Designing fiscal rules for commodity exporters

Carlos J. García *
ILADES - Universidad Alberto Hurtado

Jorge E. Restrepo *
Central Bank of Chile

Evan Tanner *
IMF Institute, International Monetary Fund

July 2008

Abstract
We compare welfare levels under alternative fiscal rules for small open commodity exporters whose fiscal income varies with the world commodity price (in a dynamic, stochastic, general equilibrium model). Between the extremes of a procyclical balanced budget policy and an acyclical spending rule, there lies a continuum of rules. The acyclical rule benefits households that do not enjoy access to capital markets by providing a financial cushion that they themselves cannot provide, while also boosting their mean consumption. However, households that enjoy full access to capital markets suffer under this rule, since the government limits their natural role in smoothing consumption and accumulating assets.

JEL Codes: E32, E61, E62, E63, F41.

Keywords: welfare, small open economy, fiscal rules, rule of thumb consumers, government spending.

* García and Restrepo: visiting scholars at the IMF Institute. We wish to thank Jordi Gali, Carlos Vegh, José de Gregorio and seminar participants at Ilades-Georgetown University, University of Chile, Central Bank of Chile, and the Catholic University of Chile. William Baeza helped code several simulations. All remaining errors are the authors’ own.

* Corresponding author: Carlos García cgarcia@uahurtado.cl; Facultad Economía y Negocios, Universidad Alberto Hurtado: Erasmo Escala 1835, Santiago, Chile; Tel 56-2-6920374; fax 56-2-6920303.

* Email: jrestrepo@bcentral.cl, Central Bank of Chile, Agustinas 1180, Santiago, Chile.

* Email: etanner@imf.org, International Monetary Fund, 700 19th Street, N.W., Washington DC 20431 USA.
Most available evidence suggests that, in emerging/developing economies, fiscal policy is procyclical (Kaminsky, Reinhart, and Végh, 2004; Talvi and Végh, 2005). Government consumption typically increases and taxes often fall during expansions, while the opposite often happens during recessions. Moreover, in countries whose exports are concentrated in one or a few primary resource-based commodities, government expenditures often move closely with the world prices of these exports. Thus, as Figure 1 suggests, in such countries, higher volatility of government spending is associated with higher commodity prices volatility. Of course, past fiscal indiscipline may play a role in procyclical fiscal behavior. Procyclical spending cuts often occur not only when commodity prices fall, but also after a buildup of public debt. Moreover, as Figure 2 suggests, spending shocks often have broader spillover effects, insofar as higher volatility in government spending is typically linked to higher volatility in economic growth. 

In this way, fiscal volatility may affect consumer welfare. For example, fiscal shocks may affect private consumption. Households that do not enjoy access to capital markets -- “hand-to-mouth” or “non-Ricardian” households – are especially vulnerable in this aspect. Without their own financial buffer stocks, such households cannot smooth their consumption. Hence, when government spending falls, their disposable income and consumption fall with it. By contrast, households that do have access to capital markets -- “Ricardian” optimizers – are better positioned to cushion themselves against such shocks.

Governments may wish to protect these more vulnerable “hand-to-mouth” households from fiscal volatility. Ideally, they would do so through a sequence of taxes and transfers whose magnitudes would yield exactly the sequence of consumption these households were in fact “Ricardian.” However, such a policy may be difficult to implement, since the government may not know what household preferences are. As a more practical alternative, some commodity exporting countries have simply chosen to reduce fiscal volatility by implementing a fiscal rule that breaks the link between current commodity prices and public spending. While such rules may be ad-hoc in nature, they may be easier to communicate and implement than other more complicated policies (like the tax / transfer scheme).

---

1 In a related vein, Talvi and Végh (2005) argue that pressures to increase public spending in countries that face large swings in their tax base, as is the case in many developing countries, are the cause of running a procyclical fiscal policy. Gavin et al. (1996) and Gavin and Perotti (1997) have attributed this procyclical bias to the fact that developing countries are rationed from international credit markets in bad times.
Figure 1
Volatility of the commodity price growth and Government spending growth

Source: International Monetary Fund, United Nations and Saudi Arabian Monetary Agency; Major commodities by country are: Copper (Chile, Zambia), Soy (Paraguay), Gold (Peru, South Africa), Natural Gas and Oil (Iran), Oil (all other countries).

Figure 2
Volatility of GDP and Government spending growth

Source: See Figure 1.
But, such a rule may have unintended consequences for the other segment of the economy – the “Ricardian” households who do enjoy access to capital markets. That is, such rules may have economic effects which “Ricardian” households cannot fully neutralize.

This paper examines the effects of alternative fiscal rules on welfare. How does fiscal volatility affect the well being of households? We use a general equilibrium model of a small open economy to do so. Our model has several “New Keynesian” features that generate real effects of both monetary and fiscal policy in the short term. Like many such “New Keynesian” models, this one assumes that prices are sticky. Monetary policy is captured in a Taylor-type rule.

Likewise, government expenditures raise total output through their impacts on the demand side. Thus, a positive shock to public expenditures will cause total expenditures (and hence GDP) to increase. To contain inflation, interest rates must also rise. Moreover, additional government expenditures, on average, raise imports (due to a real appreciation) hence squeezing out private consumption of domestic goods.

As mentioned above, our model features two kinds of households, namely “rule of thumb” and “Ricardian.” Both consume and supply their labor to firms. Note that, in standard real business cycle (RBC) models wherein all households are Ricardian, more public spending reduces after-tax lifetime income and consumption. By contrast, “hand-to-mouth” agents spend all their current income. Hence, their consumption is positively correlated with government spending (as in Galí et al (2007)).

There are several kinds of firms, including a public enterprise that exports a resource-based commodity (copper, oil, etc.”). Proceeds from the resource-based commodity are an important element of government revenue (in addition to domestic lump-sum taxes). Thus, shocks to the world-determined price of commodity will have important implications for fiscal policy.

Government expenditures they are assumed to be inherently useless to anyone except the government itself; they appear in neither utility nor production functions. Of course, if expenditures are useless, the government might instead just transfer commodity revenues directly to consumers. Under such a policy, the government would directly affect consumer wealth and hence welfare. However, we wish to examine another, less direct, channel of transmission for government expenditures, namely their impact on the volatility of demand. Our setup is well-suited to do so.

---

2 For evidence in support of such assumptions rigid prices see Woodford (2003).

3 The effect of shocks, as revealed by impulse response functions, yield results that are consistent with the traditional Mundell-Fleming model, and with those found in the other recent papers, both empirical and theoretical (Galí et al., 2007), a positive public spending shock increases output, consumption, interest rates, and the real value of the currency.
Our fiscal rules lie on a continuum between two extremes. At one extreme, under a balanced budget regime, expenditures are inherently procyclical since they must equal revenues in any period. At the other extreme, under a structural balance, expenditures are acyclical: they are linked to steady-state (rather than current) commodity revenue. More broadly, it is shown that a continuum of fiscal rules can be summarized in terms of two critical parameters; the balanced budget and structural balance extremes are special cases of such parameters that represent differing levels of procyclicality in expenditures.

But, as discussed in the paper, under a structural balance, there is an inherent asymmetry in the model. A rise in the price of the commodity reduces the risk premium and hence government debt. The impact of such a beneficial shock outweights the adverse impact of a fall in copper prices of an equivalent magnitude. For this reason, while the steady-state balance is by definition zero, the expected balance exceeds zero; the government accumulates assets under a structural balance rule. Such a fiscal rule has intriguing interpretations: also provides for an extra financial cushion for the “hand-to-mouth” households – a kind of publicly-provided precautionary savings that such households are unable to provide for themselves. ⁴

The model is simulated with dynamic stochastic general equilibrium (DSGE) techniques. Welfare, measured in terms of steady-state consumption (Lucas, 1987), is compared across regimes. Importantly, the source of cross-regime welfare differences should lie in both the mean of consumption (first moment) and its variability (second moment).

Until recently, simulations in general equilibrium models were typically based on first-order log-linear approximations which did not allow meaningful welfare comparisons; (see for example Kim and Kim, 2003). As a remedy, we use an algorithm developed by Schmitt-Grohé and Uribe (2004), whose second-order approximations permit us to assess the impact of policy-induced variability on the mean of consumption and other key economic variables. This type of approximation has been previously used to evaluate several issues, including the benefits of capital mobility and international risk sharing (Kim and Kim, 2003) and the relative merits of fixed-versus-floating exchange rate regimes (Elekdag and Tchakarov, 2004)).

The simulations reveal that some agents are better off under the acyclical government spending rule. As expected, macroeconomic aggregates are less volatile under the acyclical regime than under the balanced budget regime, since the government smooths its expenditures under the former. This is especially beneficial for non-Ricardian consumers who normally are unable to smooth out the volatility coming from the shocks and transmitted to the economy by the balanced-budget regime. In fact, volatility of income affects negatively mean consumption and, therefore, welfare. In addition, non-commodity exporters benefit. Also, under the acyclical rule, the government saves the windfalls. In so doing, it

⁴ Such an asymmetry generally presumes that there is an element of prudence (a non-zero third moment) in their utility function; see for example Carroll and Kimball (2006). A refinement of this argument is due to Huggett and Ospina (2001).
avoids some of the undesired currency appreciation – a Dutch disease that typically plagues non-commodity exporters.

By contrast, volatility of consumption by Ricardian households is nearly (but not completely) invariant to regime, since these households are able to partially undo what the government does, by borrowing and lending (saving). However, consumption of Ricardian agents is larger under a balanced-budget regime than under a perfectly acyclical one. This is because, under a balanced budget regime, Ricardian households save more. This is consistent with a precautionary savings motive implied the prudence parameter (third moment) in the utility function. And, since the public budget is balanced, all asset acquisitions are foreign. Hence, both their assets and their consumption must rise over time. Under the acyclical regime, Ricardian agents become indebted to the government. Put differently, under the acyclical spending rule, the government has usurped a role which belonged to the private sector under a balanced budget regime, namely insurance. Thus, over time, asset and consumption levels for Ricardian households are, on average, lower under the acyclical regime than under the balanced budget regime.

Regarding regimes that lie between the two extremes, we conclude that a regime that lies close to the acyclical spending rule but retains some cyclicality might be the best compromise. Overall welfare increases substantially if the government makes the rule slightly less acyclical by increasing somewhat the (backward-looking) response of public expenditures to the accumulated debt level. Such a regime is desirable when the Ricardians (who gain from some cyclicality) are able to compensate the “hand-to-mouth” households for their loss, through a lump sum transfer or otherwise. As a whole, the acyclical rule has a larger positive effect on society’s welfare, in this context, the more myopic or financially restricted consumers are.

The remainder of the paper is organized as follows. In Section II, we present the model in its entirety. In Section III, we discuss the calibration of the parameters, present the simulation results and analyze the models' dynamics. In Section IV, we present the welfare analysis. Finally in section V we summarize and conclude.
II. THE MODEL

Our real business cycle model most closely resembles one developed by Smets and Wouter (2002), but also draws on work by Woodford (2003), Clarida et al. (1999), and Galí et al. (2007). However, our model, for a small open economy, also includes: hand-to-mouth consumers (as in Galí et al., 2007), capital and investment with adjustment costs, raw materials, government, GHH preferences and, instead of overlapping generations, a representative “Ricardian” agent. Our structure is also similar to the one proposed by Gali and Monacelli (2005) regarding the modeling of a representative agent with two goods (domestic and foreign) by using CES consumption baskets, and price stickiness à la Calvo (1983). Nevertheless we close the small open economy as in Schmitt-Grohé and Uribe (2003). Another essential reference among recent models for emerging economies is Laxton and Pesenti (2003) GEM model. Nevertheless, they have a very complex and more realistic structure to describe the relationship between final goods, intermediate goods and raw and semi finished materials. Besides, they assume habit in consumption, a different price setting, nontradable goods and adjustment cost for the demand of imports and nontradable goods.

A. Households

We assume a continuum of infinitely lived households indexed by $i \in [0,1]$. Following Galí et al. (2007), a fraction of households $\lambda$ consume their current labor income; they do not have access to capital markets and hence neither save nor borrow. Such agents have been termed “hand-to-mouth” consumers. The remainder $1-\lambda$ save, have access to capital markets, and are able to smooth consumption. Therefore, their intertemporal allocation between consumption and savings is optimal (Ricardian or optimizing consumers). Both segments optimize on the intratemporal margin in labor markets.

Consumption by Ricardian Households

The representative household maximizes expected utility

$$E_0 \sum_{t=0}^{\infty} \beta^t U \left( C^o_t(i), N^o_t(i) \right),$$

Subject to the budget constraint

$$PC^o_t(i) = W_t(i)N^o_t(i) + B^o_t(i) - S_t B^o_t(i) + D^o_t(i) + P^o_t - R^o_{t+1}(i) + S_t(\Phi(B^o_t)R^o_t)^{-1}B^o_{t+1}(i),$$

where $C^o_t(i)$ is consumption, $D^o_t(i)$ are dividends from ownership of firms, $\Phi(B^o_t)$ is the country risk premium, $S_t$ is the nominal exchange rate, $B^o_t(i)$ denotes private net foreign assets, where we define a positive value of $B^o_t(i)$ as debt, $W_t(i)$ is nominal wage, $N^o_t(i)$ is
the number of hours of work, $B^*_i(i)$ is government debt held by households, $R_i$ and $R^*_i$ are the gross nominal return on domestic and foreign assets (where $R_i = 1 + i_i$ and $R^*_i = 1 + i^*_i$) and $T_i$ are lump-sum taxes.

Our utility function (Correia et al, 1995) yields realistic values for consumption volatility:

$$U(C, N) = \frac{(C - \psi N)^{1-\sigma} - 1}{1 - \sigma}.$$  

Note that $1/\sigma$ is the intertemporal elasticity of substitution in consumption and $1/(\varphi - 1)$ is the elasticity of labor supply to wages. The value of $\psi$ is calibrated to obtain a realistic fraction of steady state hours worked. Note also that the rate of relative prudence is

$$\left( C^0(i) - \psi N^0(i)^\varphi \right) U_{ccc} / U_{cc} = -(1 + \sigma).$$

This statistic is important for precautionary savings and it will be useful for explaining below the most important results of this article. As other authors have noted (Carroll and Kimball (2006)), for any individual agent, unless this statistic is non-zero, the level of consumption (and hence savings) will be invariant to volatility. The first-order condition for consumption is:

$$\left( C^0(i) - \psi N^0(i)^\varphi \right)^\sigma = \beta E_i \left( C^0_{r+1}(i) - \psi N^0_{r+1}(i)^\varphi \right)^\sigma R_i \left( \frac{P_r}{P_{r+1}} \right)^{\sigma} \right).$$

From the first order conditions it is also possible to derive the interest parity condition:

$$\frac{S^*_i}{P^*_i} = E_i \left[ \left( \frac{S_{r+1}}{P_{r+1}} \right)^R_i \Phi \left( B^*_i \right) \right].$$

Consumption by Hand-to-Mouth Households

For “Non-Ricardian” households, utility is:

$$U(C^*_i(i), N^*_i(i)).$$

We assume that these households neither save nor borrow (Mankiw (2000)). As a result, their level of consumption is given by their disposable income:

$$P_i C^*_i(i) = W_i(i) N^*_i(i) - P_i T_i.$$
Labor supply

Symmetric with the goods markets (discussed below) we assume a continuum of monopolistically competitive households that supply a differentiated labor service to the intermediate-goods-producing sector and a labor aggregator combines as much household-labor as is demanded by firms, with a constant-returns technology. The aggregate labor index has the CES form:

\[ N_t = \left[ \frac{1}{0} N_t(i)^{\frac{1}{\theta_w}} di \right]^{-\theta_w} \]  

(9)

where \( N_t(i) \) is the quantity of labor used from each household. The representative labor aggregator minimizes the cost of producing a chosen amount of the aggregate labor index, given each household’s wage rate \( W_t(i) \). Then, she sells units of labor index at their unit cost \( W_t \) (with no profit), to the production sector:

\[ W_t = \left[ \frac{1}{0} W_t(i) N_t(i)^{\frac{1}{\theta_w}} di \right]^{-\theta_w} \]  

(10)

Note that, while prices are sticky, wages are completely flexible. Nominal wages are set by households so as to maximize their intertemporal objective function (1) subject to the intertemporal budget constraint (2) and to the total demand for their labor services, which is given by:

\[ N_t(i) = \left[ \frac{W_t(i)}{W_t} \right]^{\frac{1}{\theta_w}} N_t \]  

(11)

As a result the supply of each household is given by

\[ W_t(i) = (1 + \theta_w) \phi \psi N_t(i)^{\theta_w} \]  

(12)

where \((1 + \theta_w)\) is a mark-up over the current ratio of the marginal disutility of labor and the marginal utility of an additional unit of consumption. For rule-of-thumb households, wages are set at the average wage level of optimizing households.
Demands for domestic and imported consumption goods

Consumption is a CES aggregate of consumption of domestic $C^D_t(i)$ and imported goods $C^F_t(i)$. Where $\eta_c$ is the elasticity of substitution between domestic and foreign goods and $\alpha_c$ is the steady-state share of imported goods in total consumption:

$$C_t = \left( \alpha_c \frac{1}{\eta_c} \left( C^D_t \right)^{\eta_c} + \left( 1 - \alpha_c \right) \frac{1}{\eta_c} \left( C^F_t \right)^{\eta_c} \right)^{\frac{\eta_c}{\eta_c-1}}$$

The demand for each set of differentiated domestic and imported goods, as derived from expenditure minimization, is:

$$C^D_t = \alpha_c \left( \frac{P^D_t}{P_t} \right)^{-\eta_c} C_t$$

$$C^F_t = \left( 1 - \alpha_c \right) \left( \frac{P^F_t}{P_t} \right)^{-\eta_c} C_t$$

A weighted average of either domestic or imported differentiated goods composes each type of good, which also consists of a Dixit-Stiglit index:

$$C^K_t = \left( \int C^K_j \left( P^K_j \right)^{\eta_c} d\eta \right)^{\frac{1}{\eta_c}}$$

$$C^K_t(j) = \left( \frac{P^K(j)}{P^K_t} \right)^{\eta_c} C^K_t$$

for $K = D$ (domestic) and $F$ (foreign). $P_t$, the aggregate consumer price index or CPI is defined as:

$$P_t = \left( \alpha_c (P^D_t)^{1-\eta_c} + \left( 1 - \alpha_c \right) (P^F_t)^{1-\eta_c} \right)^{\frac{1}{1-\eta_c}}$$
Where the respective price index is:

\[ P^K_t = \left( \int_0^1 P^K_t(j)^{1-\varepsilon} \, dj \right)^{\frac{1}{1-\varepsilon}}, \]  

(19)

where \( K = D \) (domestic), \( F \) (foreign).

**B. Firms**

**Domestic intermediate-goods firms**

We assume a continuum of monopolistically competitive firms, indexed by \( j \in [0,1] \) producing differentiated intermediate goods. The production function of the representative intermediate-good firm, indexed by \( (j) \), corresponds to a CES combination of capital \( K_i (j) \) and labor \( N_i (j) \), to produce \( Y^D_i (j) \) and is given by:

\[ Y^D_i (j) = A \left( \alpha K_i (j)^{\frac{\sigma}{\sigma-1}} + (1-\alpha)N_i (j)^{\frac{\sigma}{\sigma-1}} \right)^{\frac{\sigma}{\sigma-1}}, \]  

(20)

where \( A \) the technology parameter, and \( \sigma \), the elasticity of substitution between capital and labor, are both greater than zero.

The firms’ costs are minimized taking as given the rental price of capital, \( R^k_t \) and the wage, \( W_t \) subject to the production function (technology). The relative factor demands are derived from the first-order conditions:

\[ \frac{R^k_t}{W_t} = \left( \frac{\alpha}{1-\alpha} \right) \left( \frac{N_t (j)}{K_t (j)} \right)^{\frac{1}{\sigma}}. \]  

(21)

Thus, marginal cost is given by:

\[ MC^D = \frac{1}{A} \left[ \alpha^{\sigma} \left( R^k_t \right)^{1-\sigma} + (1-\alpha)^{\sigma} \left( W_t \right)^{1-\sigma} \right]^\frac{1}{1-\sigma}. \]  

(22)

When firm \( (j) \) receives a signal to optimally set a new price à la Calvo (1983), it maximizes the discounted value of its profits, conditional on the new price:

\[ \max \sum_{k=0}^{\infty} \theta^k \mathbb{E} \left\{ N_{t+k} Y^D_{t+k} (j)(P^D_t (j) - MC^D_{t+k}) \right\} \]  

(23)
subject to

$$Y_{t+k}^D(j) \leq \left( \frac{P_{t+1}^{D*}}{P_t^D} \right)^{-\varepsilon_D} Y_{t+k}^D$$ (24)

Where the probability that a given price can be re-optimized in any particular period is constant and is given by \((1 - \theta_D)\) and \(\varepsilon_D\) is the elasticity of substitution between any two differentiated goods. \(P_{t+1}^{D*}\) must satisfy the first order condition:

$$\sum_{k=0}^{\infty} \theta_D^k \mathbb{E}_t \left[ \Lambda_{t+1+k}^D Y_{t+k}^D(j) \left( P_{t+1}^{D*}(j) - \frac{\varepsilon_D}{\varepsilon_D - 1} MC_{t+k}^D \right) \right] = 0$$ (25)

where the discount factor \(\Lambda_{t+1+k}\) is:

$$\Lambda_{t+1+k} = \beta \left( \frac{C_{t+k}^0}{C_t^0} \right)^{-\sigma} \left( \frac{P_t}{P_{t+k}} \right).$$

Firms that did not receive the signal will not adjust their prices. Those who do reoptimize choose a common same price \(P_{t+1}^{D*}\). Finally the dynamics of the domestic price index \(P_t^D\) is described by the equation:

$$P_t^D = \left[ \theta_D (P_{t-1}^D)^{-\varepsilon_D} + (1 - \theta_D) (P_{t+1}^{D*})^{-\varepsilon_D} \right]^{\frac{1}{1 - \varepsilon_D}}$$ (26)

**Intermediate-goods importing firms**

As in the domestic sector, price setting in the import sector reflects little exchange rate pass-through in the short run (as in Monacelli, 2005, and Smets and Wouters, 2002). Such an assumption provides realistic simulations (impulse-responses) with little cost in complexity. This sector consists of firms that import a homogenous good from abroad and turn it into a differentiated foreign good for the home market using a linear production technology. Import firms are only allowed to change their price when they receive a random price-change signal. Thus, the dynamics of the import price index is also described by an equation similar to (24) but, in this case, the firms that are allowed to reset their prices in response to variations in the exchange rate or the foreign price and optimally charge the import price abroad expressed in domestic currency.

$$P_t^F = \left[ \theta_F (P_{t-1}^F)^{-\varepsilon_F} + (1 - \theta_F) (S P_{t+1}^{F*})^{-\varepsilon_F} \right]^{\frac{1}{1 - \varepsilon_F}}$$ (27)
Note that \((1 - \theta_F)\) and \(\varepsilon_F\) have the same definition as before but here they apply to the intermediate-goods importing firms.

**Final goods distribution**

There is a perfectly competitive aggregator, which distributes the final good using a constant return to scale technology. It is valid for both \(K = D\) (domestic) and \(F\) (imported) goods:

\[
Y_t^K = \left( \frac{1}{\int_0^1 Y_t^K (j)^{1 - \varepsilon} P_t^d} \right)^{\varepsilon_{K}}
\]

(28)

\(Y_t^K (j)\) is the quantity of the intermediate good (domestic or imported) included in the bundle that minimizes the cost of any amount of output \(Y_t\). The aggregator sells the final good at its unit cost \(P_t\) with no profit:

\[
P_t^K = \left( \frac{1}{\int_0^1 P_t^K (j)^{1 - \varepsilon} P_t} \right)^{1/\varepsilon_{K}}
\]

(29)

where \(P_t\) is the aggregate price index. Finally, demand for any good \(Y_t^K (j)\) depends on its price \(P(j)\), which is taken as given, relative to the aggregate price level \(P_t\):

\[
Y_t^K (j) = \left( \frac{P(j)}{P_t} \right)^{-\varepsilon_{K}} Y_t^K
\]

(30)

**Optimizing investment firms and Tobin’s Q**

There are firms that produce homogenous capital goods and rent them to the intermediate-goods firms. Note that only the Ricardian households own them. Firms invest the amount so as to maximize firm value:

\[
V^I(K_t^o) = R_i^t K_t^o - P_t^t I_t^o + E_t (V^{t+1}(K_{t+1}^o))
\]

subject to a capital accumulation constraint that includes an adjustment cost function \(\phi(\cdot)\).

\[
K_{t+1}^o = (1 - \delta) K_t^o + \phi \left( \frac{I_t^o}{K_t^o} \right) K_t^o
\]

(32)

The first-order conditions are:
Equation (35) corresponds to Tobin’s Q: the marginal cost of an additional unit of investment should be equal to the present value of the marginal increase in equity that it generates.

**Demand for investment goods**

Overall investment is equal to a CES aggregate of domestic and imported goods. Where $\eta_i$ is the elasticity of substitution between domestic and foreign goods and $\alpha_i$ is the steady-state share of domestic goods in total investment.

$$I_i = \left( \alpha_i \frac{1}{\eta_i} \left( I^D_i \right)^{\eta_i - 1} + (1 - \alpha_i) \frac{1}{\eta_f} \left( I^F_i \right)^{\eta_f - 1} \right)$$

Demands for investment goods, domestic and imported respectively, are derived from expenditure minimization, namely:

$$I^D_i = \alpha_i \left( \frac{P^D_i}{P_i} \right)^{-\eta_i} I_i$$

$$I^F_i = (1 - \alpha_i) \left( \frac{P^F_i}{P_i} \right)^{-\eta_f} I_i$$

A weighted average bundle of either domestic or imported differentiated goods thus comprises each type of investment good (a Dixit-Stiglitz index):

$$I^K_i = \left( \frac{1}{0} I^K_i (j)^{\varepsilon_k} \, dj \right)^{\varepsilon_k - 1}$$

$$I^K_i (j) = \left( \frac{P^K_i (j)}{P^K_i} \right)^{-\varepsilon_k} I^K_i$$

for $k=D, F$. The aggregate price of investment (investment deflator) is defined as:

$$Q^o_i \phi \left( \frac{I^o_i}{K^o_i} \right) - \frac{P^D_i}{P_i} = 0$$

$$Q^o_i = E \left[ \frac{1}{R_i} \left( \frac{P_{t+1}}{P_i} \right) \left[ R^h_{t+1} + Q^o_{t+1} \left( 1 - \delta + \phi_{t+1} - \frac{I^o_{t+1}}{K^o_{t+1}} \phi_{t+1} \right) \right] \right]$$
Each composite good is itself a bundle of differentiated goods

C. Exports

The demand for domestic exports from foreign countries is modeled as follows

\[ P_t^i = \left( \alpha_i (P_t^D)^{1-\eta_i} + (1 - \alpha_i)(P_t^F)^{1-\eta_i} \right)^{\frac{1}{1-\eta_i}} \]  \hspace{1cm} (40)

Exports of good \( J \) depend on its own relative price:

\[ X_t^D(j) = \left( \frac{P_t^D(j)}{P_t^D} \right)^{-\varepsilon_D} X_t^D \]  \hspace{1cm} (41)

There is a demand for each set of differentiated domestic goods, which by assumption depends on total consumption abroad, and on the home price of domestic goods relative to its price in the foreign country:

\[ X_t^D = \left( \frac{P_t^D}{S_t^{D*}} \right)^{-\eta_D} C_t^{D*} \]  \hspace{1cm} (43)

D. Aggregation

The weighted sum of consumption by Ricardian and rule-of-thumb agents makes aggregate consumption

\[ C_t = \lambda C_t^r + (1 - \lambda)C_t^\sigma = \int_0^1 C_t^r(i) \, di + \int_{\lambda}^1 C_t^\sigma(i) \, di \]  \hspace{1cm} (44)

Since only Ricardian households invest and accumulate capital, total investment is equal to \( (1 - \lambda) \) times optimizing investment:

\[ I_t = (1 - \lambda)(I_t^r) \]  \hspace{1cm} (45)

Likewise, the aggregate capital stock is equal to:

\[ K_t = (1 - \lambda)(K_t^r) \]  \hspace{1cm} (46)

Again, only optimizing households hold financial assets:
Foreign assets (or debt) include fiscal $B_t^G$, and private held assets $B_t^o$:

$$B_t = (1 - \lambda)(B_t^o)$$

(47)

Hours worked are given by a weighted average of labor supplied by each type of consumer:

$$N_t = \lambda N_t' + (1 - \lambda)N_t^o$$

(49)

Finally, in equilibrium each type of consumer works the same number of hours:

$$N_t = N_t' = N_t^o$$

(50)

E. Monetary policy

Even while this paper focuses on fiscal policy, price stability requires there also be an active central bank. Thus, in abbreviated way, we also include monetary policy: the central bank sets the nominal interest rate according to the following rule:

$$R_t = \mathcal{R} \left( \frac{\Pi_t}{\Pi} \right)^{\phi} \left( \frac{Y_R}{Y_R^{ss}} \right)^\phi$$

(51)

where $\mathcal{R}$ is the steady state nominal interest rate, $\Pi_t$ is total inflation, $\Pi$ is steady state total inflation (assumed to be is zero), $Y_R$ is GDP without the natural resource and $Y_R^{ss}$ its steady state value.

F. Fiscal policy

The government budget constraint is:

$$P_i T_i + \tau_{cu} (S_i P_i^{cu} Q^{cu}) + R_t^{-1} B_{t+1}^G + S_i (\Phi (B_t^*) R_t^*)^{-1} B_{t+1}^{G*} = B_t^G + S_i B_t^{G*} + P_t^G G_t$$

(52)

where $\tau_{cu}$ is the share of the production of the natural resource (copper) owned by the government, $P_i^{cu}$ is the world price of copper, and $Q^{cu}$ is the quantity supplied (assumed to be constant). Supply is assumed to be price-invariant in the business cycle (short run)
horizon. The country risk premium is a positive function of foreign debt: \( \phi(B^*_t) \). Also, \( B^G_t \) denotes public domestic debt \( P_t, T_t \) corresponds to government nominal (lump-sum) tax revenues, and \( P_t^G G_t \) is public spending. For simplicity, we assume that the government maintains fixed proportions of domestic and external debt: \( S_t B^G_t = v_b B^G_t \). When rearranged, (52) also shows the evolution of total government debt:

\[
R^{-1} t B^G_{t+1} + S_t (\Phi(B^*_t) R_{t+1}^{*}) B^G_{t+1} - B^G_t + S_t B^G_t^{*} + \left( P_t^G G_t - P_t T_t - \tau_{cu} (S_t P_t^c Q^c_u) \right)
\]

(53a)

Total public revenues are \( T_t = P_t T_t + \tau_{cu} (S_t P_t^c Q^c_u) \); their corresponding structural (steady state) values are \( \bar{T}_t = P_{ss} T_{ss} + \tau_{cu} (S_{ss} P_{ss}^c Q^c_u) \). For simplicity, we assume that \( P_t T_t = P_{ss} T_{ss} \) in all periods. By contrast, since exchange rates and copper prices vary, so do copper revenues. The fiscal rule establishes that public spending should be equal to steady-state (structural) government revenues less interest payments is:

\[
P_t^G G_t = \bar{T}_t - \left[ \frac{R_{t-1}}{R_t} + \frac{R_t^* \Phi(B^*_t) - 1}{R_t \Phi(B^*_t)} v_b + \mu_s \right] B^G_t
\]

(54)

Under this rule, the government adjusts spending (not taxes) so as to ensure intertemporal solvency. Whenever a negative shock results in lower current than permanent revenues the government will run a debt-financed fiscal deficit. By manipulating (54) and substituting it into (53), we obtain see how debt evolves under the fiscal rule (54):

\[
R^{-1} t B^G_{t+1} + S_t (\Phi(B^*_t) R_{t+1}^{*}) B^G_{t+1} - B^G_t + S_t B^G_t^{*} + \left[ \tau_{cu} (S_{ss} P_{ss}^c Q^c_u) - \tau_{cu} (S_t P_t^c Q^c_u) \right] - \left[ \frac{R_{t-1}}{R_t} + \frac{R_t^* \Phi(B^*_t) - 1}{R_t \Phi(B^*_t)} v_b + \mu_s \right] B^G_t
\]

(53b)

As an illustrative exercise, consider a special case where \( \mu_s = 0 \) and \( v = 0 \) (no foreign debt):

\[
B^G_{t+1} = B^G_t
\]

(53c)
In this case, if copper prices equal their steady state value $\tau_{cu} (S_{ss} P_{ss}^u Q^u) = \tau_{cu} (S_{ss} P_{ss}^u Q^u)$, total government debt stays constant. If there is an adverse shock to copper prices and $\mu_x = 0$, the level of debt will go up to the point where revenues and expenditures are once again equated. Put differently, if $\mu_x = 0$, government debt follows a random walk: the debt will remain at its new level forever unless there is another shock. Thus, if $\mu_x = 0$, the model will not converge. For that reason, $\mu_x$ must be set to a number greater than zero, even if only slightly so. That is, government must pay a little more than the interest on its debt to slowly amortize the principal and converge back to the steady-state level. The larger this coefficient $\mu_x$ is, the more procyclical is the rule up to a point where the debt level would become the target.

As a further generalization, consider a rule that includes an additional term: $\alpha (IT_t - \overline{IT})$, where $IT_t$ and $\overline{IT}$ represent current and steady state total revenues (tax plus copper), respectively. Note that that the government could spend part of the difference between its current and permanent revenues. If $\alpha_x = 0$, the rule will be equal to (54) and the government will spend only the equivalent to its permanent revenue. If $\alpha_x = 1$, the government will have a perfectly procyclical rule, spending its current income each period (balanced budget rule):

$$P_t^G G_t = \overline{IT} - \left[ \frac{R_t - 1}{R_x} + \frac{R_x \Phi(B_t^x) - 1}{R_t \Phi(B_t^x)} v_b + \mu_x \right] B_t^G + \alpha_x (IT_t - \overline{IT}) \quad (54')$$

Finally, the price of copper follows an autoregressive process (in log term) $P_t^u = (P_{t-1}^u)^\rho \exp(v_t)$. In that way, we are able to introduce shocks $v_t$ to it. Since government expends mostly its structural income, a permanent (or extremely persistent) positive shock to the price of the natural resource, will generate higher public spending.

**Government demand for domestic and imported goods.**

The government demands domestic and imported goods. Where $\eta_G$ is the elasticity of substitution between domestic and foreign goods and $\alpha_d$ is the steady-state share of domestic goods in total government expenditure.

$$G_t = \left\{ \alpha_d \frac{1}{\eta_G} (G_t^D)^{\frac{\eta_G - 1}{\eta_G}} + (1 - \alpha_d) \frac{1}{\eta_F} (G_t^F)^{\frac{\eta_F - 1}{\eta_F}} \right\}^{\frac{\eta_G}{\eta_G - 1}} \quad (55)$$
The demands for domestic and imported goods derived from expenditure minimization are given by:

\[ G_t^D = \alpha_o \left( \frac{P_t^D}{P_t^G} \right)^{-\eta_o} G_t \]  (56)

\[ G_t^I = (1 - \alpha_o) \left( \frac{P_t^F}{P_t^G} \right)^{-\eta_o} G_t \]  (57)

Each type of good (domestic, imported) consumed by the government is composed of a weighted average of differentiated goods, which also consists of a Dixit-Stiglitz index:

\[ G_t^K = \left( \int_0^1 G^K(j, \epsilon^{-1}) \frac{\epsilon^{-1}}{\epsilon^{-1}} dj \right) \]  (58)

\[ G_t^K(j) = \left( \frac{P^K(j)}{P^K} \right)^{-\epsilon} G_t^K \]  (59)

for \( K = D \) (domestic), \( F \) (foreign). The aggregate price deflator of government spending is:

\[ P_t^G = \left( \alpha_o (P_t^D)^{-\eta_o} + (1 - \alpha_o)(P_t^F)^{-\eta_o} \right)^{\frac{1}{\eta_o}} \]  (60)

Domestic and imported goods are themselves bundles of differentiated goods.

**G. Market-clearing conditions**

The factor market-clearing conditions are total employment by all firms \( j \):

\[ N_t = \int_0^1 N_t(j) dj \]  (61)

and full capital utilization

\[ K_t = \int_0^1 K_t(j) dj \]  (62)

The good market-clearing condition is:

\[ Y_t^D(j) = (C_t^D(j) + I_t^D(j) + G_t^D(j) + X_t^D(j)) \]  (63)
Using equation (17) and (30), (39), (43) and (59), we obtain:

\[ Y^D_i(j) = \left( \frac{P^D_i(j)}{P^D} \right)^{\epsilon_0} (C^D_i + I^D_i + G^D_i + X^D_i) \quad (64) \]

Equation (64) should be plugged into equation (28), which is:

\[ Y^K_i = \left( \int_0^{\epsilon_1} \left( \frac{\epsilon_0}{\epsilon_1} \right)^{\epsilon_0} \, dj \right)^{\epsilon_0} \]

for K=D, F. In turn, this yields

\[ Y^D_i = \left( \int_0^{\epsilon_1} \left( \frac{P^D_i(j)}{P^D} \right)^{\epsilon_0} (C^D_i + I^D_i + G^D_i + X^D_i)^{\epsilon_0} \, dj \right)^{\epsilon_0} \]

Adding up and simplifying yields the symmetric equilibrium for the domestic market:

\[ Y^D_i = (C^D_i + I^D_i + G^D_i + X^D_i) \quad (65) \]

where total supply domestic goods equals total demand of the domestic produced good for consumption, investment, government spending and exports. Finally, the economy-wide budget identity can be expressed as:

\[ P_i C_i = -P^E_i G_i - P^I_i I_i + P^D_i Y^D_i + (P^E_i Y^E_i - S_i P^F_i Y^E_i) + S_i \left( \Phi \left( B^*_i \right) R^*_i \right)^{-1} B^*_{i1} - S_i B^*_{i2} + \tau_a (S_i P^{cu} Q^{cu}) \quad (66) \]

Equation (66) has an intuitive interpretation. First note that GDP is the (approximately) sum of domestically produced goods plus value added on the distribution of imports, plus copper exports:

\[ P_i Y_i = P^D_i Y^D_i + (P^E_i Y^E_i - S_i P^F_i Y^E_i) + (S_i P^{cu} Q^{cu}) \quad (67) \]

---

5 We assume for simplicity that there are no private copper exports because these demand no resources. We treat them as if transferred from abroad.
Thus, according to the national income accounting identity, consumption must equal GDP minus investment (I) and government expenditures G plus foreign debt (positive values of $B_t^r$), which is written:

$$S_t \left( \Phi \left( B_t^r \right) R_t^r \right)^{-1} B_{t+1}^r - S_t B_t^r$$  \hspace{1cm} (68)

The risk premium ensures that the economy returns to the steady state\(^6\), thus this variable increases with the foreign debt.

### III. CALIBRATION AND DYNAMICS

The system of stochastic difference equations is solved using a second order approximation method developed by Schmitt-Grohé and Uribe (2004).\(^7\) We choose Chile as our benchmark country for the calibration because it has been a leader within commodity exporters in implementing an acyclical fiscal rule.\(^8\) Since most of the parameters have never been obtained using Chilean data, we calibrate the model taking sensible values from different studies (see Table 1).\(^9\) For example, the discount factor $\beta$ is 0.99 close to the values found elsewhere in the literature. The risk aversion coefficient $\sigma$ is greater than one (2.0) as the evidence indicates for small open economies\(^10\). Thus, the relative prudence coefficient is: $\left( C_t^0(i) - \psi N_t^0(i)^\rho U_{ccc}/U_{cc} = -(1+\sigma) = -3. \right.$ This ensures that Ricardian agents will save more as economic volatility rises.\(^11\)

The elasticity of substitution across intermediate goods, $\varepsilon_D$ and $\varepsilon_F$, is 6, in order to have a mark-up of 20%, the fraction of firms that keep their prices unchanged each period, $\theta_p$ and $\theta_F$, is 0.75 and the rate of depreciation $\delta$ is 0.025. All these values are standard in the literature on the New-Keynesian models (Woodford, 2003; Galí and Monacelli, 2005; and Galí et al., 2007).

For the labor market, we suppose the same mark up as in the good market, i.e. $\theta_w$ is 0.2. The value of $\varphi$ (=1.7) comes from Correia et al. (1995), who introduced GHH utility function in RBC models for small open economies to explain the higher volatility of the

---

\(^7\) The method was calculated by using the software Dynare (Juillard, 1996).
\(^8\) The steady state values are consistent with the obtained for the Chilean Economy where foreign debt is around 50 percent of the GDP. See for example Restrepo and Soto (2006)
\(^9\) We assume that each period corresponds to one quarter.
\(^10\) See Agénor and Montiel (1996), Table 10.1, page 353.
\(^11\) For our chosen utility function, there is no closed form solution linking consumption and volatility. An approximation is found in Talmain (1998).
consumption observed in these countries. As they do, we choose a value for $\psi$ (=7.02) to ensure that hours worked in steady state coincide with actual data in our benchmark country. The value of the investment adjustment cost $\phi$ is $1/15$, which is half of the value of Correia et al (1995). Half of households are hand-to-mouth, i.e. $\lambda$ is 0.5, which is within the range of values considered in other studies (Mankiw, 2000; and Galí et al., 2007). We assume that government spending is heavily biased towards domestic goods. Indeed, the share of domestic goods in the government consumption basket $\alpha_G$ is 0.99.

This allows us to replicate the evidence that in many commodity exporters there is a negative impact of government spending on real exchange rate (Edwards, 1989). We do not have information about the values of the elasticity of substitution between domestic and foreign goods ($\eta_C$, $\eta_I$, and $\eta_G$), thus we assume values close to one as it is used in Galí and Monacelli (2005). For the same reason we choose values for $\alpha_C$ and $\alpha_I$ close to 0.5, value also similar used by Galí and Monacelli (2005) to define the degree of openness.
Even though public debt is not exactly zero in Chile, we assume it to be so our model’s steady state. This assumption helps us to compare the acyclical rule with the balanced budget regime because so both policies share the same steady state. When public debt outside from its steady value, we assume that $v_b$ is 0.21, i.e., 21 percent is held abroad, this value is taking directly from the Chilean data. In our baseline simulation, the size of the coefficient in the monetary rule with respect to inflation $\phi_p$ is 1.5, that is the standard value for the Taylor rule, and with respect to output $\phi_y$ is zero for simplicity. For the same reason the elasticity of substitution between capital and labor $\sigma_s$ is 1.0. Thus $\sigma_s$ is the capital share and is assumed 0.4 given that this value in Chile is higher than other countries (in the US is 0.33). The elasticity of domestic exports to the real exchange rate $\eta^*$ is 1.0 in line with estimations for developing countries (Ghei and Pritchett (1999)).

The autoregressive coefficient of the real price of copper $\rho$ is 0.8 obtained from quarterly data from 1973 through 2005. Finally, we choose small values for the debt weight $\mu_x (=0.01)$ in the acyclical rule and the elasticity of the interest rate to external debt (0.001). Both coefficients warrant the stability of the model. The first one makes public debt a

**Table 1: Baseline Parameters**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount factor ($\beta$)</td>
<td>0.99</td>
</tr>
<tr>
<td>Risk aversion coefficient ($\sigma$)</td>
<td>2.00</td>
</tr>
<tr>
<td>Disutility parameters, worked hours (N)</td>
<td></td>
</tr>
<tr>
<td>$\phi$</td>
<td>1.70</td>
</tr>
<tr>
<td>$\psi$</td>
<td>7.02</td>
</tr>
<tr>
<td>Weight of rule-of-thumb consumers ($\lambda$)</td>
<td>0.50</td>
</tr>
<tr>
<td>Rate of depreciation ($\delta$)</td>
<td>0.025</td>
</tr>
<tr>
<td>Investment adjustment cost $\phi$</td>
<td>1/15</td>
</tr>
<tr>
<td>Elasticity of substitution across intermediate goods ($\varepsilon_D, \varepsilon_F$)</td>
<td>6.00</td>
</tr>
<tr>
<td>Parameter of CES production function ($\alpha$)</td>
<td>0.40</td>
</tr>
<tr>
<td>Fraction of firms that keep their prices unchanged ($\theta_D, \theta_H$)</td>
<td>0.75</td>
</tr>
<tr>
<td>Real wage mark-up ($1+\theta_w$)</td>
<td>1.20</td>
</tr>
<tr>
<td>Elasticity of substitution between capital and labor ($\sigma_c$)</td>
<td>1.00</td>
</tr>
<tr>
<td>Response of monetary authority to inflation ($\phi_p$)</td>
<td>1.50</td>
</tr>
<tr>
<td>Response of monetary authority to output ($\phi_y$)</td>
<td>0.00</td>
</tr>
<tr>
<td>Autoregressive coefficient of copper price</td>
<td>0.80</td>
</tr>
<tr>
<td>Share of the production of the natural resource owned by the government ($\tau_m$)</td>
<td>0.50</td>
</tr>
<tr>
<td>Amount produced of the natural resource ($Q^m$)</td>
<td>0.45</td>
</tr>
<tr>
<td>Weight of domestic good in consumption ($\alpha_C$)</td>
<td>0.60</td>
</tr>
<tr>
<td>Weight of domestic good in investment ($\alpha_I$)</td>
<td>0.50</td>
</tr>
<tr>
<td>Weight of domestic good in government expenditure ($\alpha_G$)</td>
<td>0.99</td>
</tr>
<tr>
<td>Foreign-domestic good (consumption) elasticity of substitution ($\eta_C$)</td>
<td>0.99</td>
</tr>
<tr>
<td>Foreign-domestic good (investment) elasticity of substitution ($\eta_I$)</td>
<td>0.99</td>
</tr>
<tr>
<td>Foreign-domestic good (government) elasticity of substitution ($\eta_G$)</td>
<td>0.99</td>
</tr>
<tr>
<td>Acyclical rule, debt weight ($\mu_x$)</td>
<td>0.01</td>
</tr>
<tr>
<td>The share of external public debt over total public debt $v_b$</td>
<td>0.21</td>
</tr>
<tr>
<td>Elasticity of interest rate to external debt</td>
<td>0.001</td>
</tr>
<tr>
<td>Elasticity of domestic export to real exchange rate ($\eta^*$)</td>
<td>1.00</td>
</tr>
</tbody>
</table>
stationary variable. The second one forces the current account to be stationary as well as net foreign assets.

IV. EFFECTS OF A COMMODITY (COPPER) PRICE SHOCK

To illustrate how the model economy works under alternative fiscal rules, the result of several shock experiments are reported in this section. The simulations are performed by simulating 100 artificial economies, 1000 period each, hitting the economy with a random shock to the price of copper each period.  

First, Figure 3 shows the responses of a large set of macroeconomic variables to a temporary shock of one standard deviation (20%) to the price of copper. In each of the small charts, each line shows impulse responses for a different fiscal rule. Our baseline, the balanced budget, is shown by the black line, while the blue line represents the alternative acyclical spending rule.

The balanced-budget rule is strongly procyclical: government expenditures grow jointly with revenues fueling the economy. Therefore, output, consumption and hours also increase. Since the currency appreciates, imports increase and non-commodity exports decrease relative to the baseline. In this case, both inflation and the real interest rate go up: this reduces investment.

By contrast, with the acyclical rule (blue line), the countercyclical nature of the fiscal deficit is evident. At impact, government revenues show the same jump in both cases. However, government spending is acyclical: it remains almost constant over the business cycle. Since most resources are saved during a boom (reflected in a fiscal surplus), GDP and consumption increase only marginally. In addition, the relative price $P_d/P_m$ (the real value of the currency) shows a very modest appreciation, while inflation, interest rates as well as investment remain stable -- there is no crowding out.

Thus, the impulse response functions yield results that have a Keynesian flavor and are consistent with the traditional Mundell-Fleming model (with imperfect capital mobility): a positive public spending shock increases consumption, interest rates, and output but not as much as in a closed economy because the real value of the currency increases, raising imports and reducing exports on average.

---

12 We also simulated the economy 10,000 periods ahead and the qualitative results did not change.

13 The size of the shock in our simulations is just enough to obtain a standard deviation of the real price of copper similar to the empirical one (33%) for the period 1973-2005.
Figure 3
Responses to a price of copper shock (% change from steady state values)
Next, since government expenditure is much smoother with the acyclical rule; several variables also display much lower volatility under the acyclical rule. These include employment, the real wage, consumption of “hand-to-mouth” consumers, real interest rate, etc.

**Figure 4** shows the path of selected variables that resulted from 1000 averaged simulations of the model, under both rules. Importantly, note that, under the acyclical rule (blue line), the government builds up assets for some period (that is $B^g$ initially falls) and expenditures are less than revenues on average. Over time, average public debt stabilizes: $B^g \approx -30$ percent of GDP. And, for this same reason, government spending ($G$) is slightly higher under the acyclical regime: the government has a larger “war chest” out of which it can spend. This happens because the movements of the risk premium and the interest rate have an asymmetric effect on its spending and savings. With a positive shock the interest rate goes down, and government savings are larger than what it dissaves when the shock is negative, and interest rates increase.

This result is captured by the second order approximation because with this type of approximation the expected values of the variables are not equal to their non-stochastic steady-state values. Indeed, Schmitt-Ghohé and Uribe (2004) show that a second order approximation corrects the policy function, i.e., the level of each control and endogenous state variable, for the size of the variance of the shocks. In other words, even though the government spending rule is mechanical and does not come from an optimization problem, government’s behavior includes the effect of variability (variance of the shock) on its decision.

As one would expect, consumption by “hand-to-mouth” households $C^r$ differs substantially across regime. The variability of $C^r$ is substantially lower under the acyclical rule; this is the key benefit of less volatility in macroeconomic variables. At the same time, the level of $C^r$ is somewhat higher under the acyclical rule. This reflects the fact that aggregate demand rises under the acyclical regime – a consequence of higher public expenditures, including higher real wages ($w/p$) and employment ($n$), which also help explain higher $C^r$.

By contrast, consumption by Ricardian households $C^o$ does not differ as much across regimes. The variability of $C^o$ is somewhat lower under the acyclical regime; Ricardian households are able to neutralize most of the volatility inherent in a balanced budget regime. The level of $C^o$ stays flat over time under the acyclical rule but rises slightly under the balanced budget rule. This reflects different savings and asset accumulation across regimes. Under the balanced budget regime, Ricardian households save more in the initial periods. They build up assets and are hence able to maintain their consumption level.
Figure 4 Average of simulated series

- Log Government expenditures
- Public debt over GDP
- Log Consumption (Hand-to-mouth agents)
- Log Consumption (Ricardian agents)
- Log Employment
- Log Real wage
- Real interest rate
- Log Relative Price (Pd/Pm)
- Foreign debt over GDP
- Capital Stock over GDP
Quite the opposite, under the acyclical regime, the excessive accumulation of assets of the government causes a decrease in the external debt and then a small decrease in the real interest rate (see Figure 4, especially in the first 300 periods). This will be enough for that the Ricardian households choose to save less: their stock of assets falls, as must their consumption.

In the case of Ricardian households the intuition of the last result is directly related to the precautionary saving motive that is introduced by the second order approximation used to solve the model. The Ricardian agents incorporate optimally the variability of the commodity price shock as well. Thus they have a strong precautionary saving motive (measured by the rate of relative prudence), which stimulates the building up of assets when uncertainty is higher. On the contrary, when government follows an acyclical rule, the Ricardian agents will decrease their saving (or increase their debt) for the significant reduction in the volatility of the commodity price that they face. However, this rule leads also the government to save more than what would be optimal from the perspective of the Ricardian agents. Therefore as a result of this strong stabilization of the commodity price impact, these consumers decide optimally to save too little whenever the interest rate goes down so their expected consumption is lower than in the case of a balanced-budget rule.

V. Calculation of Welfare Levels

A. Methodology

We follow Kollman (2002), Kim and Kim (2003), and Elekdag and Tchakarov (2004) insofar as we also compute the second order approximations developed by Schmitt-Grohé and Uribe (2004) to solve the whole system of equations of the model. In this way, we can include the effect of the shocks on both the mean and the variance of consumption. Kim and Kim (2003) note, log-linearized business-cycle models are inappropriate for welfare analysis since they are unable to account for the effect of the variance of the shocks on economic decisions. This is a serious limitation, since welfare is generally determined by both the mean and the variance of certain variables (for example consumption). Thus, we compute the welfare gains generated by moving from one rule to the other, using the second order approximation of equations in the model. Doing so permits us to find the change in steady-state consumption (\( \tilde{x} \)) required to make any household indifferent (in expected utility terms) between the procyclical balanced budget and the acyclical spending rule. Such a calculation is in the spirit of Robert Lucas (1987). To do so, we start with a second order approximation of expected utility:

\[
E[U(C, N)] = E[U(\tilde{C}, \tilde{N}) + U_c(\tilde{C}, \tilde{N})E(C - \tilde{C}) + \\
\frac{1}{2} U_{cc}(\tilde{C}, \tilde{N})(C - \tilde{C})^2 + U_{nn}(\tilde{C}, \tilde{N})(N - \tilde{N})^2 + \frac{1}{2} U_{nn}(\tilde{C}, \tilde{N})(N - \tilde{N})^2]
\]

Thus, the specific second order approximation of the utility function (equation (3)) is:
\[
E[U(C, L)] = \frac{(\bar{C} - \psi \bar{N}^\gamma)^{1-\sigma} - 1}{1 - \sigma} + \bar{C} (\bar{C} - \psi \bar{N}^\gamma)^{1-\sigma} E(\hat{C}) - (\bar{C} - \psi \bar{N}^\gamma)^{1-\sigma} \psi \gamma \bar{N}^\gamma E(\hat{N})
\]

\[-\frac{1}{2} \sigma \bar{C}^2 (\bar{C} - \psi \bar{N}^\gamma)^{1-\sigma} \text{var}(\hat{C}) - \frac{1}{2} \sigma (\bar{C} - \psi \bar{N}^\gamma)^{1-\sigma} (\psi \gamma)^2 \bar{N}^{2\gamma} \text{var}(\hat{N})
\]

\[-\frac{1}{2} (\gamma - 1) (\bar{C} - \psi \bar{N}^\gamma)^{1-\sigma} \psi \gamma \bar{N}^\gamma \text{var}(\hat{N})
\]

Note that we use these transformations in the last expression \( \hat{X} = \frac{X - \bar{X}}{\bar{X}} \) and therefore \( \bar{X} E(\hat{X}) = E(X - \bar{X}) \) and \( \bar{X}^2 V(\hat{X}) = V(X - \bar{X}) \). Next, to simplify, we write expected utilities under the procyclical balanced budget and the acyclical spending rule as \( E[U(C, L)^{bb}] = \phi_1 \) and \( E[U(C, L)^{ss}] = \phi_2 \), respectively.

Now, note that:

\[
E[U(C + \xi, L)^{bb}] = \frac{(\bar{C}(1 + \xi) - \psi \bar{N}^\gamma)^{1-\sigma} - 1}{1 - \sigma} + (\text{other terms})^{bb} = \phi_2
\]

where \((\text{other term})^{bb} = \phi_1 - \frac{(\bar{C} - \psi \bar{N}^\gamma)^{1-\sigma} - 1}{1 - \sigma}\)

Thus, the incremental consumption required to equate expected utility across regimes, \( \xi \) is computed as:

\[
\xi = \frac{\left\{ (1 - \sigma) \left[ (\phi_2 - \phi_1) + \frac{(\bar{C} - \psi \bar{N}^\gamma)^{1-\sigma}}{1 - \sigma} \right]\right\}^{\frac{1}{1-\sigma}} - (\bar{C} - \psi \bar{N}^\gamma)}{\bar{C}}
\]

That is, \( \xi \) shows how much additional consumption would be required to make an individual just as well off under a balanced budget regime as under an acyclical spending rule.

**B. Results**

The results of this analysis are shown in **Figures 5 through 8**. To begin, Figure 5 shows the net welfare gain (measured as a percent of steady state consumption) implied in comparing acyclical versus balanced budget, against the variance of commodity prices.
As Figure 4 has already foreshadowed, “hand-to-mouth” consumers benefit from the acyclical rule, not only because their consumption stream is smoother, but also because it is slightly higher.

**Figure 5**

Welfare gain (WG) and Variance of Copper Prices ($\sigma_{copper}$).

WG: welfare under acyclical versus welfare under balanced budget regime (measured in percent of steady state consumption).
Figure 6
Variable difference between regimes
(acyclical minus balanced-budget rule for different commodity price volatilities)

*Each line is the average of the series that resulted from the simulations with the acyclical spending rule minus those obtained with the balanced-budget rule.
Figure 5 suggests that the larger the variance of the shock $\sigma_{copper}$, the more “hand to mouth” consumption. Figure 6 shows why: as $\sigma_{copper}$ grows, so does $C^r$ under the acyclical regime, both absolutely and relative to the balanced budget regime. An analogous result holds for government expenditure. Hence, “hand-to-mouth” agents benefit from the (Keynesian) demand stimulus, which results from the acyclical spending rule.

By contrast, Ricardian agents suffer somewhat under the acyclical rule relative to the balanced budget rule. Their consumption is slightly less volatile under the acyclical regime (Figure 3). However, in the more volatile balanced-budget environment, Ricardian households build their own precautionary assets, that includes capital stock, – from which they are able to later consume. Figure 6 supports these results. It shows as $\sigma_{copper}$ grows, the Ricardian agents’ consumption $C^o$ under the acyclical rule decreases. Once again, this reflects their lower asset levels that they do not build up in a more certain environment (a lower precautionary savings motive). As a result, their earnings and average consumption decreases over time.

In other words, even though the government has a mechanical acyclical rule it acts as if itself were an agent with a precautionary savings motive: it builds up a prudential asset stock that cushions spending today against shocks while also permitting it to spend more in the future. In turn, this provides a benefical externality for "hand to mouth consumers:” the government is providing a substitute for the precautionary savings that they themselves cannot do. Figure 6 illustrates, under the acyclical rule that the government does what Ricardian consumers would otherwise do under the balanced-budget regime. It accumulates a large amount of assets and ends up with larger revenues and spending. The stock of assets can amount to a large share of GDP if uncertainty increases steadily.

Nevertheless, and as we explained in Section 3.2, the mechanical acyclical rule causes an accumulation of assets that is far from being optimal from the perspective of the Ricardian consumers and hence the welfare of this kind of agents is much lower with the acyclical rule.\(^\text{14}\) These results also have important implications for the design of a general fiscal rule (see equation 54\textsuperscript{*}). Recall that $\alpha_r$ equals one minus the propensity to consume out of current income. Under a completely acyclical regime, $\alpha_r=0$; under a balanced budget regime, $\alpha_r=1$.

\(^{14}\) Note that the composition of assets is not invariant to the fiscal rule. The acyclical rule encourages more domestic investment in physical capital than the balanced budget. Lower volatility encourages more plant and equipment to be built within the country. By contrast, higher volatility under the balanced budget regime encourages Ricardian consumers to invest abroad due to the precautionary saving motive and the absence of domestic bonds.
Figure 7
Difference of welfare gains as measured in consumption units when $\mu_s$ increases
$\xi_{diff} = \text{“hand to mouth” } \xi^R - \text{Ricardian } \xi^\circ$

Figure 8
Difference of welfare gains as measured in consumption units when $\alpha_r$ increases
$\xi_{diff} = \text{“hand to mouth” } \xi^R - \text{Ricardian } \xi^\circ$
By contrast, $\mu_x$ is a debt targeting parameter: an increase in the value of $\mu_x$ implies that the government is targeting more closely the debt stock (the target is the initial value of zero), but with more volatility in government expenditures.\footnote{In a more general context, when target debt $b^*$ is, for example, 50 percent of GDP, a term like $\mu_x (b_t - b^*)$ would be necessary in the rule.}

Thus, \textbf{Figures 7 and 8} show the difference of welfare gains as measured in consumption units for both $\mu_x$ and $\alpha_r$ over a continuum of values (red line in \textbf{Figure 7}), namely $\xi^{\text{diff}} = \xi^R - \xi^o$, where $\xi^R$ and $\xi^o$ are the differences of welfare gains as measured in consumption units for “hand to mouth” and Ricardian agents, respectively.

As one would expect, raising either $\mu_x$ or $\alpha_r$ helps Ricardians but hurts hand-to-mouth units. However, raising $\mu_x$ is not identical to raising $\alpha_r$. Some simulations suggest that, if we maintain the acyclical element (keep $\alpha_r = 0$) but increase somewhat the debt targeting parameter $\mu_x$, we can benefit Ricardian agents at a very small cost to the hand to mouth agents. But, no such result can be obtained by raising $\alpha_r$ (see \textbf{Figure 8}): while Ricardian households always gain, the loss suffered by “hand-to-mouth” households is even greater. Intuitively, $\alpha_r$ is a blunter instrument than $\mu_x$. If $\mu_x$ rises, the stock of debt (or assets) must return to zero more quickly than otherwise. If $\alpha_r$ rises, more volatility is introduced directly -- through the commodity price channel. By contrast, if $\mu_x$ rises, more volatility in introduced but less directly --- through the spending channel. Hence, when comparing equivalent (basis point) increases in $\alpha_r$ and $\mu_x$, volatility increases by much less and hence “hand to mouth” households suffer much less.

Indeed, we find that, conditional on $\alpha_r = 0$, there is an optimal value of $\mu_x$ that would be agreed to by both consumers. So long as the gain to Ricardian consumers from increasing $\mu_x$ exceeds the loss suffered by “hand to mouth” agents ($\xi^{\text{diff}} < 0$), the former can compensate the latter. This is shown in \textbf{Figure 7}. For the (small) initial values of the simulations, the debt targeting parameter $\mu_x$ was very small ($\mu_x = 0.01$) – as shown on the horizontal axis, $\xi^{\text{diff}} < 0$. As $\mu_x$ rises, so does $\xi^{\text{diff}}$, the red line crosses the horizontal axis ($\xi^{\text{diff}} = 0$). From this point Ricardian cannot compensate “hand to mouth” consumers. Thus they are not willing to replace the acyclical rule by a more procyclical policy. Note however, that such an optimum ($\xi^{\text{diff}} = 0$) will only coincide with one that a social planner would choose for a special case, namely where the social planner’s weights on the utility of “hand-to-mouth” and Ricardians coincide \textit{exactly} with the values of $\lambda$ and $(1-\lambda)$, as defined above. Thus, if the social planner places a weight on “hand-to-mouth” consumers that is greater (less) than $1$, the optimal value for $\mu_x$ will fall (rise).
VI. SUMMARY AND CONCLUSIONS

We assess the welfare implications of reducing the volatility and procyclicality of government expenditures in countries that specialize in a primary resource-based commodity export, facing strong fluctuations of their fiscal income due to commodity price volatility. Public spending does contribute to aggregate demand (a Keynesian channel) and hence output. Importantly, government expenditure was assumed to be useless. This is so because our focus is to understand under this extreme assumption if there is some role for government spending in stabilizing external shocks and the business cycle.

Our policy, an acyclical spending rule, was geared to helping the most vulnerable “hand-to-mouth” consumers. We found that the policy was effective: it provided a substitute financial cushion, hence reducing the volatility of their consumption and increasing welfare. This policy boosted their mean consumption through (Keynesian) aggregate demand channels. However, others in society did not fare so well. “Ricardian” households that were able to optimize over time suffered. Initially, they saved less, a response that we would expect from agents with a precautionary saving motive. Their consumption was slightly less volatile, but over time their average consumption was also less. This result has an intuitive interpretation: effectively, under the acyclical spending rule, government limits what was the role of Ricardian households, namely to smooth consumption and accumulate assets.

An obvious alternative way of increasing consumers’ welfare is relaxing one of our modeling assumptions and allowing public spending to be useful. This is equivalent to giving back part of government spending to the consumers reducing our lump-sum taxes. However, we do not consider this case here.

What we did examine was a way to increase the welfare of the Ricardian households at a very low cost to the hand-to-mouth households. If the asset position of the government is limited through a somewhat more aggressive debt (asset) targeting stance (in our model a slightly higher $\mu_0$); Ricardians saved more initially than before, building up more assets, hence boosting consumption and welfare. In other words, with full capital mobility and financial market participants having access to a wide range of financial instruments, it may be better (welfare reaches a higher level) also include a debt / asset target – even at the expense of extra volatility in expenditures$^{16}$.

If policy makers wish to cushion society’s most vulnerable agents – those without access to capital markets and who have presumably the lowest wealth, our results show that fiscal policy should de-link public expenditures from current revenues. We conclude that the

---

$^{16}$ This could be implemented empirically through infrequent revisions of permanent income and spending.
acyclical rule, in this context, has a positive effect on the welfare of society as a whole depending on how much myopic or financially restricted consumers are.\footnote{This paper also touches upon some more general issues in optimal fiscal policy. For example, our optimal debt level is, essentially, a net credit position (in average). This is similar to a conclusion found in Aiyagari et al. (2002). In this aspect, our work recognizes that one goal of a government is to provide a financial cushion for “hand-to-mouth” households that are unable to do so for themselves (Tanner and Carey, 2005).}

REFERENCES


