrium for the above game:

- Agent's strategy:
 - The agent acquires information, accepts the contract, shares it and reports truthfully (l if $\beta = \underline{\beta}$, h if $\beta = \bar{\beta}$);
 - If the principal challenges the report after the agent reported truthfully, the agent rejects the challenge;
 - If the principal challenges the report after the agent reported falsely, he accepts the challenge;
 - If the agent does not acquire pre-contractual information, he still reports truthfully after learning his type.
- Principal's strategy:

- Accept if the agent reports l ;
- Accept if the agent shows him $\beta = \bar{\beta}$ and the agent reports h ;
- Challenge otherwise (that is when the agent reports h and he didn't show any proofs that $\beta = \bar{\beta}$).

C. Figures

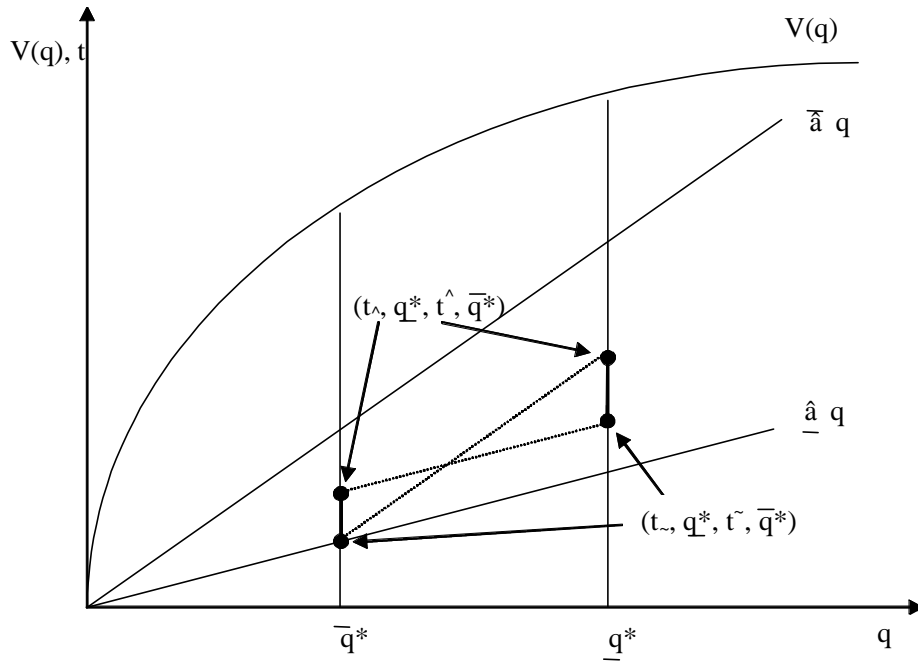


Figure C.1: Optimal Contracts when $\gamma = \infty$

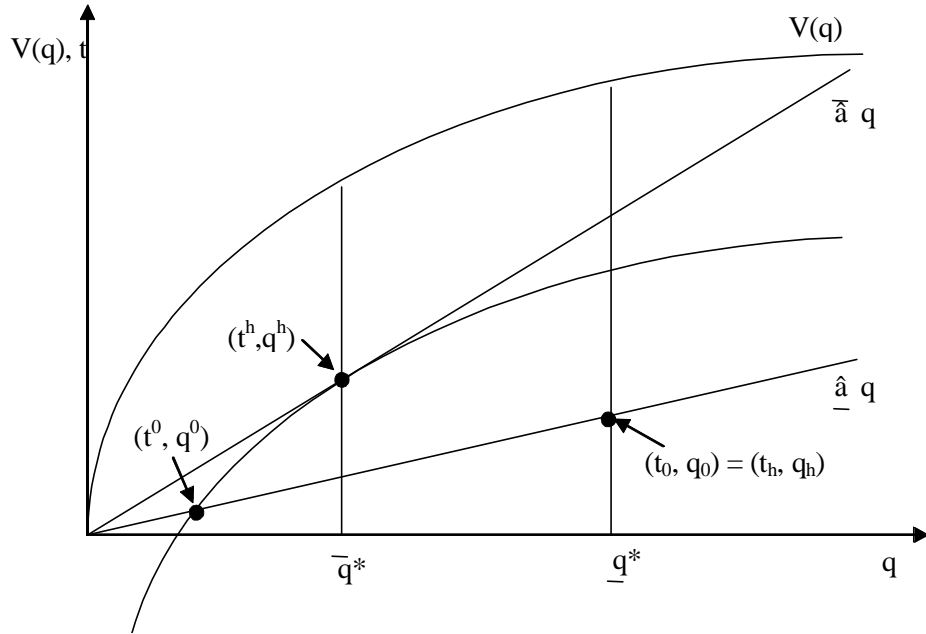


Figure C.2: The Optimal Contract when $\gamma = 0$

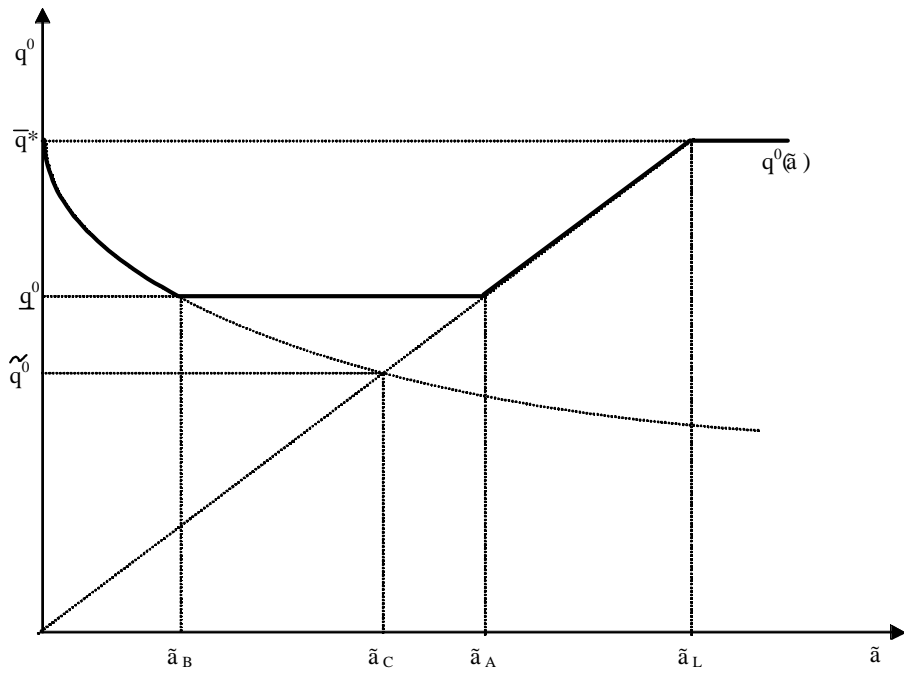


Figure C.3: Optimal q^0 - Case 1 ($\underline{q}^0 > \hat{q}^0$)

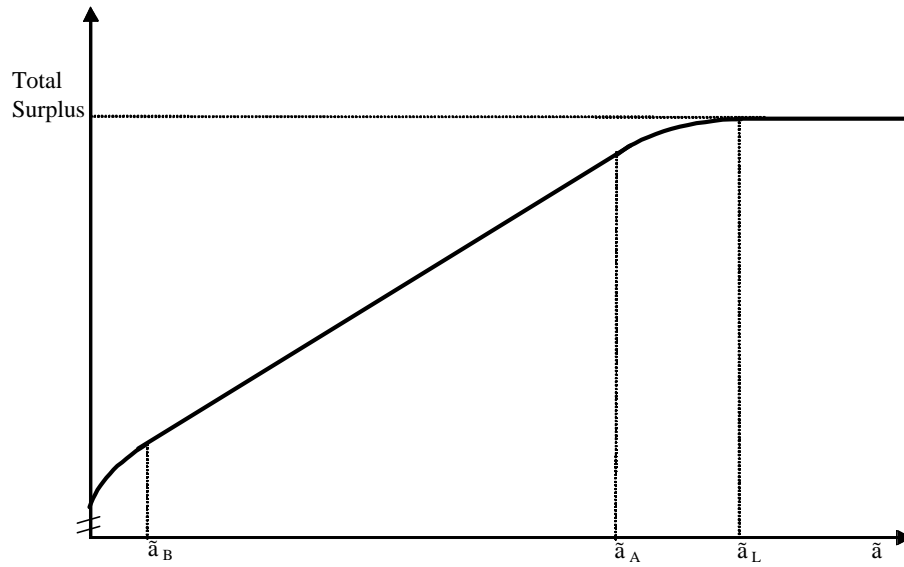


Figure C.6: Expected Profits when Inducing No Information Acquisition - Case 1 ($\underline{q}^0 > \tilde{q}^0$)

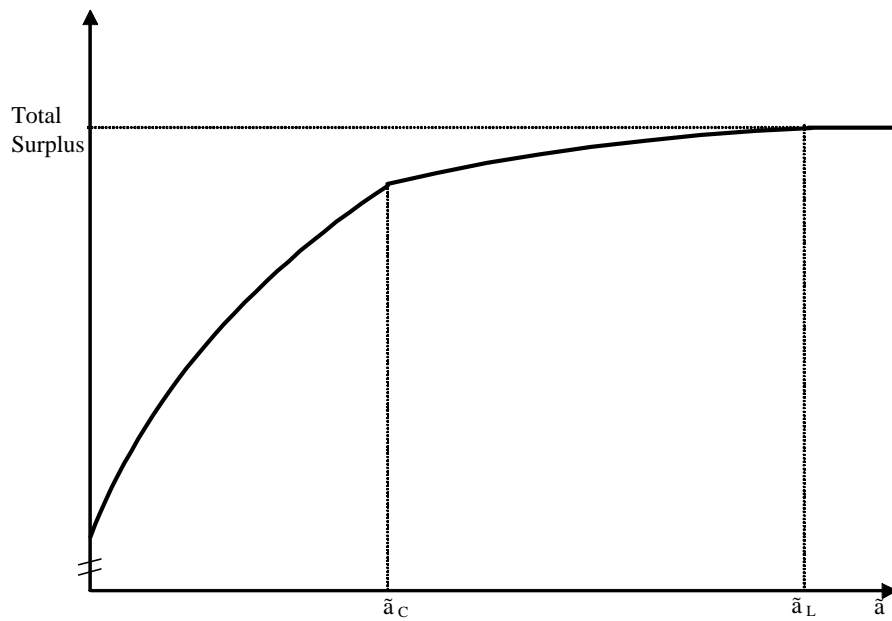


Figure C.7: Expected Profits when Inducing No Information Acquisition - Case 2 ($\underline{q}^0 \leq \tilde{q}^0$)

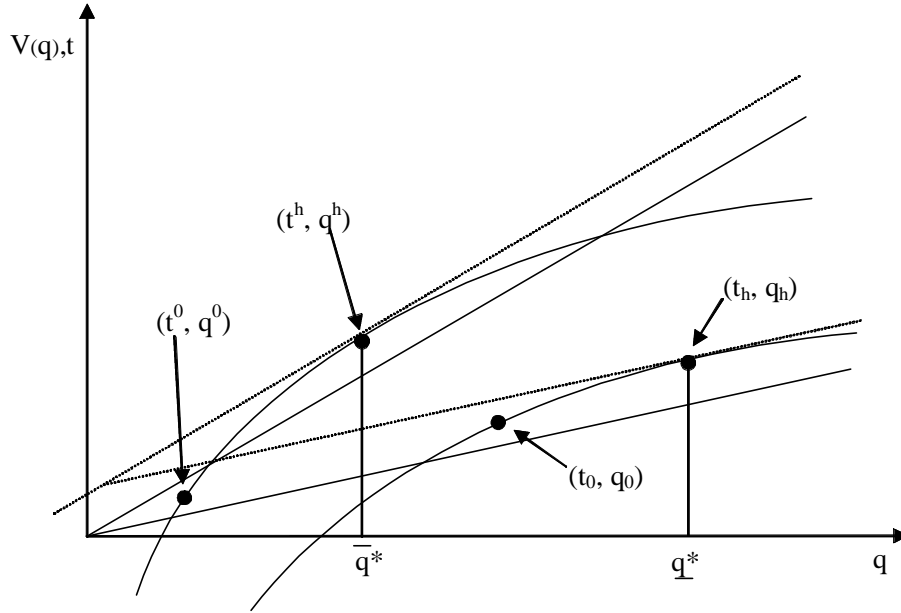


Figure C.8: Renegotiation with Hard Information

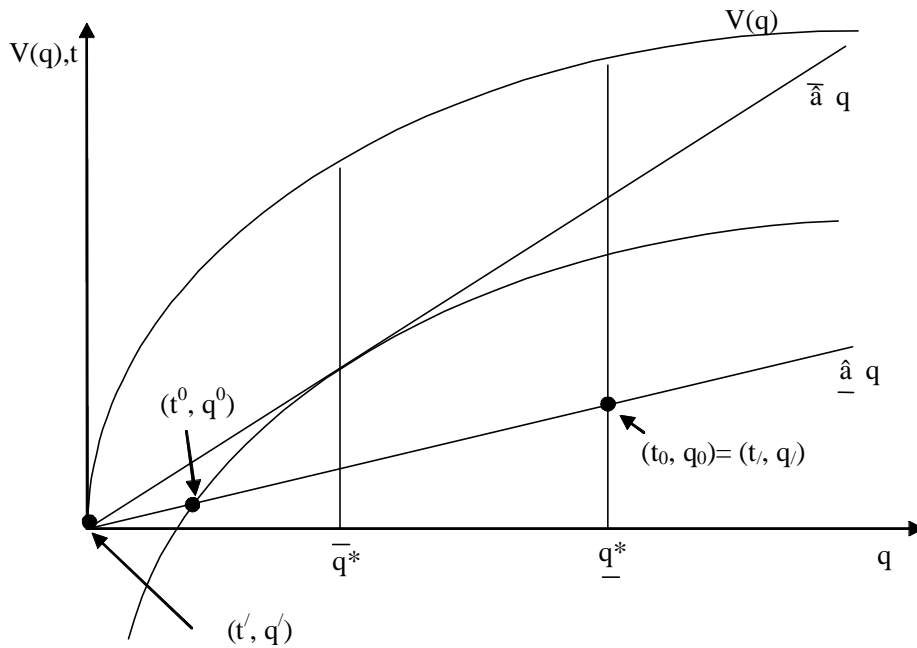


Figure C.9: Optimal Contract when Inducing Information Acquisition - (t_0, q_0, t^0, q^0) when $\gamma = 0$ and $(t/, q/, t', q')$ when $\gamma = (1 - \pi) [V(\bar{q}^*) - \bar{q}^* \bar{\beta}]$

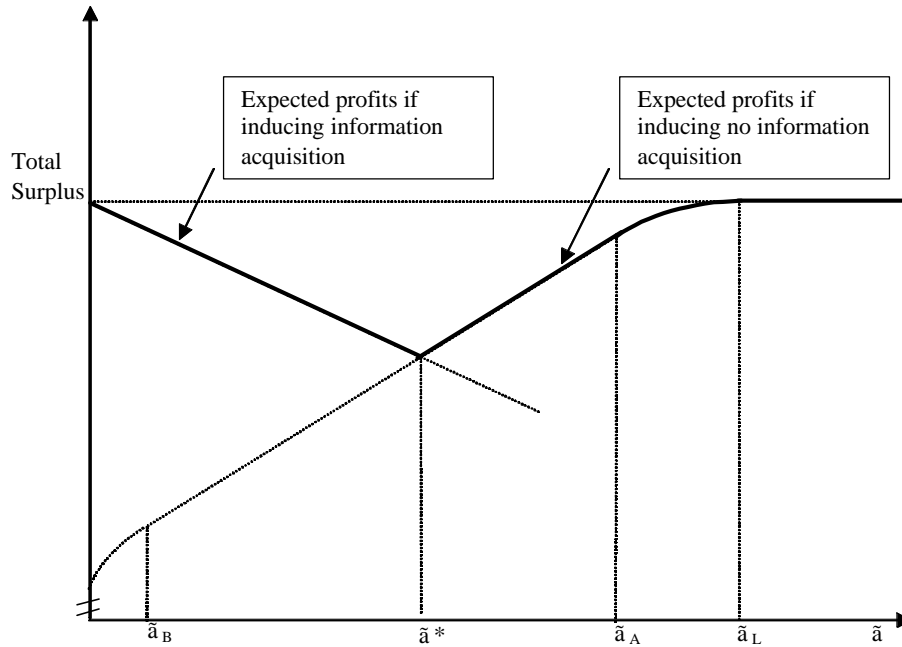


Figure C.10: Expected Profits and Critical γ - Case 1 ($\underline{q}^0 > \tilde{q}^0$)

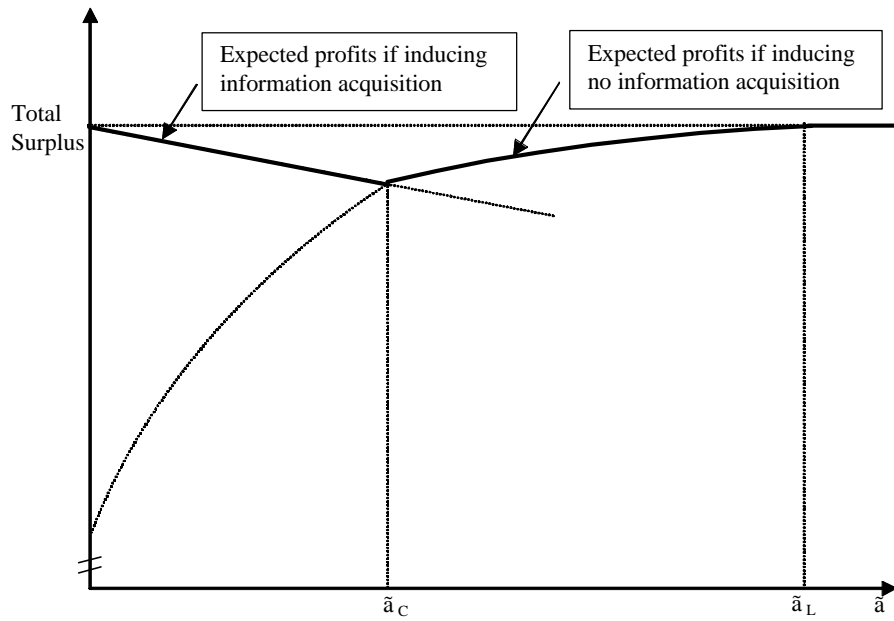


Figure C.11: Expected Profits and Critical γ - Case 2 ($\underline{q}^0 \leq \tilde{q}^0$)

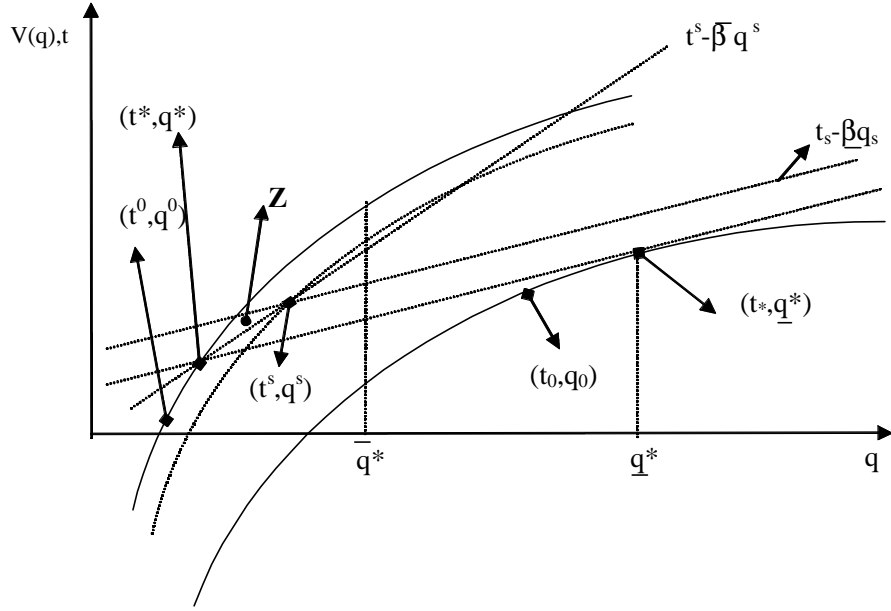


Figure C.12: Proof of Proposition 3 - $t_s - q_s \underline{\beta} > t_* - \underline{q}^* \underline{\beta} \Rightarrow V(q^s) - t^s > V(q^0) - t^0$ or the principal would reject the offer, but then a profitable deviation (point Z) exists for the type $\bar{\beta}$ agent. The Intuitive Criterion implies the principal assigns probability 1 to the agent being type $\bar{\beta}$ if she receives the ‘message’ Z instead of (t_s, q_s, t^s, q^s) . $(t_*, \underline{q}^*, t^*, q^*)$ is the unique equilibrium outcome that satisfies the Intuitive Criterion.

References

- Aghion, P., Dewatripont, M. and P. Rey. 1994. Renegotiation Design with Unverifiable Information. *Econometrica* 62, 257-282.
- Baron, D. and R. Myerson. 1982. Regulating a Monopolist with Unknown Costs. *Econometrica* 50, 911-930.
- Bergemann, D. and J. Valimaki. 2002. Information Acquisition and Efficient Mechanism Design. *Econometrica* 70, 1007-1033.
- Cho, I. and D. Kreps. 1987. Strategic Stability and Uniqueness in Signaling Games. *Quarterly Journal of Economics* 102, 381-413.
- Craswell, R. 1988. Pre-Contractual Investigation as an Optimal Precaution Problem. *Journal of Legal Studies* 17, 401-436.
- Crémer, J. 1995. Arm's Length Relationships. *Quarterly Journal of Economics* 110, 275-295.
- Crémer, J. and F. Khalil. 1992. Gathering Information Before Signing a Contract. *American Economic Review* 82, 566-578.
- Crémer, J., Khalil, F. and J. Rochet. 1998a. Strategic Information Gathering Before a Contract Is Offered. *Journal of Economic Theory* 81, 163-200.
- Crémer, J., Khalil, F. and J. Rochet. 1998b. Contracts and Productive Information Gathering. *Games and Economic Behavior* 25, 174-193.
- Dewatripont, M. and E. Maskin. 1995. Contractual Contingencies and Renegotiation. *Rand Journal of Economics* 26, 704-719.
- Hermalin, B. and M. Katz. 1991. Moral Hazard and Verifiability: The Effects of Renegotiation in Agency. *Econometrica* 59, 1735-1753.

- Kessler, A. 1998. The Value of Ignorance. *Rand Journal of Economics* 29, 339-354.
- Lewis, T. and D. Sappington. 1991. All or Nothing Information Control. *Economic Letters* 37, 111-113.
- Lewis, T. and D. Sappington. 1993. Ignorance in Agency Problems. *Journal of Economic Theory* 61, 169-183.
- Maskin, E. and J. Tirole. 1999. Unforeseen Contingencies and Incomplete Contracts. *Review of Economic Studies* 66, 83-114.
- Matthews, S. and N. Persico. 2001. Money-Back Warranties. Unpublished Manuscript.
- Sobel, J. 1993. Information Control in the Principal-Agent Problem. *International Economic Review* 34, 259-269.
- Tirole, J. 1986. Procurement and Renegotiation. *Journal of Political Economy* 94, 235-259.