Endogenous Verifiability and Optimality in Agency: A non-contingent approach

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Abstract

In the context of a principal-agent model where verification of an agent's effort is endogenously determined through strategic interactions between contracting parties, we derive a neccesary and suficient condition to achieve the first best with a non-contingent or incomplete contract.

These conditions relate the Principal's benefit, the Agent's cost, the probability of winning and the cost of litigation. Also, these conditions are found to be more general than the ones established in Ishiguro (2002) within a similar setup.

KEYWORDS: *incomplete contracts, endogenous verifiability, expectation damages.*

JEL CLASSIFICATION: D20, D86, K41, M52.

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1. Introduction

This paper presents a principal-agent model involving a moral hazard problem and endogenous verification of the agent's effort, a neccesary and suficient condition is derived to achieve the first best with a non-contingent contract.

Most agency models assume that the agent's effort is non-verifiable, and non-contractable as a consecuence. Others assume the contrary, that the agent's action is verifiable and is then contractable. In other words, they treat verifiability of the agent's effort as an exogenous variable. Other papers make verifiability contingent on some ex-ante variables, with the result that effort could be verifiable in some circunstances.

Unlike these papers, we model an environment where verification of an agent's effort is endogenously determined through strategic interactions between the contracting parties. An agent's effort can be observed perfectly by both parties. A third party, a court of law, is able to observe it just imperfectly. In case of breach, the affected party could make an additional effort, namely hire lawyers and collect evidence, in order to increase the probability that the court will be able to observe the agent's effort. However, this verification effort is inefficient because it does not add any value and could be avoided.

Ishiguro (2002) investigated the optimal contract design in a principal-agent setup where the agent could spend resources to verify its action in a court of law. Ishiguro derived a necessary and sufficent condition relating to the cost of going to court, the probability of verifying the action taken, and the cost of the optimal action under which the first best can be induced with a "three-step" wage scheme. Payment under this scheme is contingent on the level of effort, as it assigns a different wage level for each level of effort, therefore making it a complete contract. As a result the principal is the only one with incentives to breach the contract, through paying a lower wage than that established in the contract once the agent has chosen his level of effort.[1] In this setup, the agent is the only one that may want to go to court.

[1] This is so because Ishiguro (2002) additionally assumes that any payment is costlessly verifiable by court. We maintain this assumption.

We modified one angle assumption of Ishiguro's setup: we assume that the same verification technology available to the agent is also available to the principal. Our goal is then to derive a neccesary and suficient condition to achieve the first best with a non-contingent wage scheme that establishes one level of payment and effort. Once this an "incomplete contract" is signed, the incentive to breach the contract and go to court are reversed. Now, as the agent recieves a fixed salary he is the one with the incentive to exert a lower level of effort, and the principal could be ready to go to court. In this setup, we characterize when the first best can be implemented with a non-contingent contract, and show that this condition is weaker than Ishiguro's. That is, whenever the first best can be implemented with a contingent contract, it also can be implemented with a non-contingent contract, and the converse is not true.

Our result is in the same vein of Bernheim and Whinston (1998) and Willington (2003) where contracts are optimally incomplete. The rest of the paper is organized as follows: previous literature is reviewed in section two; the model is presented and solved in section tres; section four establishes the relation with Ishiguro (2002); and conclusions are presented in section five. Proofs missing from the text are including in the appendix.

2. Literature Review

In this section, we review two areas of the literature: incomplete contract theory and agency models with endogenous verifiability of agent effort. Regarding the first area, incomplete contract theory, the review is limited to studies relevant to our model.

Bernheim and Whinston (1998) remind us that in practice economic agents rarely write complete contracts in the sense of Arrow-Debreu. Furthermore, they point out that frequently contracts are excessively incomplete, meaning that variables which are verifiable, and therefore also contractible, are not taken into consideration. The reasons which are usually given for this are two fold: the existence of transaction costs,[2] and the limited rationality of agents.[3]

[2] Coase (1975) and Williamson (1975, 1985)

[3] Term first coined by Herbert Simon. In Simon (1991) he points out that most people are rational only in some actions, and emotional in the rest. As a result, their choices are not necessarily optimal but rather those that satisfy them.

Due to the existence of transaction costs agents do not specify all the states of nature in order to save transaction costs. In other words, incomplete contracts are written because it is very costly and/or impossible to forecast future scenarios. On the other hand, bounded rationality means that while agents are rational in their decision making, cognitive limits exist, namely, limitations to an agent's knowledge and/or capacity to calculate. As a result all the different states of nature are not necessarily distinguished, and the necessity to specify certain performance measures in contracts is not recognized.

Bernheim and Whinston present an additional explanation. According to them, contracts are excessively incomplete because it is optimal for the agents to sign incomplete contracts. This characteristic is called "strategic ambiguity", and exists when performance has both verifiable and non-verifiable aspects, with the result that it is optimal for the agents not to specify in their contracts some of the verifiable aspects given that they can not specify all of them. In the same vein, our model derives the conditions which make an incomplete contract optimal in a particular context of endogenous verifiability.

Willington (2003) makes a similar argument. It explores the role of a court, where it is costly to litigate, in the context of a hold-up problem with cooperative investment. The study shows that even in the extreme case that a court can not obtain any information regarding the actual investment level, a non-contingent contract can assist the parties with their implementation problem.[4]

Regarding the second area, the vast majority of existing agency models assume that certain variables are verifiable while others are not. Two main exceptions are proposed in law and economics literature, and Costly State Verification (CSV) models. The first set of models, even though these belong to another literature with distinct objectives and motivations [5], have modelled the litigation process as a rent-seeking game. Compared to these, our model is extremely simple as the decision of the court only depends on the resources that the affected party invested in the process [6].

[4]This result contrasts with the negative result of Che and Hausch (1999) who show that contracting has no value in the case of cooperative investment because no contract out performs the null contract.

[5]Such as analizing optimality of fee shifting rule and comparing judicial regimes (inquisitorial vs adversarial), amongst others.

[6]For example, in Bernardo et al. (2000) and Sanchirico (2000), court decision depends additionaly on past actions

The second exception to the above are the Costly State Verification (CSV) models which endogenize the verifiability of exogenous state variables.[7]. In contrast to this, the effort of the agent in our model is not a variable of state but an endogenous variable determined by the interaction between parties.

Kvaloy and Olsen (2004) analyze an agency model with ex-ante endogenous verifiability in a context of repeated interaction. This means that the verifiability of the agent's effort depends on the principal writing an explicit contract before the start of the relationship. In contrast to these authors, our study analyzes only one interaction and, more importantly, that verifiability is determined by the court following the breach of contract, which can only occur after commencment of the relationship.

Going one step further, Ishiguro (2002) [8] endogenizes the verifiability of the variable in an agency model, giving rise to the problem of moral hazard: the agent's effort. In this setup, the agent is able to go to court in case of breach[9]. Through exerting an additional effort during the trial, a verification effort such as hiring lawyers and collecting evidence, the agent increases the probability of the court discovering the breach. Ishiguro derived a necessary and sufficent condition relating to the cost of going to court, the probability of verifying the action taken, and the cost of the optimal action under which the first best can be induced with a "three-step" wage scheme. This wage scheme is a complete contract since payment is contingent on the level of effort, such that a different wage level is assigned for each level of effort.

Additionally, a couple of studies that analyze endogenous verifiability in the context of incomplete contracts, but with different goals. Zhang and Zhu (2000) show that even if a court can only imperfectly observe the actions of the parties, as long as the court is independent of those parties (ie. is not unduly influenced by the parties during the trial), it is still possible to achieve optimal results. In other words, courts do not need to perfectly observe the actions of the parties in order to

of the parties. In a car accident, someone that have not taken all precautions it is more likely to be found guilty.

[7] A generalization of these models could be found in Krasa and Villamil (2000).

[8] Another relevant work would be Yanagawa (1998) but we can not access to it.

[9] As mentioned, in Ishiguro's setup the principal is the only one with incentives to breach the contract, through paying a lower wage that the one established in the contract once the agent have chosen his level of effort.

reach optimal social outcomes. However achieving in an incomplete contract framework, this result is not surprising because even in the conventional principal-agent model with moral hazard, we only need the result to be correlated with the agent's effort to implement the first best.[10] Nonetheless, as we already saw Willington (2003) shows that even under the extreme case that a court can not obtain information regarding the actual investment level, a non-contingent contract can assist the parties with their implementation problem.

On the other hand, Usman (2002) introduces the problem of moral hazard in the conduct of the judge who has to exert effort in its ruling and whose actions are not observable. In this scenario, even though the cost of the effort for the judge is low, low levels of verifiability of parties actions are obtained.

3. The Model

Consider a simple agency model. A principal hire an agent to perform a task from which he obtains utility. In particular, the agent has to exert a level of effort $e \in [\underline{e}, \bar{e}]$ at a cost of $C(e)$, and will receive a transfer w from the principal. The principal gets a utility of $V(e)$.

We assume that both players: (S1) are risk neutral and (S2) have reserve utility equal to zero; therefore their payoffs are: $V(e) - t$ and $t - C(e)$. In addition, we futher assume that (S3):

$$V' > 0, \; V'' \leq 0$$
$$C' > 0, \; C'' > 0 \quad \text{with } C(0) = 0 \quad \text{and } V'(0) > C'(0),$$

In case of breach, the affected party can go to court, exert some "verification" effort $x \in [0, X^{\max}]$ so that the court will verify agent's effort with probability $P(x)$. The cost of going to court is $g(x)$ and we assume that (S4):

$$P' > 0, \; P'' < 0 \quad \text{with } P(0) = 0 \quad \text{and } P(X^{\max}) < 1,$$
$$g' > 0, \; g'' \geq 0 \quad \text{with } g(0) = 0$$

[10] Formally, we need that agent's effort improves the outcome in the sense of first order stochastic dominance. See Laffont and Martimort (2002), chapter 4.

Futher, we assume that (S5) the court only imposes expected damages in a case of breach. That is, if the court finds that one party has breached the contract, then this party has to make a monetary transfer to the other that exactly compensates the loss generated by the breach. The affected party will be as well off as if the contract were fully performed.[11] In terms of our variables, if the contract stipulated a level of effort E and the agent undertook a lower level $e < E$, then the court would impose damages for $\Delta = V(E) - V(e)$ if it discovers the breach.[12]

Complete and Incomplete Contracts

As noted earlier, the novelty in our model with respect to the existing literature is our assumption that (S6) the type of contract between principal and agent is a non-contingent one: it establishes one level of payment and, one level of effort. This means that even though ex-ante parties can specify whatever the level of effort $E \in [\underline{e}, \bar{e}]$ and an associated payment $w(E)$, once both parties have reached an agreement on a determined contract their values remains fixed (E, w).Without a loss of generality we also assume that $\underline{e} = 0$.

In addition, we also assume that (S7) the payment w received by the agent is verifiable by a court at no cost. Therefore, if the contract is just a pair (E, w),the principal would never pay $w' \neq w$.

On the other hand, the agent will always receive a fixed salary w, no matter his level of effort. Even more, in the case the principal takes him to court, his breach is only discovered with a probability $P(x)$.Consequently, the agent is the only party which holds an incentive to breach the contract and, unlike Ishiguro (2002), the principal is now the party who will seek to go to court.

Timing

The timing of the model is the following. In the first stage, the principal offers a contract (E, w) to the agent. If the agent accepts; the game passes to the second phase. In this the agent chooses his level of optimal effort based on the signed contract $e \in [\underline{e}, \bar{e}]$. In the third phase, the principal

[11] We are ruling out the posibility that the court imposes a very large penalty so that neither party would ever breach the contract. We are taking this assumption from Ishiguro's setup too (full penalty contracts are not allowed).

[12] In Ishiguro (2002), the expected damage is $\Delta = W - w$, where W is the contracted payment and $w \leq W$ is the one paid by the principal.

observes the level of effort of the Agent and, if this is different to that specified in the contract, the Principal decides whether or not to go to court. In the case that he or she decides to do so, the Principal chooses in addition its level of verification effort x.

Figure 1: Timing of the model

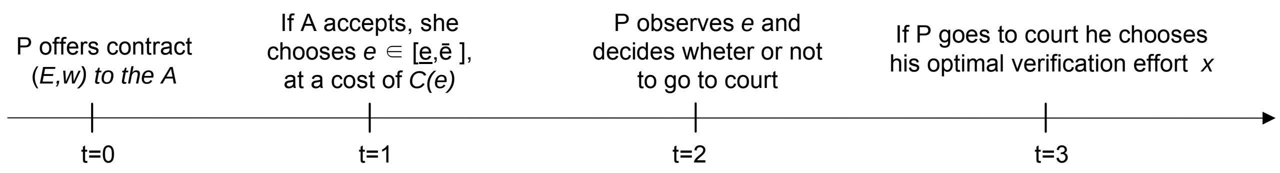
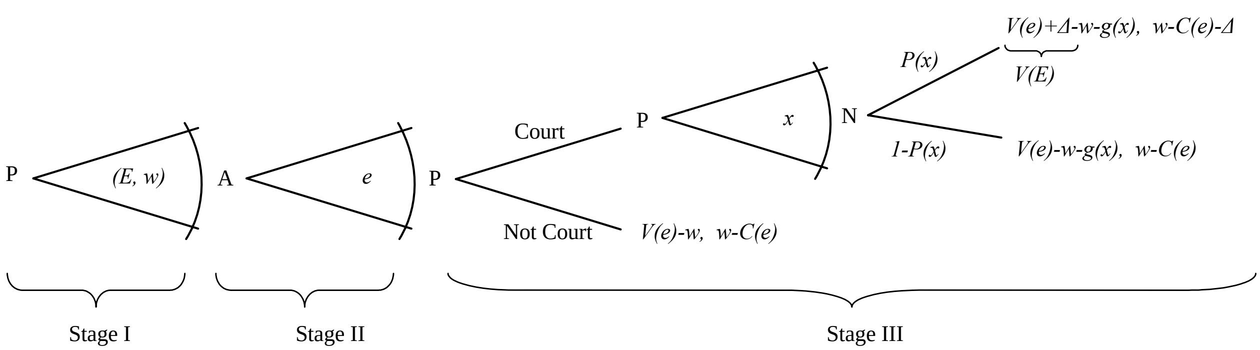


Figure 2: Estructure of the Game



4. Results

First Best

From a social point of view, any verification effort is inefficient as it provokes costs $g(x)$, which could be avoided. As a result, we define the first best as a situation in which the principal does not go to court and the benefits of the agent-principal relation are maximized. The first best level of effort is then:

$$\max_{e} \text{Principal benefit} + \text{Agent benefit} = [V(e) - W] + [W - C(e)] = V(e) - C(e)$$

$$e^{FB} : V'(e^{FB}) = C'\left(e^{FB}\right) \tag{1}$$

Inducible levels of effort through an incomplete contract

Next, we characterize the levels of effort which can be induced through an incomplete contract (e, w) such that the Principal does not go to court. We first focus in this and then characterize when the first best can be reached. Solving game by backwards induction, we can show:

Proposition 1.

Assume that $\Delta^* \equiv \min_{x \in X} \frac{g(x)}{P(x)}$ is well defined in $\Re_+$ and greater than cero. Let $\tilde{e}(\hat{e})$ be the solution of :

$$\min_{0 \le e \le \hat{e}} C(e) + \Delta(E(\hat{e}), e).P(x^*(\Delta(E(\hat{e}), e)))$$

Where:

$\Delta(E, e) = V(E) - V(e)$

$E(\hat{e})$ solves $V(E) = \Delta^* + V(\hat{e})$, and

$x^*(\Delta)$ solves $P'(x^*).\Delta = g'(x^*)$

We also define:

$$\hat{e}^{\max} = \begin{cases} \hat{e} : C(\hat{e}) = C(\tilde{e}(\hat{e})) + \Delta(E(\hat{e}), \tilde{e}(\hat{e})).P(x^*(\Delta(E(\hat{e}), \tilde{e}(\hat{e})))), \text{ if } E(\hat{e}) \le \bar{e} \\ \hat{e} : V(\bar{e}) - V(\hat{e}) = \Delta^*, \text{ otherwise} \end{cases}$$

Then, $\hat{e}$ can be induced without reaching the court stage with the contract $(E(\hat{e}), C(\hat{e}))$ if and only if $\hat{e} \leq \hat{e}^{\max}$.

Proof:

We will characterize the equilibria in which $\hat{e}$ can be induced.

Stage III

Given a contract (E, w) and an effort level chosen by the agent (e), the principal should decide his level of effort in the court. This effort will be successful with a probability of $P(x)$ and has a cost of $g(x)$.

This problem can be written as:

$$\max_{x \in X} P(x).\Delta(E, e) + V(e) - w - g(x)$$

Finding the first and second order conditions, we have:

$$FOC: \qquad P'(x^*).\Delta = g'(x^*) \tag{2}$$

$$\rightarrow x^* = x^*(\Delta) = x^*(\Delta(E, e))$$

$$SOC: \qquad P''(x^*).\Delta(E, e) - g''(x^*) < 0 \tag{3}$$

Given the assumptions on $g(.)$ and $P(.)$, x^* is uniquely determined. Note that, from (1) x^* is such that $\Delta = \dfrac{g'(x^*)}{P'(x^*)}$.Also, differentiating it, we can obtain:

$$P''(x^*).dx^*.\Delta + P'(x^*).d\Delta = g''(x^*)dx^*$$

$$P'(x^*).d\Delta = [g''(x^*) - P''(x^*)\Delta]dx^*$$

$$\frac{dx^*}{d\Delta} = x^{*\prime}(.) = \frac{P'(x^*)}{g''(x^*) - P''(x^*)\Delta} > 0 \tag{4}$$

That is, $x^*(\Delta)$ is increasing. The larger the agent's breach, $\Delta = V(E) - V(e)$, the larger the verification effort the principal will choose. Also, $x^*(\Delta)$ is decreasing in e because $\Delta(E, e)$ is decreasing in e .

Stage II

Given the contract (E, w) the agent chooses his effort level e. As he recieves a fixed salary he will always be tempted to choose $e < E$ and breach the contract. He has two "reasonable" alternatives: a)to choose the lowest e such that the principal would not go to court, or b) choose an even lower e that maximizes his payoff considering that the principal will go to court.

(a) $\hat{e}$ is the minimum level of effort such that the principal does not go to court:

Recall $\Delta(E(\hat{e}), \hat{e}) = \Delta^* = \min_{x \in X} \frac{g(x)}{P(x)} > 0$. We will show that $\hat{e}$ is the minimum level of effort that makes the principal choose not to sue the agent.

Lemma 1

The effort level $\hat{e}$ makes the principal indifferent between whether or not to sue the agent.

Proof:

Given $\Delta(E(\hat{e}), \hat{e}) = \Delta^*$.The principal's utility of not going to court is $V(\hat{e})$. On the other hand, the utility of doing so is:

$$V(\hat{e}) + \Delta(E(\hat{e}), \hat{e})P(x^*(\Delta(E(\hat{e}), \hat{e}))) - g(x^*(\Delta(E(\hat{e}), \hat{e})))$$

$$V(\hat{e}) + \Delta^* P(x^*(\Delta^*)) - g(x^*(\Delta^*))$$

Also, $\Delta^* = \min_{x \in X} \frac{g(x)}{P(x)} > 0$ that is:

$$\min_{x \in X} \frac{g(x)}{P(x)}$$

$FOC:$

$$\frac{g'(x^*)P(x^*) - P'(x^*)g(x^*)}{P^2} = 0$$

$$g'(x^*)P(x^*) - P'(x^*)g(x^*) = 0 \qquad \Rightarrow \frac{g'(x^*)}{P'(x^*)} = \frac{g(x^*)}{P(x^*)} = \Delta^* \tag{5}$$

Then, the principal's utility of going to court is also $V(\hat{e})$,because :

$$V(\hat{e}) + \left\{ \frac{g(x^*(\Delta^*))}{P(x^*(\Delta^*))} \right\} P(x^*(\Delta^*)) - g(x^*(\Delta^*)) = V(\hat{e})$$

In words, if the agent generates the breach $\Delta(E(\hat{e}), \hat{e}) = \Delta^*$ through exerting $\hat{e}$ instead of $E(\hat{e})$, the principal is indifferent wheter or not going to court. **LQQD.**

On the hand, given the contract (E, w), the effort level $e < E$ exerted by the agent, and its generated breach $\Delta = V(E) - V(e)$; the net profit of going to court of the principal $\Delta P(x^*(\Delta)) - g(x^*(\Delta))$ is decreasing in e.

$$\begin{aligned} \frac{d}{de}\left[\Delta P(x^*(\Delta)) - g(x^*(\Delta))\right] = \quad & -V'(e)\left[\Delta P'(.)x^{*\prime}(.)) + P(.) - g'(.)x^{*\prime}(.)\right] \\ & -V'(e)\left[P(.) + x^{*\prime}(.)\left\{\Delta P'(.) - g'(.)\right\}\right] \\ & -V'(e)P(.) < 0 \end{aligned}$$

As by Lemma 1 the net profit of going to court of the principal with $\hat{e}$ is zero, then if $\tilde{e} < \hat{e}$ his net profit will be positive so that he will always choose to go to court.

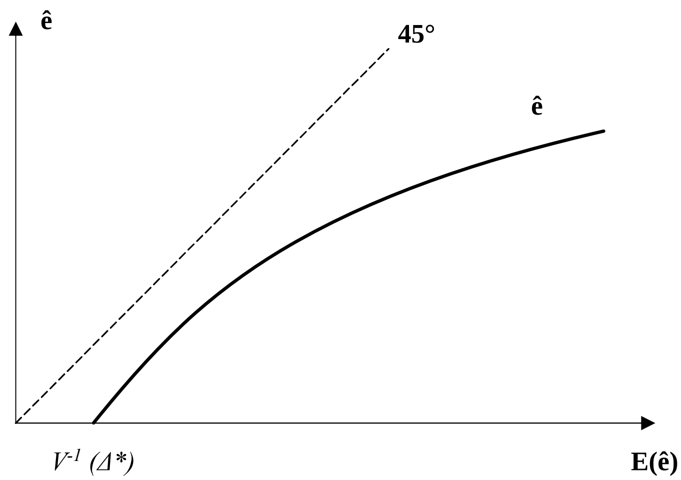
Even more if $\Delta^* > 0 \Rightarrow V(E(\hat{e})) - V(\hat{e}) > 0 \Rightarrow V(E(\hat{e})) > V(\hat{e}) \Rightarrow E(\hat{e}) > \hat{e}$. That is, $\hat{e}$ is always lower than $E(\hat{e})$.Also, differentiating $\Delta^* = \Delta(E(\hat{e}), \hat{e})$, we obtain:

$$0 = V'(E(\hat{e}))dE - V'(\hat{e})d\hat{e}$$

$$\Rightarrow \frac{dE}{d\hat{e}} = \frac{V'(\hat{e})}{V'(E)} > 0, \text{ and } \frac{d^2E}{d\hat{e}^2} = \frac{V''(\hat{e})}{V'(E)} < 0.$$

In sum, in the $\hat{e} - E(\hat{e})$ space $\hat{e}$ has a form similar to:

Figure 3: Level of effort $\hat{e}$ that can be induced given the contract $(E(\hat{e}), w)$



(b) Exerting $\hat{e}$ the agent gets a benefit higher than any other that he could get in court exerting a lower effort e :

Given the contract $(E(\hat{e}), w)$ let's suppose the agent exerts an effort lower than the one makes the principal indifferent toward going or not to court $e < \hat{e}$. The agent would choose this e so that his expected utility of going to court will be maximized.[13]

Let $\tilde{e}$ and $\Delta = \Delta(E(\hat{e}), \tilde{e}(E(\hat{e})))$ be this lower level of effort and the breach it generates, this $\tilde{e}$ must solve:

[13] We assume $C(e)$ is concave enough so that the solution to this problem is characterized by the first order condition.

$$\max_{0\leq e\leq \hat{e}} P(x^*(\Delta(E(\hat{e}),e)).\{w - C(e) - \Delta(E(\hat{e}),e)\} + [1 - P(x^*(\Delta(E(\hat{e}),e)))].\{w - C(e)\}$$

$$\Leftrightarrow$$

$$\min_{0\leq e\leq \hat{e}} C(e) + \Delta(E(\hat{e}),e)P(x^*(\Delta(E(\hat{e}),e)))$$

FOC:

$$C'(\tilde{e}) - V'(\tilde{e})[P'(\ x^*(\Delta(E(\hat{e}),\tilde{e})))x^{*\prime}(\Delta(E(\hat{e}),\tilde{e}))\Delta(E(\hat{e}),\tilde{e}) + P(x^*(\Delta(E(\hat{e}),\tilde{e})))] = 0 \tag{6}$$

$$\frac{C'(\tilde{e})}{V'(\tilde{e})} = [P'(\ x^*(\Delta(E(\hat{e}),\tilde{e})))x^{*\prime}(\Delta(E(\hat{e}),\tilde{e}))\Delta(E(\hat{e}),\tilde{e}) + P(x^*(\Delta(E(\hat{e}),\tilde{e})))] = [.]$$

$$\Rightarrow \tilde{e} = \tilde{e}(E(\hat{e}))$$

SOC:

$$C''(\tilde{e}) - V''(\tilde{e})[.] - V'(\tilde{e})\{\frac{d\Delta}{d\tilde{e}}P'(.)x^{*\prime}(.) + \Delta x^{*\prime}(.)P''(.)x^{*\prime}(.)\frac{d\Delta}{d\tilde{e}} \\ +\Delta P'(.)x^{*\prime\prime}(.)\frac{d\Delta}{d\tilde{e}} + P'(.)x^{*\prime}(.)\frac{d\Delta}{d\tilde{e}}\} \geq 0$$

$$C''(\tilde{e}) - V''(\tilde{e})\frac{C'(\tilde{e})}{V'(\tilde{e})} + V'(\tilde{e})^2\left\{2P'(.)x^{*\prime}(.) + \Delta x^{*\prime}(.)^2P''(.) + \Delta P'(.)x^{*\prime\prime}(.)\right\} \geq 0$$

Equation (6) provides us with the optimal breach $\tilde{e}(E(\hat{e})) < \hat{e}$ given the contract $(E(\hat{e}),w)$. Notice also that $\Delta > \Delta^*$ and $d\Delta = V'(E(\hat{e}))dE - V'(\tilde{e})d\tilde{e}$. Differentiating (6) we can have obtain $\frac{d\tilde{e}}{dE}$:

$$C''(\tilde{e})d\tilde{e} - V''(\tilde{e})d\tilde{e}[.] - V'(\tilde{e})d\Delta\left\{P'(.)x^{*\prime}(.) + \Delta x^{*\prime}(.)P''(.)x^{*\prime}(.) + \Delta P'(.)x^{*\prime\prime}(.) + P'(.)x^{*\prime}(.)\right\} = 0$$

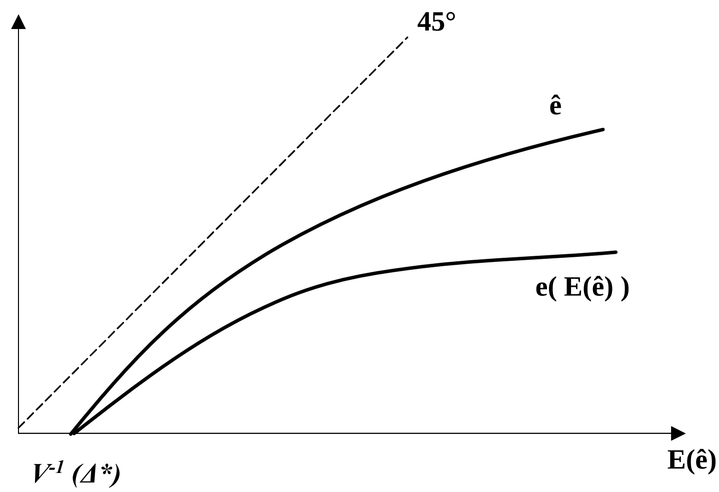
$$d\tilde{e}\left[C''(\tilde{e}) - V''(\tilde{e})\frac{C'(\tilde{e})}{V'(\tilde{e})}\right] - V'(\tilde{e})d\Delta\left\{2P'(.)x^{*\prime}(.) + \Delta x^{*\prime}(.)P''(.)x^{*\prime}(.) + \Delta P'(.)x^{*\prime\prime}(.)\right\} = 0$$

$$d\tilde{e}\left[C''(\tilde{e}) - V''(\tilde{e})\frac{C'(\tilde{e})}{V'(\tilde{e})} + V'(\tilde{e})^2\left\{.\right\}\right] - V'(\tilde{e})V'(E(\hat{e}))\left\{.\right\}dE = 0$$

$$\frac{d\tilde{e}}{dE} = \frac{V'(\tilde{e})V'(E(\hat{e}))\left\{.\right\}}{C''(\tilde{e}) - V''(\tilde{e})\frac{C'(\tilde{e})}{V'(\tilde{e})} + V'(\tilde{e})^2\left\{.\right\}}$$

Analogously to the case of $\hat{e}$, we can plot $\tilde{e}(E(\hat{e}))$. This is immediate because of $\tilde{e}(E(\hat{e})) < \hat{e}$ by definition. Therefore, $\tilde{e}(E(\hat{e}))$ could be similar to:

Figure 4: Optimal breach $\tilde{e}(E(\hat{e})) < \hat{e}$ **given the contract** $(E(\hat{e}), w)$



Notice also that:

$\Delta^* > 0 \Rightarrow \Delta^*.P(x^*(\Delta^*)) > 0 \Rightarrow \Delta.P(x^*(\Delta)) > 0$

In words, as $\hat{e}$ leaves the principal indiferent toward going or not to court, whichever lower level of effort $\tilde{e}(E(\hat{e})) < \hat{e}$ exerted by the agent will be less costly $C(\tilde{e}(E(\hat{e}))) < C(\hat{e})$ but it will also generate him an expected loss of $\Delta.P(x^*(\Delta) > 0$ because the principal will go to court.

The agent will choose $\hat{e}$ instead of $\tilde{e}(E(\hat{e}))$, if and only if:

$$\begin{aligned} w - C(\hat{e}) &\geq w - C(\tilde{e}(E(\hat{e}))) - \Delta.P(x^*(\Delta)) \\ C(\tilde{e}(E(\hat{e}))) + \Delta.P(x^*(\Delta)) &\geq C(\hat{e}) \\ \Delta.P(x^*(\Delta)) &\geq C(\hat{e}) - C(\tilde{e}(E(\hat{e}))) \end{aligned} \qquad (b)$$

Note also that if $\hat{e} = 0 \Rightarrow \tilde{e}(E(0)) = 0$ and . Replacing these values into (7):

$\Delta(E(0), 0).P(x^*(\Delta(E(0), 0))) > C(0) - C(0)$

That is, it is always posible to induce the null level of effort $\hat{e}=0$ with the contract $(E(0),0)$.[14] By continuity, it is possible to increase $\hat{e}$ holding (b). On the other hand, as $\hat{e}$ increases the right hand side (RHS) grows at a higher rate than the left hand side (LHS):

$$\begin{aligned}
\frac{d}{d\hat{e}}(LHS) &= \frac{d}{d\hat{e}}\left[\Delta P(x^*(\Delta))\right] \\
&= \Delta . P'(.)x^{*\prime}(.)\frac{d\Delta}{d\hat{e}} + P(.)\frac{d\Delta}{d\hat{e}} \\
&= \frac{d\Delta}{d\hat{e}}\left[\Delta P'(.)x^{*\prime}(.)) + P(.)\right] < 0
\end{aligned}$$

Since $\dfrac{d\Delta}{d\hat{e}} = V'(E(\hat{e})) - V'(\tilde{e}(E(\hat{e}))) < 0$ because $V_{\rm J} < 0$

On the other hand, $\dfrac{d}{d\hat{e}}(RHS)$ is given by:

$$\begin{aligned}
\frac{d}{d\hat{e}}(RHS) &= \frac{d}{d\hat{e}}\left[C(\hat{e}) - C(\tilde{e}(E(\hat{e})))\right] \\
&= C'(\hat{e}) - C'(\tilde{e}(E(\hat{e})))\frac{d\tilde{e}}{dE}\frac{dE}{d\hat{e}} \\
&= C'(\hat{e}) - C'(\tilde{e}(E(\hat{e})))\left\{\frac{V'(\tilde{e})V'(E(\hat{e}))\{.\}}{C''(\tilde{e}) - V''(\tilde{e})\dfrac{C'(\tilde{e})}{V'(\tilde{e})} + V'(\tilde{e})^2\{.\}}\frac{V'(\hat{e})}{V'(E(\hat{e}))}\right\} \\
&= C'(\hat{e}) - C'(\tilde{e}(E(\hat{e})))\left\{\frac{V'(\tilde{e})^2\{.\}}{C''(\tilde{e}) - V''(\tilde{e})\dfrac{C'(\tilde{e})}{V'(\tilde{e})} + V'(\tilde{e})^2\{.\}}\right\}
\end{aligned}$$

As $\left\{\dfrac{V'(\tilde{e})^2\{.\}}{C''(\tilde{e}) - V''(\tilde{e})\dfrac{C'(\tilde{e})}{V'(\tilde{e})} + V'(\tilde{e})^2\{.\}}\right\} < 1 \Rightarrow \dfrac{d}{d\hat{e}}(RHS) > 0$

[14] Notice that $\hat{e}=0$ also implies $\Delta^* = \Delta = V(E(0))$.Therefore $E(0) = V^{-1}(\Delta^*)$.

In sum, the difference among RHS and LHS is reduced as $\hat{e}$ increases. If they would become equal we have that for some $\hat{e}^{\max}$:

$$C(\hat{e}^{\max}) = C(\tilde{e}(E(\hat{e}^{\max}))) + \Delta(E(\hat{e}^{\max}), \tilde{e}(E(\hat{e}^{\max}))).P(x^*(\Delta(E(\hat{e}^{\max}), \tilde{e}(E(\hat{e}^{\max})))$$

On the hand, if $E(\hat{e}^{\max}) > \bar{e}$ they would not equalize and (b) would stay as a strict inequality. As a result, $\hat{e}$ could only increases until $E(\hat{e}^{\max}) = \bar{e}$ so that $\hat{e}^{\max}$ solves: $V(\bar{e}) - V(\hat{e}^{\max}) = \Delta^*$.

LQQD.

We next characterize the conditions for e^{FB} to be implemented. That is, for which functions $V, C, g,$y P it will be true that $e^{FB} \leq e^{\max}$.

Corollary 1

If functions $V, C, g,$y P are differentiable and:

$$(i) \quad \text{There is } \Delta^* \in \Re \text{ such that } \min_{x \in X} \frac{g(x)}{P(x)} = \Delta^* > 0, \quad \text{and}$$

$$(ii) \quad P\left(x^*(\Delta)\right).\Delta \geq C\left(e^{FB}\right) - C(e),$$

Where: $\Delta = V(E(e^{FB})) - V(e)$ for all $\underline{e} \leq e \leq e^{FB}$

The first best can be reached with the non-contigent contract $(E(e^{FB}), C(e^{FB}))$:

Proof:

Given the contract $(E(e^{FB}), C(e^{FB}))$, the agent chooses $e = e^{FB}$ in the second stage because choosing $e \leq e^{FB}$ is more costly, as $(ii)\, P\left(x^*(\Delta)\right).\Delta \geq C\left(e^{FB}\right) - C(e)$ holds.

In the next stage, given that $\Delta = V(E) - V(e^{FB}) = \Delta^*$ the principal chooses not to go to court. **LQQD.**

5. Relation with Ishiguro's Condition

Ishiguro (2002) shows under which conditions the first best can be reached with a three-step wage scheme, a contingent contract. This conditions are called "Condition N" by the autor:

Condition N:

The first best outcome is implementable if and only if there exists $\Delta_I = (w^* - \underline{w}) \in \Re$ and $x^* \in X^* (\Delta_I)$ such that:

$$(i) \quad \psi \geq (1 - P(x^*)) \Delta_I, \text{ and}$$
$$(ii) \quad P(x^*) \Delta_I \geq C\left(e^{FB}\right) - C(\underline{e}),$$

where:

$$\begin{aligned} & \underline{e} \text{ is the lowest level of effort possible} \\ x^* \quad \in & \quad X^*(\Delta_I) \equiv \underset{x \in X}{\arg\max} P(x) \Delta_I - g(x), \text{and} \\ \psi \quad \equiv & \quad \inf_{x \in X \backslash \{0\}} \frac{g(x)}{P(x)} \end{aligned}$$

If this condition holds, it can be shown that exist payments $\underline{w} \leq \bar{w} \leq w^*$ and a effort level $\widehat{e} \in [\underline{e}, e^{FB})$ such that first best outcome can be implemented by the contract:

$$w(e) = \begin{cases} w^* & \text{if } e \geq e^{FB} \\ \overline{w} & \text{if } e \in [\widehat{e}, e^{FB}) \\ \underline{w} & \text{if } e \in [\underline{e}, \widehat{e}) \end{cases}$$

Next, we compare Condition N with the one we obtained previously, corollary 1.

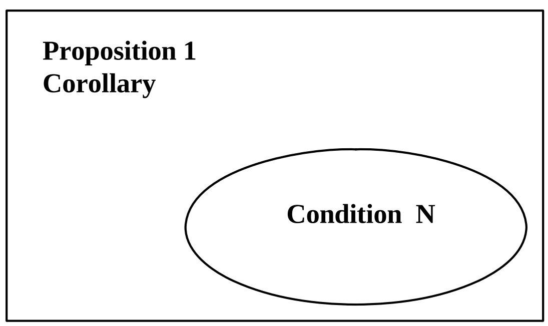
Proposition 2.

Suppose that the same verification technology is available to the principal and the functions V, C, g,y P are differentiable. If condition N holds then corollary of proposition 1 also holds. The converse is not true.

Proof: See Appendix.

That is, as both parties can go to court and the primitive functions are differentiable; then Ishiguros's Condition N implies corollary of proposition 1. In words, under the conditions stated if the first best can be implemented by a contigent contract it could also be implemented by a non-contigent contract.

Figure 5: Relation with Ishiguro's Condition



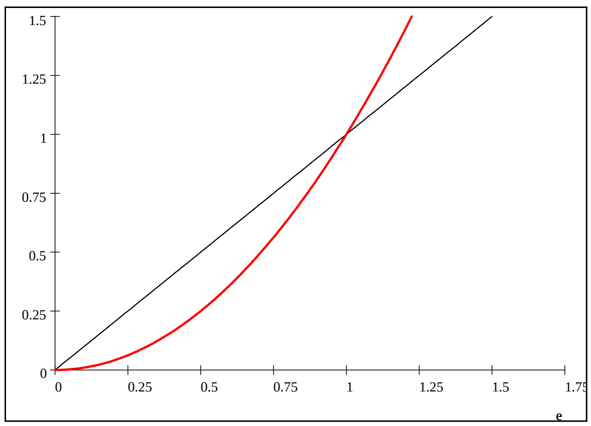
6. Numerical Examples

Consider the following functions[15]:

$$V(e) = e \quad P(x) = 1 - e^{-x}$$
$$C(e) = e^2 \quad g(x) = 0.05 + x$$

Note that $V' > 0$, $V'' = 0$, $C' > 0, C'' > 0, C(0) = 0$, and $V'(0) > C'(0)$.

Figure 6: Benefit $V(e)$ and cost $C(e)$ functions for the principal and agent

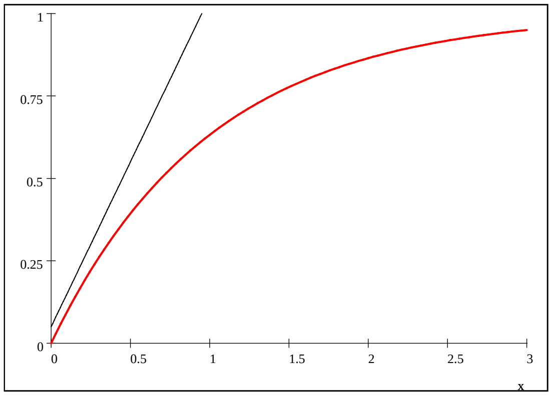


On the other hand, the minimum of $\dfrac{g(x)}{P(x)}$ is well defined and positive. Consequently, condition (i) of corollary 1 is met as well.

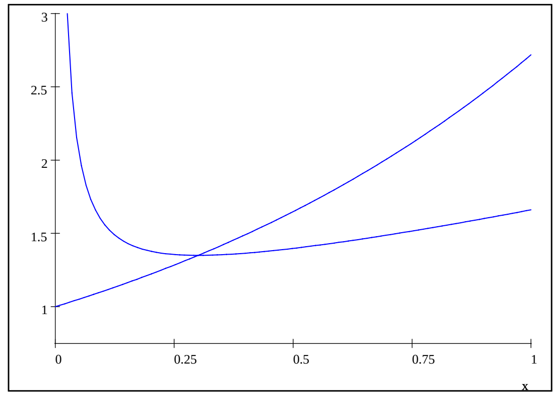
[15] Notice that for these functions, the second order condition of the next optimization problem also holds:
$$\min_{0 \le e \le \hat{e}} C(e) + \Delta(E(\hat{e}), e) P(x^*(\Delta(E(\hat{e}), e)))$$

Because: $\dfrac{d^2}{de^2}\left[C(e) + \Delta(E(\hat{e}), e) P(x^*(\Delta(E(\hat{e}), e)))\right] > 0$

Figure 7: Probability $P(x)$ and Cost $g(x)$ functions in court



$P(x)$ and $g(x)$



$\dfrac{g(x)}{P(x)}$ and $\dfrac{g'(x)}{P'(x)}$

Analytically: $\quad \min\limits_{x\,\in\,X} \dfrac{g(x)}{P(x)} \Rightarrow \Delta^* = \dfrac{1}{e^{-x^*}} = \dfrac{0.05 + x^*}{1 - e^{-x^*}}$

$\Rightarrow e^{x^*} - x^* - 1.05 = 0$

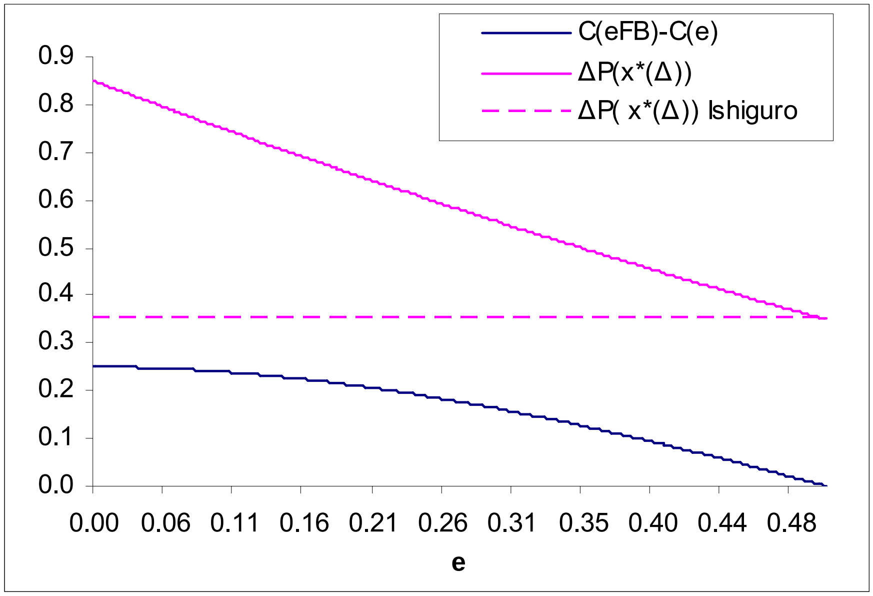
$\Rightarrow x^* = 0.3$ y $\Delta^* = \dfrac{1}{e^{-0.3}} = 1.35 > 0$

ii) FB: $V'(e^{FB}) = C'(e^{FB}) \Rightarrow e^{FB} = 0.5$

$\Rightarrow E(e^{FB}) = \Delta^* + e^{FB} = 1.35 + 0.5 = 1.85$

Notice that $\Delta = 1.85 - e$,we can plot $P\,(\,x^*(\Delta))\,.\Delta$ and $C\,(0.5) - C(e)$ for all $0 \leq e \leq 0.5$.

Figure 8: Expected loss in court $P\left(\, x^{*}(\Delta)\right).\Delta$ and gains of breach $C\left(0.5\right)-C(e)$ for the agent



We see that corollary 1 (ii) holds because $P\left(\, x^{*}(\Delta)\right).\Delta \geq C\left(0.5\right)-C(e)$ for all $0 \leq e \leq 0.5$. In particular, the non-contingent contract that implements the first best $e^{FB}=0.5$ is:

$$(E(e^{FB}), C(e^{FB})) = (1.35+0.5,\ (0.5)^2) = (1.85,\ 0.25).$$

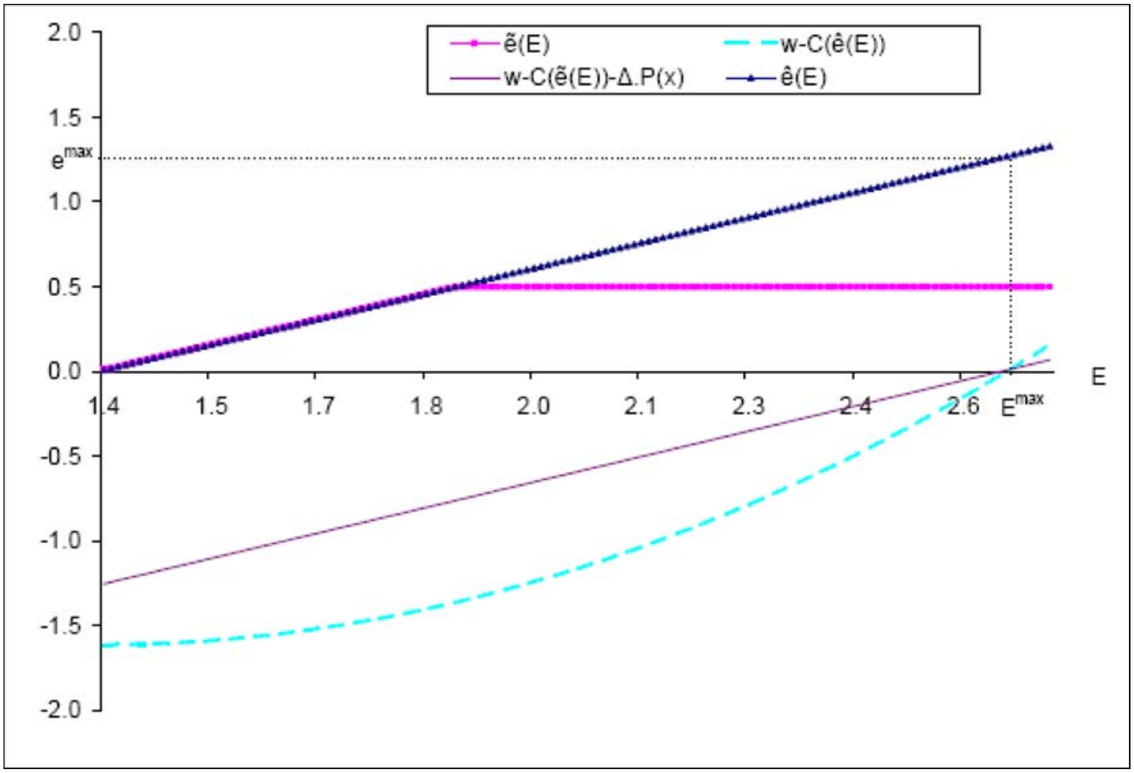
On the other hand, Condition N also holds. In this case $\psi = \inf\limits_{x\, \in\, X\setminus\{0\}} \dfrac{g\left(x\right)}{P\left(x\right)} = \min\limits_{x\, \in\, X} \dfrac{g(x)}{P(x)} = \Delta_I = \Delta^{*} = 1.35,\ x^{*}=0.3$,and:

(i) $\Delta_I \geq \left(1-P\left(x^{*}\right)\right)\Delta_I,$ holds because $P\left(x^{*}\right) \leq 1$, and

(ii) $P\left(x^{*}\right)\Delta_I = 0.35 \geq C\left(e^{FB}\right) - C(0) = 0.25$

In addition to that, we can draw the utility the agent can get exerting $\hat{e}$ and $\tilde{e}(E(\hat{e}))$ to calculate the maximum level of effort that can be induced is $\hat{e}^{\max}$. In the first quadrant we plot these effort levels and their associated payments (in negative) in the fourth, $w-C(\hat{e})$ and $w-C(\tilde{e}(E(\hat{e}))) - \Delta(E(\hat{e}), \tilde{e}(E(\hat{e}))).P(x^{*}(\Delta(E(\hat{e}), \tilde{e}(E(\hat{e})))))$ respectively.

Figure 9: Maximum level of effort $\hat{e}^{\max}$ that can be induced with the contract $(E(\hat{e}), w)$



The maximum level of effort that can be induced is $\hat{e}^{\max} = 1.27$ and is obtained with the contract $(E^{\max}, C(\hat{e}^{\max})) = (2.63, 1.61)$.

Next, we will present other group of functions such that Condition N does not hold but the corollary of proposition 1 does.[16]

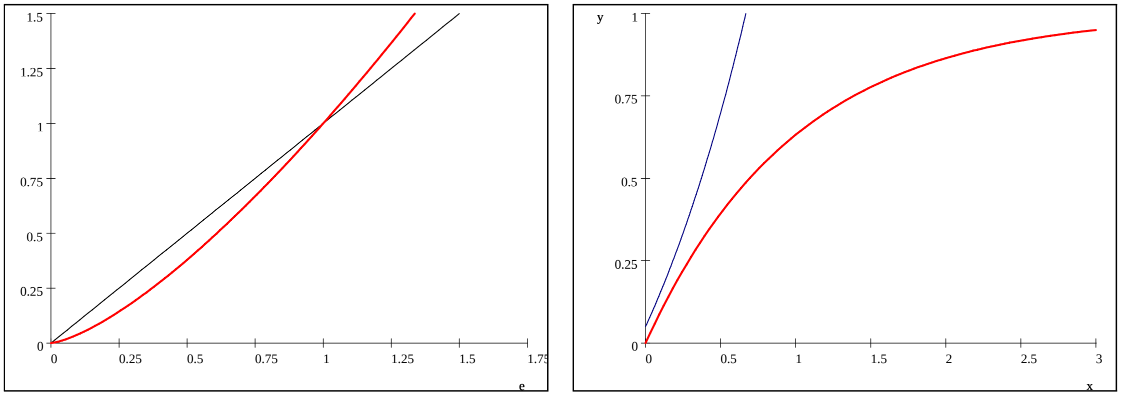
$$V(e) = e \qquad P(x) = 1 - e^{-x}$$

$$C(e) = e^{1.4} \quad g(x) = e^{x} - 0.95$$

[16] Notice that for these functions, the second order condition of the next optimization problem also holds: $\min_{0 \le e \le \hat{e}} C(e) + \Delta(E(\hat{e}), e) P(x^*(\Delta(E(\hat{e}), e)))$

Because: $\dfrac{d^2}{de^2}\left[C(e) + \Delta(E(\hat{e}), e) P(x^*(\Delta(E(\hat{e}), e)))\right] > 0$

Figure 10: Functions V, C, P and g that do not meet Condition N.

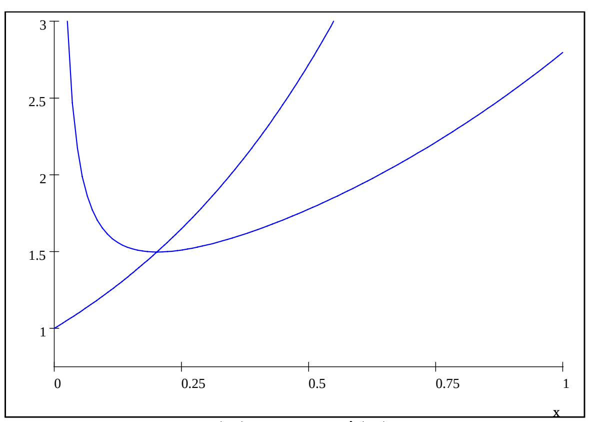


$V(e)$ and $C(e)$ $\qquad$ $P(x)$ and $g(x)$

All are differentiable and meet (S3), (S4), (i) and (ii):

$i)$ Ploting $\dfrac{g(x)}{P(x)} = \dfrac{e^x - 0.95}{1 - e^{-x}}$ we can see that Δ^*:

Figure 11: Minimum of $\dfrac{g(x)}{P(x)}$



$\dfrac{g(x)}{P(x)}$ and $\dfrac{g'(x)}{P'(x)}$

Analitically: $\displaystyle \min_{x \,\in\, X} \frac{g(x)}{P(x)} \quad \Rightarrow \Delta^* = \frac{g'(x^*)}{P'(x^*)} = \frac{g(x^*)}{P(x^*)}$

$$\Rightarrow \frac{e^{x^*}}{e^{-x^*}} = \frac{e^{x^*} - 0.95}{1 - e^{-x^*}}$$

$\Rightarrow e^{2x^*} - 2e^{x^*} + 0.95 = 0$

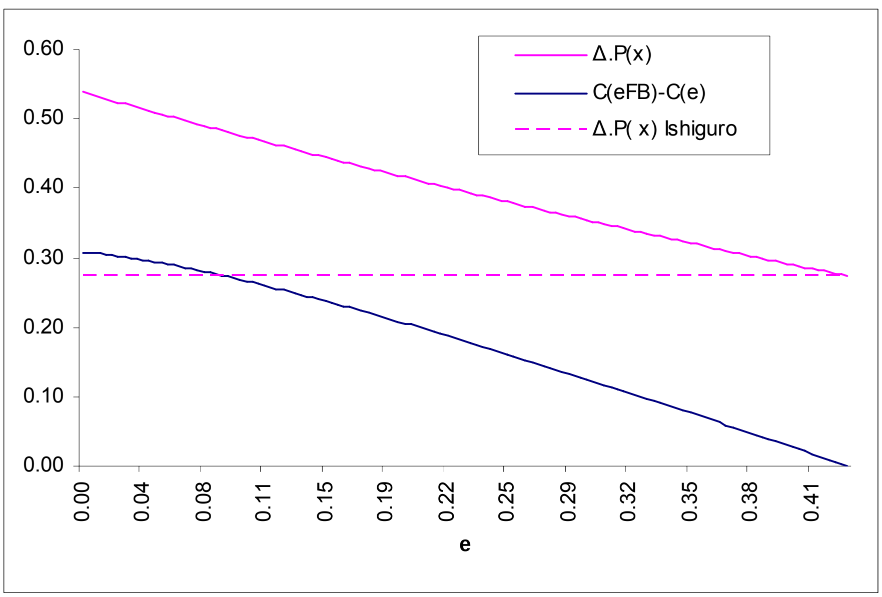
$\Rightarrow x^* = 0.201\,8$ y $\Delta^* = e^{2*0.2018} = 1.\,497\,2$

$ii)$ FB: $V'(e^{FB}) = C'(e^{FB}) \Rightarrow e^{FB} = (\frac{1}{1.4})^{1/0.4} = 0.431\,2$

As $E(e^{FB})$ solves: $\Delta^* = V(E(e^{FB})) - V(e^{FB}) \Rightarrow E = \Delta^* + e^{FB} = 1.\,497\,2 + 0.431\,2 =$ $1.\,928\,4$

Ploting $P\left(\, x^*(\Delta)\right).\Delta$ and $C\left(e^{FB}\right) - C(e)$, for all $0 \leq e \leq e^{FB}$ with $\Delta = V(E(e)) - V(e)$

Figure 12: Expected loss in court $P\left(\, x^*(\Delta)\right).\Delta$ and gains of breach $C\,(0.5) - C(e)$ for the agent



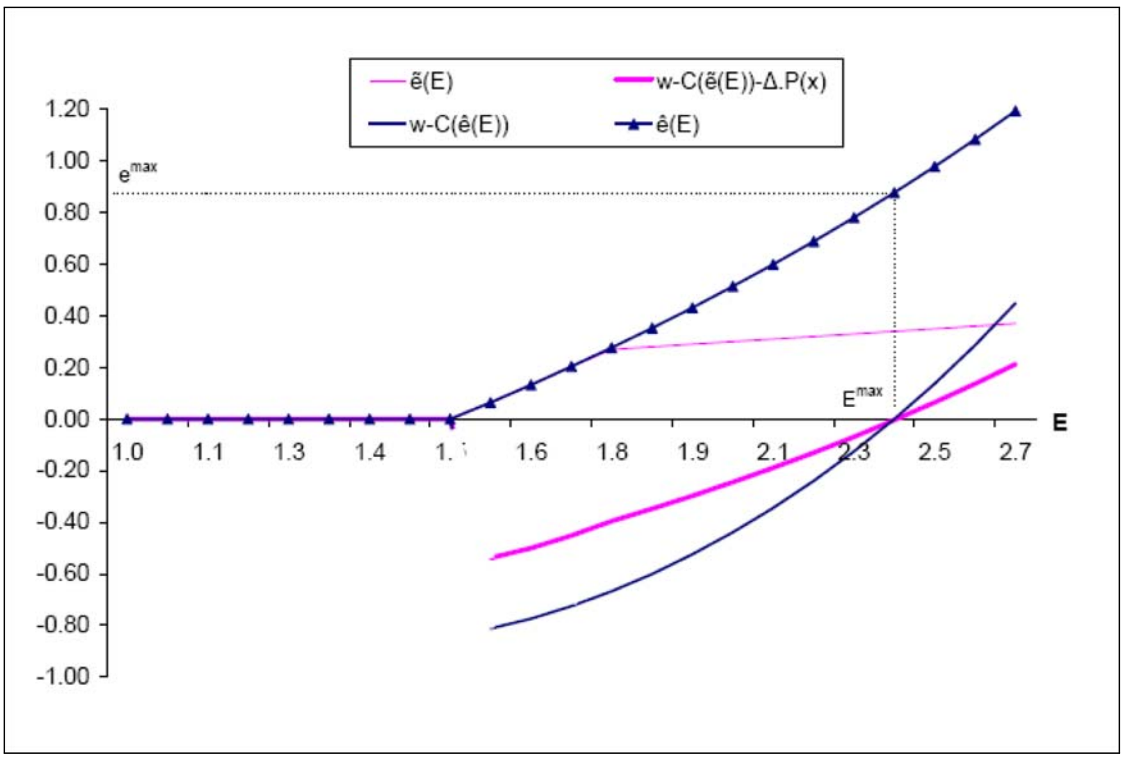
In contrast with the previous functions we can see that Condition N (ii) does not hold, because:
(ii) $P\left(x^*\right)\Delta_I = 0.2736 \leq C\left(e^{FB}\right) - C(\underline{e}) = 0.431\,2^{1.4} - 0^{1.4} = 0.308\,00$

However, this new group of functions meet corollary 1 (ii), because:
$P\left(\, x^{*}(\Delta)\right).\Delta \geq C\left(e^{FB}\right) - C(e)$ for all $0 \leq e \leq e^{FB}$.

Therefore, within these new functions the first best can not be implemented by a contingent contract but it can be induced through a the incomplete contract: $(E(e^{FB}), C(e^{FB})) = (1.997\,2, 0.308\,00)$.

Finally, we again plot $\hat{e}$ and $\tilde{e}^{*}(E(\hat{e}))$ and their payments (in negative) in the in the first and fourth quadrant to graphically find the maximum level of effort that can induce $\hat{e}^{\max}$.In this case, we have that $\hat{e}^{\max} = 0.88$ and it is induced by the contract $(E(\hat{e}^{\max}), C(\hat{e}^{\max})) = (2.38, 0.83)$.

Figure 13: Maximum level of effort $\hat{e}^{\max}$ that can be induced with the contract $(E(\hat{e}), w)$



7. Conclusions

This paper investigates the conditions under which an incomplete contract, a non-contingent wage scheme that establishes one level of payment and effort, can be optimal in the context of an agency model where the verification of an agent's effort is endogenously determined.

Previously, Ishiguro (2002) investigated the optimal contract design in such a setup and derived a necessary and sufficent condition under which the first best can be induced with a ”three-step” wage scheme. In other words, he proposed a complete contract, a different wage level for each level of effort to implement the first best.

We modified Ishiguro's (2002) setup by allowing the principal to go to court and assuming he has the same verification technology available. As a consequence, the incentives to breach the contract are reversed and now is the principal the one who has the incentives to go to court. In this "modified" setup, we derive a neccesary and suficient condition to achieve the first best with a non-contingent or incomplete contract. Assuming the functions are differentiable, these conditions relate the Principal's benefit, the Agent's cost, the probability of winning and the cost of the litigation. Moreover, this condition is found to be more general than the one established in Ishiguro (2002) within his setup.

In equilibrium, the contract is always breached by the agent. Knowing this in advance, the principal writes a contract that specifies a level of effort higher than the one he is trying to induce and a wage that exactly compensates this "induced" level of effort. Therefore, the optimal contract is such that it gives the incentives to the agent to make the effort level that the principal is trying to induce. At the same time, it prevents the principal from going to court because his expected benefit equals his cost of doing so.

Likewise in Willington (2003), this result goes through the lines proposed by Bernheim y Whinston (1998) in the sense that a model is presented in which incomplete contracts is optimal. In simple words, this results may give a plausible explanation of why in real life situations contracts are much simplier than the ones suggested in the literature. It may be that parties do not need these sophisticated mechanisms because with simple contracts they may achieve optimal outcomes.

8. Appendix

Proof of Proposition 2.

a) Proposition 2 (i) $\Leftrightarrow$ Condition N (i) if the same verification technology is available to the principal and the functions V, C, g,y P are differentiable

a.1) Proposition 2 (i) $\Rightarrow$Condition N (i)

Let $\min \dfrac{g(x)}{P(x)} = \Delta^* \in \Re$ and $\Delta^* > 0$ so that corollary of proposition 1 (i) holds. Given that all functions are differentiable and $\psi \equiv \inf\limits_{x \,\in\, X\backslash\{0\}} \dfrac{g(x)}{P(x)} \Rightarrow \psi = \Delta^*$.

Additionally, given that $\Delta^* > 0$

$$\Rightarrow 0 < x^*(\Delta^*)$$

$$\Rightarrow 0 < P(x^*(\Delta^*))$$

$$\Rightarrow 0 > -P(x^*(\Delta^*))$$

$$\Rightarrow 1 > 1 - P(x^*)$$

$$\Rightarrow \Delta^* > [1 - P(x^*)]\Delta^*$$

$$\Rightarrow \psi \geq [1 - P(x^*)]\Delta^*$$

Therefore, we found one $\Delta_I = \Delta^* = \min \dfrac{g(x)}{P(x)}$ so that Condition N (i) holds.

a.2) Condition N (i)$\Rightarrow$Condition 2 (i)

By Condition N (i) we have $\Delta_I \in \Re$ and $\Delta_I > 0$ so that $\psi \geq (1 - P(x^*))\,\Delta_I$ holds. Within a differentiable environment $X^*(\Delta)$ becomes a singleton $\Rightarrow$ $X^*(\Delta) = x^*$ and $\Delta_I = \dfrac{g'(x^*)}{P'(x^*)} =$

$\min \dfrac{g(x)}{P(x)} = \Delta^*$

Therefore, Condition 1 (i) always holds because $\Delta_I = \Delta^*$.[17]

[17] Notice, however, that optimal contracts implied by Δ_I and $\Delta^*$are different because $\Delta_I = w^* - \underline{w}$ and $\Delta^* = V(E) - V(e^{FB})$.

b) Condition N (ii) $\Rightarrow$ Proposition 1 (ii)

By Condition N (ii) we have $P\,(\;x^{*}(\Delta^{*}))\,.\Delta^{*} \geq C\left(e^{FB}\right) - C(\underline{e})$.

Also, for all e in $\underline{e} \leq e \leq e^{FB}$ we have $\Delta = V(E) - V(e) \quad > \quad \Delta^{*} = V(E) - V(e^{FB}) > 0$

$\Rightarrow\; P\,(\;x^{*}(\Delta))\,\Delta \geq P\,(\;x^{*}(\Delta^{*}))\,\Delta^{*}$and $\;C\left(e^{FB}\right) - C(\underline{e}) \geq C\left(e^{FB}\right) - C(e)$

$\Rightarrow P\,(\;x^{*}(\Delta))\,\Delta \geq C\left(e^{FB}\right) - C(e)$.Proposition 1 (ii)

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