Fiscal Rules for Commodity Exporters: Prudence and Procyclicality

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Abstract

This paper compares welfare levels under alternative fiscal rules for small open, commodity exporter, economies whose fiscal income varies with the world commodity price (in a dynamic, stochastic, and general equilibrium setting). Between the extremes of a procyclical balanced budget policy and an acyclical spending rule, there is a continuum of rules. Thus, the best degree of spending stabilization is found. The acylical rule benefits households that do not enjoy access to capital markets by providing a financial cushion that they themselves cannot provide, boosting their mean consumption. However, households that enjoy full access to capital markets suffer under this rule, since the government reduces their role in smoothing consumption and accumulating assets.

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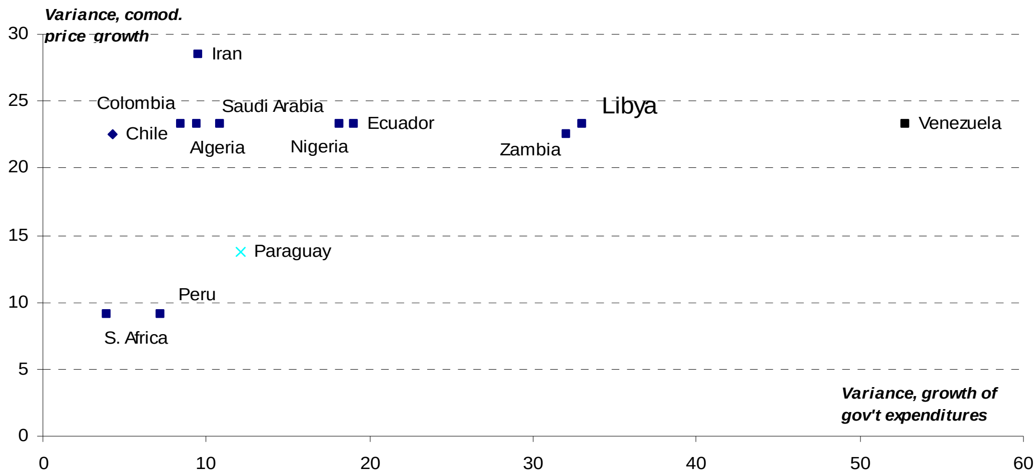
I. Introduction

Most available evidence suggests that, in emerging/developing economies, fiscal policy is *procyclical* (Kaminsky, Reinhart, and Végh, (2004), Talvi and Végh, (2005)). Government consumption typically increases and taxes often fall during expansions, while the opposite often happens during recessions. Moreover, in countries whose exports are concentrated in one or a few primary resource-based commodities, government expenditures often move closely with the world prices of these exports. Thus, as Figure 1 suggests, in such countries, higher volatility of government spending is associated with higher commodity price volatility. Of course, past fiscal indiscipline may play a role in procyclical fiscal behavior. Procyclical spending cuts often occur not only when commodity prices fall, but also after a buildup of public debt. Moreover, as Figure 2 suggests, spending shocks often have broader spillover effects, insofar as higher volatility in government spending is typically linked to higher volatility in economic growth.[2]

Figure 1. Volatility of the Commodity Price Growth
and Government Spending Growth



Source: International Monetary Fund, United Nations and Saudi Arabian Monetary Agency; Major commodities by country are: Copper (Chile, Zambia), Soy (Paraguay), Gold (Peru, South Africa), Natural Gas and Oil (Iran). Oil (all other countries).

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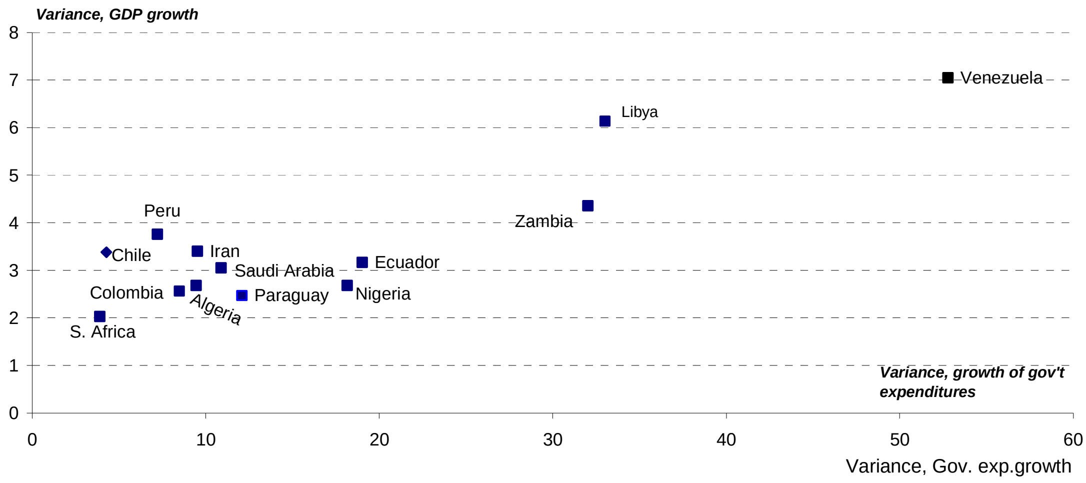
[2] In a related vein, Talvi, and Végh (2005) argue that pressures to increase public spending in countries that face large swings in their tax base, as is the case in many developing countries, are the cause of running a procyclical fiscal policy. Gavin et al (1996) and Gavin and Perotti (1997) have attributed this procyclical bias to the fact that developing countries are rationed from international credit markets in bad times.

Figure 2. Volatility of GDP and Government Spending Growth



Source: See Figure 1.

In this way, fiscal volatility may affect consumer welfare. For example, fiscal shocks may affect private consumption. Households that do not enjoy access to capital markets—"hand-to-mouth" or "non-Ricardian" households—are especially vulnerable in this aspect. Without their own financial buffer stocks, such households cannot smooth their consumption. Hence, when government spending falls, their disposable income and consumption fall with it. By contrast, households that do have access to capital markets—"Ricardian" optimizers—are better positioned to cushion themselves against such shocks. To the extent that these households have a precautionary motive (i.e. their utility function has a third moment), they will save and acquire a cushion of assets.

Governments may wish to protect these more vulnerable "hand-to-mouth" households from fiscal volatility. This implies that the government links its expenditures to steady-state, not current, revenues. Moreover, the government would acquire the precautionary cushion of assets on behalf of these consumers—something they cannot do for themselves. Ideally, government policy should be a sequence of taxes and transfers whose magnitudes would yield exactly the hypothetical sequence of consumption by households if were in fact "Ricardian." However, such a policy may be difficult to implement, since the government may not know what household preferences are. As a more practical alternative, some commodity exporting countries have simply chosen to reduce fiscal volatility by implementing a *fiscal rule* that breaks the link between current commodity prices and public spending. While such rules may be ad-hoc in nature, they may be easier to communicate and implement than other more complicated policies (like the tax / transfer scheme). This reflects Kydland and Prescott's (1977) suggestion that a fiscal rule should be transparent and easily understood so as to transparent and easily understood.

The goal of this paper is to examine how both the level of government demand and its volatility affect consumer welfare. Our perspective is more theoretical than empirical, even though we use a calibrated model. In our model, the role of the government is to some extent limited because there are two sources of government revenue: lump-sum taxes (assumed to be constant and policy invariant) and export revenue, which comes–manna like–from a resource-based commodity (in our model, copper) whose world-determined price fluctuates randomly.

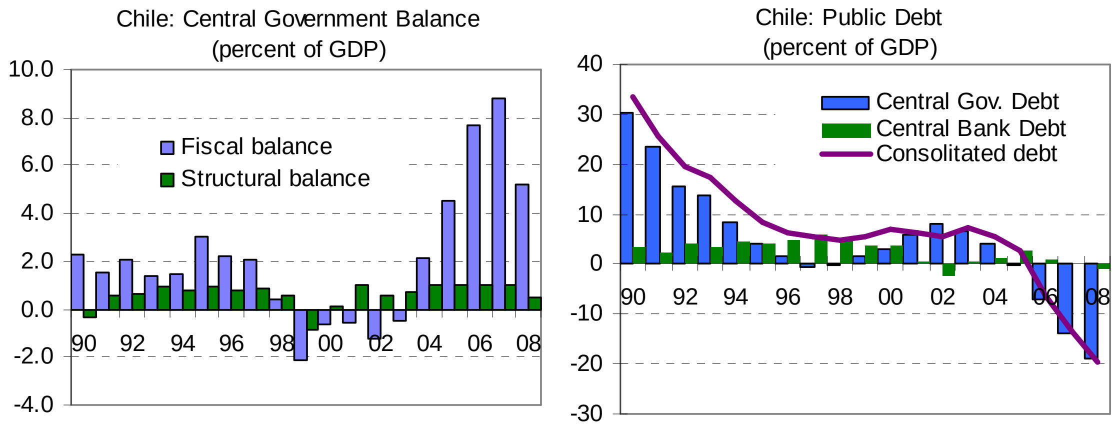
The government serves merely as a conduit for the manna. Its only choice is how much to spend and *when* to spend it on purchases of imports and domestic goods / services. Since the latter are supplied by households, government spending raises domestic household income. Otherwise, government expenditures are assumed to be inherently useless; they appear in neither utility nor production functions.

The fiscal rule determines how much the government spends and when. Hence, a fiscal rule affects both the level and volatility of government spending. We emphasize several desirable characteristics of a fiscal rule. First, a fiscal rule should be easily understood, consistent with Kydland and Prescott's (1977) idea. .Second, the rule should reduce volatility and provide a precautionary cushion of assets for the most vulnerable (i.e. "hand-to-mouth") households. (The idea that the government should be a net creditor is not new; see for example Ayiagari, Marcet, Sargent, and Seppäla (2002)).Third, and in a related vein, the government's net asset position – debtor or creditor – must be bounded. The government's net creditor (or debtor) position should not grow without limit. [3]

A *balanced budget* rule is perhaps the easiest rule to understand: expenditures must always equal revenues. However, such a rule is inherently procyclical: it brings volatility that is detrimental to vulnerable households. By contrast, some countries, for example Chile has adopted an *acyclical* (or *structural surplus*) rule in which expenditures are linked to *steady-state* (rather than current) commodity revenue, see Figure 3. This policy permitted Chile to reduce its public debt and even attain a net credit position in recent years. During the 2009 recession, the country had ample fiscal space for countercyclical spending.

[3] This is related to, but not the same as, the "no-Ponzi game" condition which specifies that the *present value* of net assets must tend to zero.

Figure 3. Chile: Central Government Balance and Public Debt



Source: Ministry of Finance and Central Bank of Chile

We show that an acyclical rule has an inherent bias toward attaining a bounded net creditor position. In adopting such a rule, the government effectively acts as if it had a precautionary savings motive. Importantly, such a rule has an important effect on the consumers' saving-spending decisions and therefore the degree of acyclicality can be calibrated to obtain welfare improvements. The net assets position serves as an extra financial cushion for the "hand-to-mouth" households—a kind of publicly provided precautionary savings that such households are unable to provide for themselves. However, under this policy, the government also reaps a 'dividend' that helps it to boost spending. In order to ensure that public asset growth is bounded, i.e. the "dividend" is bounded as well, we assume in the rule that government spending decreases when fiscal debt rises.

Under a balanced budget rule, spending fluctuates around a fixed mean. By contrast, under the acyclical rule, the government spends less in the early years and more later– and it does so on a smoother path. Also, these two types of rules are easily shown to be special cases of a more general fiscal rule – two specific points on a *continuum* of fiscal rules. We use this continuum to measure the impact on welfare of different types of acyclical rule.

Welfare is measured in terms of steady-state consumption (Lucas, 1987, Schmitt-Grohé and Uribe, 2007; Bergin et al, 2007), and compared across regimes. Importantly, the source of cross-regime welfare differences should lie in both the mean of consumption (first moment) and its variability (second moment)[4].

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4 Traditionally, simulations in general equilibrium models have been based on first-order log-linear approximations which did not allow meaningful welfare comparisons under uncertainty (see for example Kim and Kim (2003)). As a remedy, we follow the literature using an algorithm developed by Schmitt-Grohé and Uribe (2004), whose second-order

(continued…)

The simulations reveal that some agents will prefer one rule over the other. As expected, macroeconomic aggregates are less volatile under the acyclical regime than under the balanced budget regime, since expenditures follow a smoother path under former.[5] This especially benefits the non-Ricardian consumers who are unable to smooth out volatility on their own.

By contrast, Ricardians are better off under the balanced budget regime. Since they have access to capital markets and they can do their own smoothing, public efforts to smooth are redundant. Moreover, Ricardian households, unlike non-Ricardian ones, can benefit from a stream of government spending that is higher (in the initial years) and more volatile. Only Ricardians can save: they smooth their consumption stream and build up assets that fund higher consumption in the outer years[6]. Under the acyclical rule, the government postpones its spending, generating a negative wealth effect on the Ricardian consumers.

The remainder of the paper is organized as follows. In Section II, we present the model in its entirety. In Section III, we discuss the calibration of the parameters, present the simulation results and analyze the models' dynamics. In Section IV, we present the welfare analysis. Finally in Section V we summarize and conclude.

II. The Model

Our New Keynesian model most closely resembles one developed by Smets and Wouters (2002), but also draws on work by Woodford (2003), Clarida et al (1999), and Galí et al (2007). However, our model of a small open economy also includes: hand-to-mouth consumers (as in Galí et al, 2007), capital and investment with adjustment costs, raw materials, government, Greenwood, Hercowitz and Huffman (GHH 1988) preferences and a representative "Ricardian" agent (rather than overlapping generations). Our structure also

approximations permit us to assess the impact of policy-induced variability over other key economic variables, including consumption. This type of approximation has been previously used to evaluate several issues, including the benefits of capital mobility and international risk sharing (Kim and Kim, (2003); the relative merits of fixed-versus-floating exchange rate regimes (Elekdag and Tchakarov, 2007; Bergin et al, 2007); optimal monetary and fiscal rules (Schmitt-Grohé and Uribe, 2007).

[5] Also, under the acyclical rule, the government avoids some of the undesired currency appreciation—a Dutch disease that typically plagues non commodity exporters.

[6] Such an asymmetry generally presumes that there is an element of prudence (a non-zero third moment) in their utility function; see for example Carroll and Kimball (2006). A refinement of this argument is due to Huggett and Ospina (2001).

follows Gali and Monacelli's (2005) model of a representative agent with two goods (domestic and foreign) by using constant elasticity of substitution (CES) consumption baskets and price stickiness à la Calvo (1983). We close the small open economy by introducing a risk premium, following Schmitt-Grohé and Uribe (2003). Another essential reference among recent models for emerging economies is the general equilibrium model (GEM, Laxton and Pesenti, 2003). They have a very complex and more realistic structure to describe the relationship between final goods, intermediate goods and raw and semi finished materials.[7]

A. Households

We assume a continuum of infinitely lived households indexed by $i \in [0,1]$. Following Galí et al. (2007), a fraction of households λ consume their current labor income; they do not have access to capital markets and hence neither save nor borrow. Such agents have been termed "hand-to-mouth" consumers. The remainder $1-\lambda$ save, have access to capital markets, and are able to smooth consumption. Therefore, their intertemporal allocation between consumption and savings is optimal (Ricardian or optimizing consumers). Both segments optimize on the intratemporal margin in labor markets.

Consumption by Ricardian Households

The representative household maximizes expected utility

$$. \; E_o \sum_{t=0}^{\infty} \beta^t U\left(C_t^o(i), N_t^o(i)\right), \tag{1}$$

Subject to the budget constraint

$$\begin{aligned} P_t C_t^o(i) &= W_t(i) N_t^o(i) + B_t^o(i) - S_t B_t^{o*}(i) + D_t^o(i) - P_t T_t \\ &\quad - R_t^{-1} B_{t+1}^o(i) + S_t (\Phi(B_{t+1}^*) R_t^*)^{-1} B_{t+1}^{o*}(i), \end{aligned} \tag{2}$$

where $C_t^o(i)$ is consumption, $D_t^o(i)$ are dividends from ownership of firms, $\Phi(B_{t+1}^*)$ is the country risk premium, S_t is the nominal exchange rate, $B_t^{o*}(i)$ denotes private net foreign assets, *where we define a positive value of* $B_t^{o*}(i)$ *as debt*, $W_t(i)$ is nominal wage, $N_t^o(i)$ is the number of hours of work, $B_t^o(i)$ is government debt held by households, R_t and R_t^* are the gross nominal return on domestic and foreign assets (where $R_t = 1 + i_t$ and $R_t^* = 1 + i_t^*$) and T_t are lump-sum taxes.

[7] Laxton and Pesenti (2003) also assume habit formation in consumption, different price setting, nontradable goods and adjustment costs for the demand for imports and nontradable goods.

Our utility function (Correia et al, 1995) yields realistic values for consumption volatility:

$$U(C,N)=\frac{(C-\psi N^{\varphi})^{1-\sigma}-1}{1-\sigma} \tag{3}$$

Note that $1/\sigma$ is the intertemporal elasticity of substitution in consumption and $1/(\varphi-1)$ is the elasticity of labor supply to wages. The value of ψ is calibrated to obtain a realistic fraction of steady state hours worked. Note also that the rate of relative prudence is $\left(C_t^0(i)-\psi N_t^{\,0}(i)^{\varphi}\right) U_{CCC}/U_{CC}=-(1+\sigma)$. This statistic is important to explain precautionary savings -- one of the most important results of this article. As other authors have noted (Carroll and Kimball, 2006), *for any individual agent*, unless this statistic is non-zero, the level of consumption (and hence savings) will be invariant to volatility. The first-order condition for consumption is:

$$\left(C_t^0(i)-\psi N_t^{\,0}(i)^{\varphi}\right)^{-\sigma}=\beta\; E_t\left(\left(C_{t+1}^0(i)-\psi N_{t+1}^{\,0}(i)^{\varphi}\right)^{-\sigma} R_t\left(\frac{P_t}{P_{t+1}}\right)\right) \tag{4}$$

From the first order conditions it is also possible to derive the interest parity condition:

$$\frac{S_t}{P_t}=E_t\left[\left(\frac{S_{t+1}}{P_{t+1}}\right)\frac{R_t^*\Phi\left(B_{t+1}^*\right)}{R_t\left(\frac{P_t}{P_{t+1}}\right)}\right] \tag{5}$$

Consumption by Hand-to-Mouth Households

For "Non-Ricardian" households, utility is:

$$U(C_t^r(i),N_t^r(i)). \tag{6}$$

We assume that these households neither save nor borrow (Mankiw, 2000). As a result, their level of consumption is given by their disposable income:

$$P_tC_t^r(i)=W_t(i)N_t^r(i)-P_tT_t. \tag{8}$$

Labor Supply

Symmetric with the goods markets (discussed below), the continuum of monopolistically competitive households supply a differentiated labor service to the intermediate-goods-producing sector and a labor aggregator combines as much household-labor as is demanded by firms, with a constant-returns technology. The aggregate labor index has the CES form:

$$N_t = \left[\int_0^1 N_t(i)^{\frac{1}{1+\theta_\omega}} di \right]^{1+\theta_w} \tag{9}$$

where $N_t\ (i)$ is the quantity of labor used from each household. The representative labor aggregator minimizes the cost of producing a chosen amount of the aggregate labor index, given each household's wage rate $W_t\ (i)$. Then, she sells units of labor index at their unit cost W_t (with no profit), to the production sector:

$$W_t = \left[\int_0^1 W_t(i)^{-\frac{1}{\theta_\omega}} di \right]^{-\theta_\omega} \tag{10}$$

Note that, while prices are sticky, wages are completely flexible. Nominal wages are set by households so as to maximize their intertemporal objective function (1) subject to the intertemporal budget constraint (2) and to the total demand for their labor services, which is given by:

$$N_t(i) = \left[\frac{W_t(i)}{W_t} \right]^{-\frac{1+\theta_\omega}{\theta_\omega}} N_t \tag{11}$$

As a result the supply of each household is given by

$$W_t(i) = \left(1+\theta_w\right) \varphi\psi N_t\ (i)^{\varphi-1} \tag{12}$$

where $\left(1+\theta_w\right)$ is a mark-up over the current ratio of the marginal disutility of labor and the marginal utility of an additional unit of consumption. For rule-of-thumb households, wages are set at the average wage level of optimizing households.

Demand for Domestic and Imported Consumption Goods

Consumption is a CES aggregate of consumption of domestic $C_t^D(i)$ and imported goods $C_t^F(i)$, where η_C is the elasticity of substitution between domestic and foreign goods and α_C is the steady-state share of imported goods in total consumption:

$$C_t = \left(\alpha_c^{\frac{1}{\eta_C}} \left(C_t^D\right)^{\frac{\eta_c - 1}{\eta_C}} + (1-\alpha_c)^{\frac{1}{\eta_C}} (C_t^F)^{\frac{\eta_c - 1}{\eta_C}} \right)^{\frac{\eta_C}{\eta_C - 1}} \tag{13}$$

The demand for each set of differentiated domestic and imported goods, as derived from expenditure minimization, is:

$$C_t^D = \alpha_c \left(\frac{P_t^D}{P_t} \right)^{-\eta_c} C_t \tag{14}$$

$$C_t^F = (1-\alpha_c) \left(\frac{P_t^F}{P_t} \right)^{-\eta_c} C_t \tag{15}$$

A weighted average of either domestic or imported differentiated goods composes each type of good, which also consists of a Dixit-Stiglitz index:

$$C_t^K = \left(\int_0^1 C_t^K(j)^{\frac{\varepsilon_K - 1}{\varepsilon_K}} dj \right)^{\frac{\varepsilon_K}{\varepsilon_K - 1}}. \tag{16}$$

$$C_t^K(j) = \left(\frac{P_t^K(t)}{P_t^K} \right)^{-\varepsilon_K} C_t^K. \tag{17}$$

for K= D (domestic) and F (foreign). P_t..the aggregate consumer price index or CPI is defined as:

$$P_t = \left(\alpha_c (P_t^D)^{1-\eta_C} + (1-\alpha_c)(P_t^F)^{1-\eta_C} \right)^{\frac{1}{1-\eta_C}} \tag{18}$$

where the respective price index is:

$$P_t^K = \left(\int_0^1 P_t^K(j)^{1-\varepsilon_K} dj \right)^{\frac{1}{1-\varepsilon_K}}. \tag{19}$$

where K= D (domestic), F (foreign).

B. Firms

Domestic intermediate-goods firms

We assume a continuum of monopolistically competitive firms, indexed by $j \in [0,1]$ producing differentiated intermediate goods. The production function of the representative intermediate-good firm, indexed by (j) corresponds to a CES combination of capital $K_t(j)$ and labor $N_t(j)$, to produce $Y_t^D(j)$ and is given by:

$$Y_t^D(j) = A_t\left[\alpha K_t(j)_t^{\frac{\sigma_s-1}{\sigma_s}} + (1-\alpha)N_t^{\frac{\sigma_s-1}{\sigma_s}}(j)\right]^{\frac{\sigma_s}{\sigma_s-1}} \tag{20}$$

where A_t the technology parameter, and σ_s the elasticity of substitution between capital and labor, are both greater than zero.

The firms' costs are minimized taking as given the rental price of capital, R_t^k and the wage, W_t subject to the production function (technology). The relative factor demands are derived from the first-order conditions:

$$\frac{R_t^k}{W_t} = \left(\frac{\alpha}{1-\alpha}\right)\left(\frac{N_t(j)}{K_t(j)}\right)^{\frac{1}{\sigma_s}} \tag{21}$$

Thus, marginal cost is given by:

$$MC^D = \frac{1}{A_t}\left[\alpha^{\sigma_s}(R_t^k)^{1-\sigma_s} + (1-\alpha)^{\sigma_s}(W_t)^{1-\sigma_s}\right]^{\frac{1}{1-\sigma_s}} \tag{22}$$

When firm (j) receives a signal to optimally set a new price à la Calvo (1983), it maximizes the discounted value of its profits, conditional on the new price:

$$\max \sum_{k=0}^{\infty} \theta_D^k E_t\left\{\Lambda_{t,t+k} Y_{t+k}^D(j)(P_t^{D*}(j) - MC_{t+k}^D)\right\} \tag{23}$$

subject to:

$$Y_{t+k}^D(j) \leq \left(\frac{P_t^{D*}(j)}{P_t^D}\right)^{-\varepsilon_D} Y_{t+k}^D \tag{24}$$

Where the probability that a given price can be reoptimized in any particular period is constant and is given by $(1-\theta_D)$ and ε_D is the elasticity of substitution between any two differentiated goods. The price P_t^{D*} must satisfy the first order condition:

$$\sum_{k=0}^{\infty} \theta_D^k E_t\left\{\Lambda_{t,t+k} Y_{t+k}^D(j)\left(P_t^{D*}(j) - \frac{\varepsilon_D}{\varepsilon_D - 1} MC_{t+k}^D\right)\right\} = 0 \tag{25}$$

where the discount factor $\Lambda_{t,t+k}$ is:

$$\Lambda_{t,t+k} = \beta^k \left(\frac{C^0_{t+k} - \psi N^{0\ \varphi}_{t+k}}{C^0_t - \psi N^{0\varphi}_t} \right)^{-\sigma} \left(\frac{P_t}{P_{t+k}} \right).$$

Firms that did not receive the signal will not adjust their prices. Those who do reoptimize choose a common same price, P_t^{D*}. Finally, the dynamics of the domestic price index P_t^D is described by the equation:

$$P_t^D = \left[\theta_D (P_{t-1}^D)^{1-\varepsilon_D} + (1-\theta_D)(P_t^{D*})^{1-\varepsilon_D} \right]^{\frac{1}{1-\varepsilon_D}} \tag{26}$$

Intermediate-goods importing firms

As in the domestic sector, price setting in the import sector reflects little exchange rate pass-through in the short run (as in Galí and Monacelli, 2005, and Smets and Wouters, 2002). Such an assumption, while simplistic, provides realistic simulations (impulse response functions). This sector consists of firms that import a homogenous good from abroad and turn it into a differentiated foreign good for the home market using a linear production technology. Import firms are only allowed to change their price when they receive a random price-change signal. Thus, the dynamics of the import price index is also described by an equation similar to (24). But in this case, firms reset their price in response to variations in the exchange rate or the foreign price; they optimally charge the import price abroad expressed in domestic currency.

$$P_t^F = \left[\theta_F (P_{t-1}^F)^{1-\varepsilon_F} + (1-\theta_F)(S_t P_t^{F*})^{1-\varepsilon_F} \right]^{\frac{1}{1-\varepsilon_F}} \tag{27}$$

Note $(1-\theta_F)$ and ε_F have the same definition as before but here they apply to the intermediate-goods importing firms.

Final goods distribution

Total final output is expressed with a CES aggregator function (across firms). There is a perfectly competitive aggregator, which distributes the final good using a constant return to scale technology. It is valid for both K= D (domestic) and F (imported) goods:

$$Y_t^K = \left(\int_0^1 Y_t^K(j)^{\frac{\varepsilon_K - 1}{\varepsilon_K}} dj \right)^{\frac{\varepsilon_K}{\varepsilon_K - 1}} \tag{28}$$

$Y_t^K(j)$ is the quantity of the intermediate good (domestic or imported) included in the bundle that minimizes the cost of any amount of output Y_t. The aggregator sells the final good at its unit cost P_t with no profit:

$$P_t^K = \left(\int_0^1 P_t^K(j)^{1-\varepsilon_K} dj \right)^{\frac{1}{1-\varepsilon_K}} \tag{29}$$

where P_t is the aggregate price index. Finally, demand for any good $Y_t^K(j)$ depends on its price $P(j)$, which is taken as given, relative to the aggregate price level P_t:

$$Y_t^K(j) = \left(\frac{P(j)}{P_t} \right)^{-\varepsilon_K} Y_t^K \tag{30}$$

Optimizing investment firms and Tobin's Q

There are firms that produce homogenous capital goods and rent them to the intermediate-goods firms. Firms are owned exclusively by Ricardian households. Firms invest the amount so as to maximize firm value:

$$V^t(K_t^o) = R_t^k K_t^o - P_t^I I_t^o + E_t(V^{t+1}(K_{t+1}^o)) \tag{31}$$

subject to a capital accumulation constraint that includes an adjustment cost function $\phi(.)$.

$$K_{t+1}^o = (1-\delta)K_t^o + \phi\left(\frac{I_t^o}{K_t^o} \right) K_t^o \tag{32}$$

The first-order conditions are:

$$Q_t^o \phi'\left(\frac{I_t^o}{K_t^o} \right) - \frac{P_t^D}{P_t} = 0 \tag{33}$$

$$Q_t^o = E_t \left\{ \frac{1}{R_t} \left(\frac{P_{t+1}}{P_t} \right) \left[\frac{R_{t+1}^k}{P_{t+1}} + Q_{t+1}^o \left((1-\delta) + \phi_{t+1} - \frac{I_{t+1}^o}{K_{t+1}^o} \phi_{t+1}' \right) \right] \right\} \tag{34}$$

Equation (35) corresponds to Tobin's Q: the marginal cost of an additional unit of investment should be equal to the present value of the marginal increase in equity that it generates.

Demand for investment goods

Overall investment is equal to a CES aggregate of domestic and imported goods. Where η_I is the elasticity of substitution between domestic and foreign goods and α_I is the steady-state share of domestic goods in total investment.

$$I_t = \left(\alpha_I^{\frac{1}{\eta_I}} (I_t^D)^{\frac{\eta_I - 1}{\eta_I}} + (1-\alpha_I)^{\frac{1}{\eta_I}} (I_t^F)^{\frac{\eta_I - 1}{\eta_I}} \right)^{\frac{\eta_I}{\eta_I - 1}} \tag{35}$$

Demand for investment goods, domestic and imported respectively, is derived from expenditure minimization, namely:

$$I_t^D = \alpha_I \left(\frac{P_t^D}{P_t^I} \right)^{-\eta_I} I_t \tag{36}$$

$$I_t^F = (1-\alpha_I) \left(\frac{P_t^F}{P_t^I} \right)^{-\eta_I} I_t \tag{37}$$

A weighted average bundle of either domestic or imported differentiated goods thus comprises each type of investment good (a Dixit-Stiglitz index):

$$I_t^K = \left(\int_0^1 I_t^K(j)^{\frac{\varepsilon_K - 1}{\varepsilon_K}} dj \right)^{\frac{\varepsilon_K}{\varepsilon_K - 1}} \tag{38}$$

$$I_t^K(j) = \left(\frac{P_t^K(j)}{P_t^K} \right)^{-\varepsilon_K} I_t^K \tag{39}$$

for k=D, F . The aggregate price of investment (investment deflator) is defined as:

$$P_t^I = \left(\alpha_I (P_t^D)^{1-\eta_I} + (1-\alpha_I)(P_t^F)^{1-\eta_I} \right)^{\frac{1}{1-\eta_I}} \tag{40}$$

Each composite good is itself a bundle of differentiated goods

C. Exports

The demand for total domestic (non-copper) exports from foreign countries is:

$$X_t^D = \left(\int_0^1 X_t^D(j)^{\frac{\varepsilon_D - 1}{\varepsilon_D}} dj \right)^{\frac{\varepsilon_D}{\varepsilon_D - 1}} \tag{41}$$

Exports of good *J* depend on its own relative price:

$$X_t^D(j) = \left(\frac{P_t^D(j)}{P_t^D} \right)^{-\varepsilon_D} X_t^D \tag{42}$$

There is a demand for each set of differentiated domestic goods, which in turn depends on both total consumption abroad and on the home price of domestic goods (relative to its price in the foreign country):

$$X_t^D = \left[\left(\frac{P_t^D}{S_t P_t^{D*}} \right) \right]^{-\eta^*} C_t^{D*} \tag{43}$$

D. Aggregation

Total consumption is a weighted sum of consumption by Ricardian and rule-of-thumb agents:

$$C_t = \lambda C_t^r + (1-\lambda) C_t^o = \int_0^\lambda C_t^r(i) di + \int_\lambda^1 C_t^o(i) di \tag{44}$$

Since only Ricardian households invest and accumulate capital, total investment is equal to $(1-\lambda)$ times optimizing investment:

$$I_t = (1-\lambda)(I_t^o) \tag{45}$$

Likewise, the aggregate capital stock is:

$$K_t = (1-\lambda)(K_t^o) \tag{46}$$

Again, only optimizing households hold financial assets:

$$B_t = (1-\lambda)(B_t^o) \tag{47}$$

Foreign assets (or debt) include fiscal B_t^{G*}. and private held assets B_t^{o*}:

$$B_t^* = B_t^{G*} + (1-\lambda)B_t^{o*} \tag{48}$$

Hours worked are given by a weighted average of labor supplied by each type of consumer:

$$N_t = \lambda N_t^r + (1-\lambda)N_t^o \tag{49}$$

Finally, in equilibrium each type of consumer works the same number of hours:

$$N_t = N_t^r = N_t^o \tag{50}$$

E. Monetary policy

Even while this paper focuses on fiscal policy, price stability requires there also be an active central bank. Thus, in abbreviated way, we also include monetary policy: the central bank sets the nominal interest rate according to the following rule:

$$R_t = \overline{R}\left(\left(\frac{\Pi_t}{\overline{\Pi}}\right)^{\phi_\pi}\left(\frac{YR_t}{\overline{YR}_t}\right)^{\phi_y}\right) \tag{51}$$

where $\overline{R}$ is the steady state nominal interest rate, Π_t is total inflation, $\overline{\Pi}$ is steady state total inflation (assumed to be zero), $YR_{t.}$ is GDP without the natural resource and $\overline{YR}$ is steady state value.

F. Fiscal Policy

The government budget constraint is:

$$IT_t + R_t^{-1}B_{t+1}^G + S_t(\Phi(B_{t+1}^*)R_t^*)^{-1}B_{t+1}^{G*} = B_t^G + S_tB_t^{G*} + P_t^GG_t \tag{52}$$

where IT_t is total revenue (copper and otherwise), $B_{t.}^G$ is domestic public debt, $S_tB_{t.}^{G*}=v_bB_t^G$ is public foreign debt (a fixed proportion of domestic public debt) and $P_t^GG_t$ is public spending.

There are two sources of revenue. A domestic (non-copper) lump-sum tax that is assume to to always be in steady state: $P_tT_t = \overline{PT}, \forall t$. By contrast, copper revenue, which varies each period, is defined as $\tau_{cu}(S_tP_t^{cu}Q^{cu})$ where τ_{cu} is the share of the production of the natural resource (copper) owned by the government, P_t^{cu} is the world price of copper, and Q^{cu} is the quantity supplied. We assume that copper prices vary exogenously according to

$P_t^{cu} = \overline{P^{cu}}(1+\varepsilon_t)$, where $\varepsilon_t \sim N(0,\sigma_{cu})$. By contrast, the quantity of copper is assumed constant: $Q_t^{cu} = \overline{Q^{cu}}$.

Simple fiscal rules

Copper revenue is essentially "manna from heaven." The government purchases goods and services with this manna. A *fiscal rule* determines the intertemporal allocation of such spending.

This paper highlights fiscal rules that meet several criteria. The rule should be transparent and easily understood, as Kydland and Prescott (1977) emphasized. And, the government's net asset position – debtor or creditor – must be bounded. Neither net debt nor assets may grow without limit.

Our benchmark is a *balanced-budget* (*BB*) rule:

$$P_t^G G_t(BB) = IT_t - \tilde{r}_t \tilde{B}_t^G \tag{53a}$$

where $\tilde{r}_t$ is a weighted average (effective) interest rate on total debt (domestic plus foreign), namely:

$$\tilde{r}_t \equiv \frac{\tilde{R}_t - 1}{\tilde{R}_t} \equiv \left[\frac{R_t - 1}{R_t} + \frac{R_t^* \Phi(B_{t+1}^*) - 1}{R_t^* \Phi(B_{t+1}^*)} v_b \right]$$

and $\tilde{B}^G$ is a weighted average of domestic and foreign-held government debt. While transparency is a subjective criterion, most would agree that a balanced budget rule is easy to understand. Also, by definition, government debt is bounded at a constant (zero) under this rule.

However a drawback of this rule is that it exposes vulnerable consumers to market (copper price) volatility. We thus propose an alternative acyclcical (*AC*) or structural balance rule that provides a cushion against market volatility – especially for the non-Ricardian (or "hand-to-mouth") consumers who are unable to smooth their consumption stream. Under this rule, spending is linked one-to-one with steady-state (or structural - permanent) government revenues less interest payments, but with a small adjustment factor for the debt level (μ_x).

The AC rule is:

$$P_t^G G_t(AC) = \overline{IT} - \left[\mu_x + \frac{R_t - 1}{R_t} \right] \tilde{B}_t^G(AC) \tag{53b}$$

Essentially, under the *AC* rule, government spending tracks steady-state revenues more closely than current revenues. Under AC, spending is substantially less procyclical than under BB. But, procyclicality is not entirely eliminated under AC. When there is a adverse shock to copper prices $\varepsilon_t < 0$, there is an *incipient* rise in future debt $\tilde{B}_{t+1}^{G} < 0$. This raises the risk premium. According to the AC rule, expenditures must fall. Conversely, for a beneficial shock to copper prices $\varepsilon_t > 0$, spending increases.

Are public assets bounded under the AC rule? To investigate conditions under which the government's creditor position is bounded, consider a simplified case with no foreign debt ($B_t^{G*} = 0, \forall t$). Equation (52) is thus:

$$R_t^{-1} B_{t+1}^{G} = B_t^{G} + P_t^{G} G_t - IT_t$$

Substituting this equation (budget constraint) in the fiscal rule (53b), noting that, in this special case, $\tilde{R}_t \equiv (R_t - 1)/R_t$, and rearranging, we see that debt evolves between any two periods, t and t+1 according to:

$$B_{t+1}^{G}(AC) = \left(1 - R_t \mu_x\right) B_t^{G}(AC) + R_t \left(\overline{IT} - IT_t\right)$$

This equation converges if only if $0 < \left|1 - R\mu_x\right| < 1$. Clearly, μ_x must be non-zero. Otherwise, government debt would follow a random walk. To see this, consider a special case where $\mu_x = 0$ and $v = 0$ (no foreign debt):

$$B_{t+1}^{G} = B_t^{G} \qquad (53c)$$

In this case, if copper prices equal their steady state value $[\overline{\tau_{cu} SP^{cu} Q^{cu}} = \tau_{cu}(S_t P_t^{cu} Q^{cu})]$, total government debt stays constant. If there is an adverse shock to copper prices and $\mu_x = 0$, the level of debt will go up to the point where revenues and expenditures are once again equated. Put differently, if $\mu_x = 0$, government debt will remain at its new level forever unless there is another shock. Thus, the model will not converge.

What determines the behavior of public debt with the acyclical rule?

If we take expectation of the last expression, in the simplified case, with not government foreign debt, we are using for illustration purposes, we get:

$$E_t\left(B_{t+1}^{G}(AC)\right) = B_t^{G}(AC) - E_t\left(R_t\right)\mu_x B_t^{G}(AC),$$

where the change in the public debt depends negatively on its stock times the expression: $-E_t\left(R_t\right)\mu_x$.

$$E_t\left(\Delta B_{t+1}^G(AC)\right) = -E_t\left(R_t\right)\mu_x B_t^G(AC) .$$

We know that the interest rate $E(R_t)$ also depends on B_{t+1}^{*} for our assumption on how the economy, through the risk premium, goes back to equilibrium after a shock takes the current account out of balance (see equation 68 below). To be precise, we assume that, if the economy faces a negative shock and the government increases its debt level, there will be a higher interest rate (due to a hike in the risk premium), which will increase not only interest payments but also debt amortizations in the following periods by this amount $E_t\left(R_t\right)\mu_x$, as is clear in the last equation.[8]

Conversely, with a positive shock to the commodity price, the interest rate will post a reduction due to the fall in the risk premium, which is associated to a lower debt level or even a positive asset position. More specifically, the difference equation for the government debt level tells us that, to warrant convergence, spending should increase by the expression: $E_t\left(R_t\right)\mu_x$ whose absolute value is smaller than in the former case.[9]

In summary, spending cuts are larger than spending hikes as a result of the interaction between the mechanism that governs the behavior of the risk premium and the parameter that warrants stability of public debt (μ_x) in our rule. In other words, $\left|E_t\left(R_t \middle| shock\ to\ P_t^{cu} > 0\right)\mu_x\right|$ < $\left|E_t\left(R_t \middle| shock\ to\ P_t^{cu} < 0\right)\mu_x\right|$.The difference between these two expressions depends on the elasticity of the risk premium to the level of debt, which determinates the wedge between the costs of external funds and the opportunity cost of the country's own savings (the interest rate that the country obtains on its assets).[10]

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[8] The assumption of a risk premium hike whenever the government debt goes up after a negative shock to the commodity price is valid even in the case in which all government debt is domestic, because the shock is temporary and Ricardian equivalence does not hold in this model due to the presence of hand-to-mouth consumers. Therefore, a higher government debt should imply a current account deficit. For the more general case in which the government holds both types of debt (foreign and domestic), this is also true as is clear in the impulse response of the current account (figure 4).

[9] In that case, the economy will take longer to run out of those assets and get back to equilibrium because, as the equation shows, a lower interest rate debilitates the positive increase in spending that is implied by the AC rule.

[10] Mendoza and Oviedo (2004) find that in emerging economies under uncertainty, the "aversion to a collapse in outlays leads the government to respect a "natural debt limit" equal to the annuity value of the primary balance in a fiscal crisis."

(continued…)

The expected level of assets depends on the volatility of shocks, the elasticity of the risk premium to the level of debt, and the parameter μ_x (see Appendix 1 for an example).

The small procyclical adjustments are asymmetric if copper price volatility is sufficiently high: the small contractions during bad times are slightly greater than the small expansions during good times. This ensures that the government's position converges to one of a net creditor. If the government wants to smooth spending to provide a cushion against market volatility, it needs to have this buffer against the asymmetry built in the debt dynamics.In so doing, it performs a kind of precautionary saving that hand-to-mouth agents cannot do for themselves. Otherwise, the government will be forced to cut spending in the presence of negative shocks being unable to implement the acyclical spending rule. The welfare implications of such a policy are explored in the next section.

A more general fiscal rule

It may be easily seen that rules *BB* and *AC* are merely two options along a continuum of a general rule (*GR*) whose form is:

$$P_t^G G_t(GR) = \overline{IT} - \left[R_t + \mu_x\right] B_t^G + \alpha_r (IT_t - \overline{IT}) \tag{54}$$

Thus, for *BB*, $\alpha_\gamma = 1, \mu_x = 0$; for an *AC* regime, $\alpha_\gamma = 0$, and $0 < \mu_x < R^{-1}$. For other intermediate rules, $0 < \alpha_\gamma < 1$, and $0 < \mu_x < R^{-1}$. We may thus think of alternative pairings of $[\alpha_\gamma, \mu_x]$ as ways of introducing both the mean and variance of government spending in a continuous fashion.

Government demand for domestic and imported goods

The government demands domestic and imported goods, according to:

$$G_t = \left(\alpha_G^{\frac{1}{\eta_G}} (G_t^D)^{\frac{\eta_G - 1}{\eta_G}} + (1 - \alpha_G)^{\frac{1}{\eta_G}} (G_t^F)^{\frac{\eta_G - 1}{\eta_G}} \right)^{\frac{\eta_G}{\eta_G - 1}} \tag{55}$$

where η_G is the elasticity of substitution between domestic and foreign goods and α_G is the steady-state share of domestic goods in total government expenditure.

The demand for domestic and imported goods derived from expenditure minimization is given by:

$$G_t^D = \alpha_G \left(\frac{P_t^D}{P_t^G} \right)^{-\eta_G} G_t \tag{56}$$

$$G_t^F = (1-\alpha_G) \left(\frac{P_t^F}{P_t^G} \right)^{-\eta_G} G_t \tag{57}$$

Each type of good (domestic, imported) consumed by the government is composed of a weighted average of differentiated goods, which also consists of a Dixit-Stiglitz index:

$$G_t^K = \left(\int_0^1 G_t^K(j)^{\frac{\varepsilon_K - 1}{\varepsilon_K}} dj \right)^{\frac{\varepsilon_K}{\varepsilon_K - 1}} \tag{58}$$

$$G_t^K(j) = \left(\frac{P_t^K(j)}{P_t^K} \right)^{-\varepsilon_K} G_t^K \tag{59}$$

for K= D (domestic), F (foreign). The aggregate price deflator of government spending is:

$$P_t^G = \left(\alpha_G (P_t^D)^{1-\eta_G} + (1-\alpha_G)(P_t^F)^{1-\eta_G} \right)^{\frac{1}{1-\eta_G}} \tag{60}$$

Domestic and imported goods are themselves bundles of differentiated goods

G. Market-Clearing Conditions

The factor market-clearing conditions are total employment by all firms j:

$$N_t = \int_0^1 N_t(j) dj \tag{61}$$

and full capital utilization

$$K_t = \int_0^1 K_t(j) dj \tag{62}$$

The good market-clearing condition is:

$$Y_t^D(j) = (C_t^D(j) + I_t^D(j) + G_t^D(j) + X_t^D(j)) \tag{63}$$

Using equation (17) and (30), (39), (43) and (59), we obtain:

$$Y_t^D(j) = \left(\frac{P_t^D(j)}{P_t^D}\right)^{-\varepsilon_D} (C_t^D + I_t^D + G_t^D + X_t^D) \tag{64}$$

Equation (64) should be plugged into equation (28), which is:

$$Y_t^K = \left(\int_0^1 Y_t^K(j)^{\frac{\varepsilon_K - 1}{\varepsilon_K}} dj\right)^{\frac{\varepsilon_K}{\varepsilon_K - 1}}$$

for K=,D, F. In turn, this yields

$$Y_t^D = \left[\int_0^1 \left(\frac{P_t^D(j)}{P_t^D}\right)^{-\varepsilon_D} \left(C_t^D + I_t^D + G_t^D + X_t^D\right)^{\frac{\varepsilon_D - 1}{\varepsilon_D}} dj\right]^{\frac{\varepsilon_D}{\varepsilon_D - 1}}$$

Adding up and simplifying yields the symmetric equilibrium for the domestic market:

$$Y_t^D = (C_t^D + I_t^D + G_t^D + X_t^D) \tag{65}$$

where the total supply of domestic goods equals total demand of the domestic produced good for consumption, investment, government spending and exports. Finally, the economy-wide budget identity can be expressed as:

$$\begin{aligned} P_t C_t = & -P_t^G G_t - P_t^I I_t + P_t^D Y_t^D + (P_t^F Y_t^F - S_t P_t^{*F} Y_t^F) + \\ & S_t \left(\Phi\left(B_{t+1}^*\right) R_t^*\right)^{-1} B_{t+1}^* - S_t B_t^* + \\ & \tau_{cu}(S_t P_t^{cu} Q^{cu}) \end{aligned} \tag{66}$$

Equation (66) has an intuitive interpretation. First note that GDP is the (approximately) sum of domestically produced goods plus value added on the distribution of imports, plus copper exports: [11]

$$P_t Y_t = P_t^D Y_t^D + (P_t^F Y_t^F - S_t P_t^{*F} Y_t^F) + (S_t P_t^{cu} Q^{cu}) \tag{67}$$

Thus, according to the national income accounting identity, consumption must equal GDP minus investment (I) and government expenditures G plus foreign debt (positive values of B_t^*), which is written:

$$S_t \left(\Phi\left(B_{t+1}^* \right) R_t^* \right)^{-1} B_{t+1}^* - S_t B_t^* \tag{68}$$

The risk premium ensures that the economy returns to the steady state[12], thus this variable increases with the foreign debt.

III. Calibration and Dynamics

We choose Chile as our benchmark country for the calibration because it has been a leader within emerging commodity exporters in implementing an acyclical fiscal rule.[13] Unfortunately, many parameters have never been obtained using Chilean data. For this reason, we calibrate the model taking sensible values from different studies (see Table 1).[14] For example, the discount factor β is 0.99 close to the values found elsewhere in the literature. The risk aversion coefficient σ is greater than one (2.0) as the evidence indicates for small open economies.[15] Thus, the relative prudence coefficient is: $\left(C_t^0(i) - \psi N_t^{\,0}(i)^{\varphi}\right) U_{CCC} / U_{CC} = -(1+\sigma)$=-3. This ensures that Ricardian agents will save more as output volatility rises.[16]

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[11] We assume for simplicity that there are no private copper exports; we treat them as if they were transfers from abroad.

[12] See Schmitt-Grohé and Uribe (2004).

[13] The steady state values are consistent with those obtained for the Chilean Economy where foreign debt is around 50 percent of the GDP. See for example Restrepo and Soto (2006).

[14] We assume that each period corresponds to one quarter.

[15] See Agénor and Montiel (1996), Table 10.1, page 353.

[16] For our chosen utility function, there is no closed form solution linking consumption and volatility. An approximation is found in Talmain (1998).

The elasticity of substitution across intermediate goods, ε_D and ε_F, is 6, in order to have a mark-up of 20%, the fraction of firms that keep their prices unchanged each period, θ_D and θ_F, is 0.75 and the rate of depreciation δ is 0.025. All these values are standard in the literature on the New Keynesian models (Woodford (2003), Galí and Monacelli (2005) and Galí et al (2007)).

Table 1. Baseline Parameters

Discount factor (β)	0.99
Risk aversion coefficient (σ)	2.00
Disutility parameters, worked hours (N)	
φ	1.70
ψ	7.02
Weight of rule-of-thumb consumers (λ)	0.50
Rate of depreciation (δ)	0.025
Investment adjustment cost ϕ	1/15
Elasticity of substitution across intermediate goods (ε_D, ε_F)	6.00
Parameter of CES production function (α)	0.40
Fraction of firms that keep their prices unchanged (θ_D, θ_F)	0.75
Real wage mark-up ($1+\theta_W$)	1.20
Elasticity of substitution between capital and labor (σ_S)	1.00
Response of monetary authority to inflation (φ_π)	1.50
Response of monetary authority to output (φ_{yr})	0.00
Autoregressive coefficient of copper price	0.80
Share of the production of the natural resource owned by the government (τ_{cu})	0.50
Amount produced of the natural resource (Q^{cu})	0.45
Weight of domestic good in consumption (α_C)	0.60
Weight of domestic good in investment (α_I)	0.50
Weight of domestic good in government expenditure (α_G)	0.99
Foreign-domestic good (consumption) elasticity of substitution (η_C)	0.99
Foreign-domestic good (investment) elasticity of substitution (η_I)	0.99
Foreign-domestic good (government) elasticity of substitution (η_G)	0.99
Acyclical rule, debt weight (μ_X)	0.01
The share of external public debt over total public debt v_b	0.21
Elasticity of interest rate to external debt	0.001
Elasticity of domestic export to real exchange rate (η*)	1.00

For the labor market, we suppose the same mark up as in the good market, i.e. θ_w is 0.2. The value of φ (=1.7) comes from Correia et al (1995), who introduced GHH utility function in RBC models for small open economies to explain the higher volatility of the consumption observed in these countries. As they do, we choose a value for ψ (=7.02) to ensure that hours worked in steady state coincide with actual data in our benchmark country. The value of the investment adjustment cost ϕ is 1/15, which is half of the value of Correia et al (1995). Half of households are hand-to-mouth, i.e. λ is 0.5, which is within the range of values considered in other studies (Mankiw (2000) and Galí et al (2007)). We assume that government spending is heavily biased towards domestic goods. Indeed, the share of domestic goods in the government consumption basket α_G is 0.99.

This allows us to replicate a stylized fact: in many commodity exporting countries, increases in government spending cause real appreciations (Edwards, 1989). We do not have information about the values of the elasticity of substitution between domestic and foreign goods (η_C, η_I, and η_G), thus we assume values close to 1 one (following Galí and Monacelli, 2005). For the same reason we choose values for α_C and α_I close to 0.5 (also following Galí and Monacelli, 2005) as a measure of openness.

Even though public debt is not exactly zero in Chile, we assume it to be so in our model's steady state. This assumption helps us to compare the acyclical rule with the balanced budget regime: to do so, both policies must share the same steady state. Also, we assume that 21 percent of public debt is held by foreigners (v_b=0.21); this value comes directly from historic Chilean data. In our baseline simulation, the coefficient in the monetary rule with respect to inflation ϕ_π is 1.5, which is a standard one for Taylor rules. The interest rate response with respect to the output gap ϕ_{yr} is assumed to be zero. Likewise, the elasticity of substitution between capital and labor σ_S is 1.0. Thus α is the capital share and is assumed to be 0.4 given that this value in Chile is higher than in other countries (in the US, α=0.33). The elasticity of domestic exports to the real exchange rate η^* is 1.0 in line with estimations for developing countries (Ghei and Pritchett, 1999).

The autoregressive coefficient of the real price of copper ρ is 0.8 obtained from quarterly data from 1973 through 2005. We choose small values for the debt weight μ_X (=0.01) in the acyclical rule and the elasticity of the interest rate to external debt (0.001). Both coefficients warrant the stability of the model. The first one makes public debt a stationary variable. The second one forces the current account to be stationary as well as net foreign assets.

IV. EFFECTS OF A COMMODITY (COPPER) PRICE SHOCK

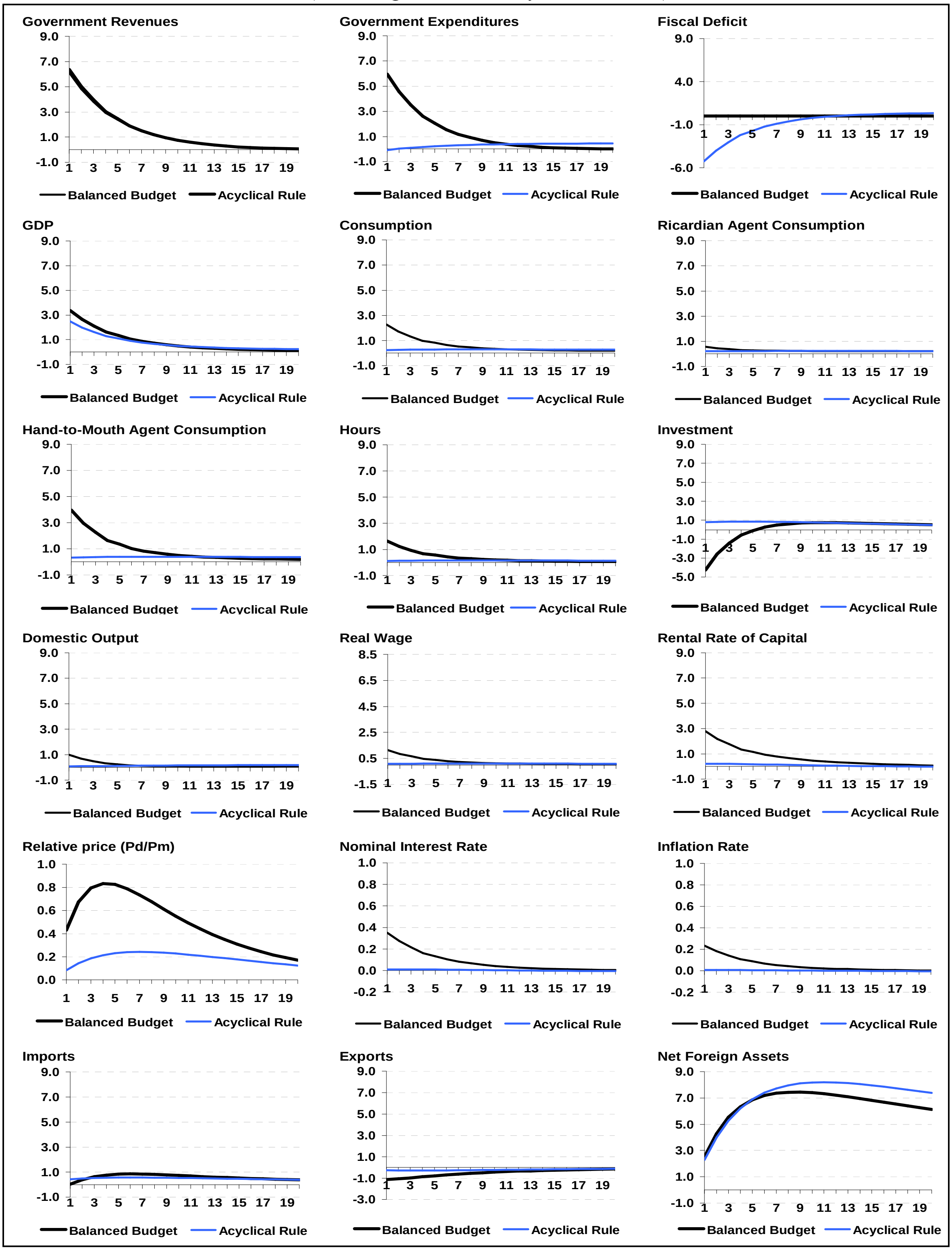
To illustrate how the model economy works under the two extreme alternative fiscal rules, we discuss several shock experiments below. We simulate 100 artificial economies, 1000 period each, hitting the economy with a random shock to the price of copper each period.[17]

[17] We also simulated the economy 10.000 periods ahead and the qualitative results did not change.

To begin, Figure 4 shows responses of several macroeconomic variables to one standard deviation (20%) copper price shock.[18] In each of the small charts, the black and grey-blue lines represent impulse responses under balanced budget and acyclical structural balance, respectively.

[18] The size of the shock in our simulations is just enough to obtain a standard deviation of the real price of copper similar to the empirical one (33%) for the period 1973-2005.

Figure 4. Responses to a Price of Copper Shock
(% change from steady state values)



The balanced-budget rule (black line) yields patterns that are generally procyclical. When copper prices increase, so do government revenues, expenditures, hand-to-mouth private consumption, hours worked, and output. Likewise, the real value of the currency appreciates. Note that, to gauge real appreciation, we look at the relative price of the domestic good $P_d/P_{m.}$ Since we assume that it is a good proxy for the inverse of the real exchange rate. The real appreciation also reduces non-commodity exports. Inflation rises, as does the real interest rate (via the Taylor rule). Note however, that investment falls: capital expenditures are crowded out by the copper price boom.

By contrast, the acyclical rule (grey-blue line) displays behavior that is less procyclical. By definition, the behavior of government revenues is invariant to regime. However, government spending shows virtually no response to the shock. Instead the public balance is positively related to the copper price shock. GDP and consumption increase only slightly. In addition, the currency shows a much more modest appreciation than under the balanced budget rule, and exports remain largely unchanged. Inflation and interest rates remain unaffected, as does investment; there is no crowding out. Thus, for most variables, volatility is greater under the balanced budget rule than under the acyclical rule. One important exception to this observation is consumption by Ricardian households, who are able to almost fully neutralize the otherwise volatile effects of government policy.

Some other crucial differences between the rules are shown in Figure 5a-b. There is an important difference regarding the evolution of government debt. Under the balanced budget regime, government debt is always, by definition, equal to its initial value, namely zero. By contrast, under the acyclical rule, the government accumulates assets. Over time, the average public debt stabilizes: $B^g \approx -30$ percent of GDP. Our simulations of the AC rule confirm the explanation given in section II.F (fiscal policy) and Appendix 1. Our structural balance rule, which provides a cushion against market volatility, needs to have these assets as a buffer against the asymmetry produced by the volatility of shocks, the risk premium and the mechanism that warrants the stationarity of public debt.

Predictably, consumption by "hand-to-mouth" households C^r differs substantially across regimes. Under the acyclical rule, the variability of C^r is substantially smaller than under the balanced budget rule. This feature will be critical for our welfare comparisons between rules (below). At the same time, the level of C^r is somewhat higher under the acyclical rule. This reflects the fact that aggregate demand rises under the acyclical regime—a consequence of higher public expenditures, including higher real wages (*W/P*) and employment (*N*), which also help explain higher C^r.

On the contrary, the variability of consumption by Ricardian households C^o does not differ much across regimes. Ricardian households are able to neutralize most of the volatility inherent in a balanced budget regime. The level of C^o stays flat over time under the acyclical rule, but rises slightly under the balanced budget rule. This reflects different saving rates and asset accumulation across regimes. Under the balanced budget regime, Ricardian households save more in the initial periods. They build up assets and hence are able to maintain their consumption level.

Figure 5a. Average of Simulated Series

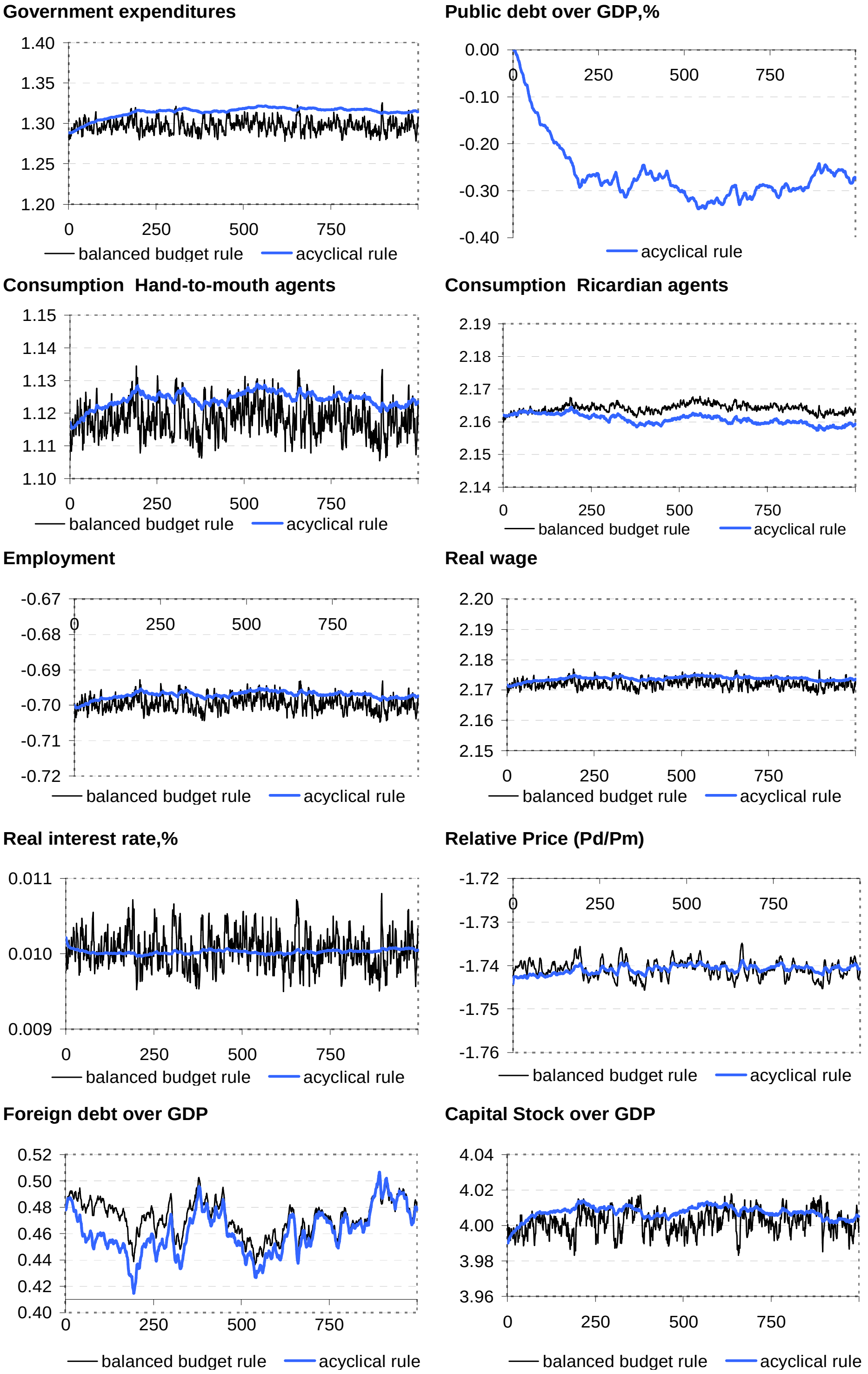
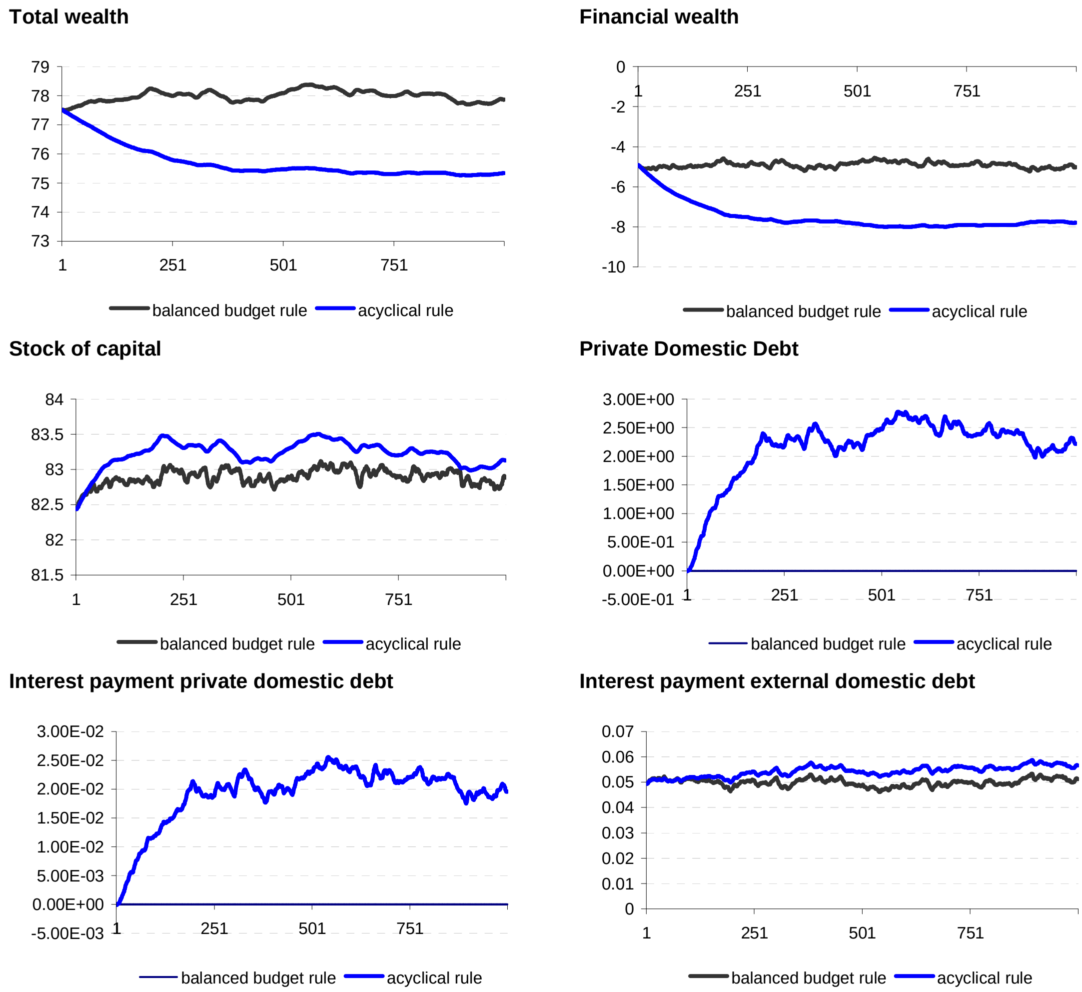


Figure 5b. Average of Simulated Series



Under the acyclical regime, asset accumulation by the government is mirrored by a decrease in private financial wealth, which belongs to Ricardians. Since the government saves, agents have lower income in the early periods and accumulate debt to finance their consumption. In the long run, consumption by Ricardians is lower because they need to pay interests on their debts.

The intuition of the last result is directly related to the fact that Ricardian agents know how to save optimally, incorporating the variability of the commodity price shock. Thus, they have a strong precautionary saving motive (measured by the rate of relative prudence), which stimulates asset build-up when uncertainty is higher. The government saves following a mechanical acyclical rule which is not optimal from the perspective of the Ricardian agents. As a consequence of the government's asset build-up, these consumers save less. This behavior can be observed in Graph 5.a and 5.b where, in our acyclical rule, while total and financial wealth decrease, government assets grow. On the contrary, Ricardians would prefer to receive those resources and administer them by themselves rather than giving that role to the government, which postpones its spending infringing a negative wealth effect on the Ricardian consumers.

V. Calculation of Welfare Levels

A. Methodology

We follow Kollman (2002), Kim and Kim (2003), Elekdag and Tchakarov (2004), Bergin et al (2007) insofar as we also compute the second order approximations developed by Schmitt-Grohé and Uribe (2004) to solve the whole system of equations of the model. In this way, we can include the effect of the volatility on the mean of consumption. Kim and Kim (2003) note, log-linearized business-cycle models are inappropriate for welfare analysis since they are unable to account for the effect of the variance of the shocks on economic decisions. Thus, we compute the welfare gains generated by moving from one rule to the other, finding the change in steady-state consumption (ξ) required to make any household indifferent (in expected utility terms) between the procyclical balanced budget and the acyclical spending rule. Such a calculation is in the spirit of Lucas (1987). To do so, we start taking unconditional expectations of a second order approximation of expected utility:

$$\begin{aligned} E\left[U(C,N)\right] = {} & E[U(\bar{C},\bar{N}) + U_c(\bar{C},\bar{N})E(C_t-\bar{C}) + \\ & \frac{1}{2}U_{cc}(\bar{C},\bar{N})(C_t-\bar{C})^2 + U_n(\bar{C},\bar{N})(N_t-\bar{N}) + \frac{1}{2}U_{nn}(\bar{C},\bar{N})(N_t-\bar{N})^2] \end{aligned} \quad (69)$$

The specific second order approximation of the utility function (equation (3)) is:

$$\begin{aligned} E\left[U(C,N)\right] = {} & \frac{\left(\bar{C}-\psi\bar{N}^{\gamma}\right)^{1-\sigma}-1}{1-\sigma} + \bar{C}\left(\bar{C}-\psi\bar{N}^{\gamma}\right)^{-\sigma}E(\hat{C}_t) - \left(\bar{C}-\psi\bar{N}^{\gamma}\right)^{-\sigma}\psi\gamma\bar{N}^{\gamma}E(\hat{N}_t) \\ & -\frac{1}{2}\sigma\bar{C}^2(\bar{C}-\psi\bar{N}^{\gamma})^{-\sigma-1}\operatorname{var}(\hat{C}_t) - \frac{1}{2}\sigma(\bar{C}-\psi\bar{N}^{\gamma})^{-\sigma-1}\left(\psi\gamma\right)^2\bar{N}^{2\gamma}\operatorname{var}(\hat{N}_t) \\ & -\frac{1}{2}(\gamma-1)(\bar{C}-\psi\bar{N}^{\gamma})^{-\sigma}\psi\gamma\bar{N}^{\gamma}\operatorname{var}(\hat{N}_t) \end{aligned} \quad (70)$$

Note that we use these transformations in the last expression $\hat{X} = \frac{X-\bar{X}}{\bar{X}}$ and therefore $\bar{X}E(\hat{X}) = E\left(X-\bar{X}\right)$ and $\bar{X}^2V(\hat{X}) = V\left(X-\bar{X}\right)$. Next, to simplify, we write expected utilities under the procyclical balanced budget and the acyclical spending rule as $E\left[U(C,N)^{bb}\right] = \phi_1$ and $E\left[U(C,N)^{ss}\right] = \phi_2$, respectively.

Now, note that:

$$E\left[U(C(1+\xi^{unconditional}),N)^{bb}\right]=\frac{\left(\bar{C}(1+\xi^{unconditional})-\psi\bar{N}^{\gamma}\right)^{1-\sigma}-1}{1-\sigma}+(other\ terms)^{bb}=\phi_2 \qquad (71)$$

where $\left(other \quad term\right)^{bb} = \phi_1 - \frac{\left(\bar{C}-\psi N^{\gamma}\right)^{1-\sigma}-1}{1-\sigma}$

Thus, the incremental consumption required to equate expected utility across regimes, (ξ) is computed as:

$$\xi^{unconditional}=\frac{\left\{(1-\sigma)\left[(\phi_2-\phi_1)+\frac{\left(\bar{C}-\psi\bar{N}^{\gamma}\right)^{1-\sigma}}{1-\sigma}\right]\right\}^{\frac{1}{1-\sigma}}-\left(\bar{C}-\psi\bar{N}^{\gamma}\right)}{\bar{C}} \qquad (72)$$

That is, ξ shows how much additional consumption would be required to make an individual just as well off under a balanced budget regime as under an acyclical spending rule.

The welfare gains of moving from one rule to the other were also computed using conditional measures of utility. This strategy takes into consideration the transition when one of the rules is applied because by computing the discounted sum of expected utilities, it considers the costs of sacrificing consumption for precautionary reasons when the balance budget is put in place (Schmitt-Grohé and Uribe, 2007; Bergin et al 2007).

$$U((1+\xi^{conditional})C,N)=(1-\beta)\sum_{t=0}^{\infty}\beta^{t}E_{o}\left[U(C,N)\right] \qquad (73)$$

B. Results

Before discussing the results in details, there are some impacts of the choice of fiscal rule that are quite intuitive and easy to see.

Implication (1): *Welfare for both types of consumers will increase when government expenditures increase.* This implication is uncontroversial. It simply reflects the fact that the government has an exclusive right to spend its manna from heaven (copper revenues) when it chooses (according to the rule). More spending raises the demand for domestic (as well as imported) goods and services, whose ultimate suppliers are the economy's households – both Ricardian and non-Ricardian.

Implication (2): *Discounted welfare for both types of consumers will increase when the time profile of government expenditures is shifted towards the present.* As before, this implication is simply a consequence of the government's distribution rights for manna (copper). More spending now raises the demand for goods and services supplied by the economy's households – now.

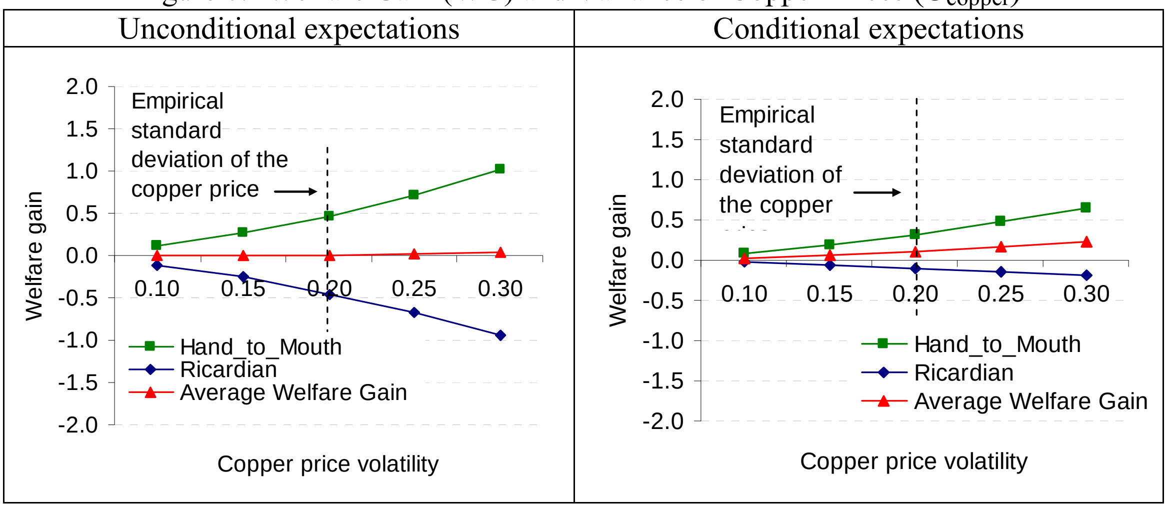
Implication (3): *Reducing the variance of government expenditures helps non-Ricardian consumers more than Ricardian ones; the latter are able to smooth their consumption stream on their own.*

Implication (4): *Reducing the variance of government expenditures reduces asset accumulation by Ricardian consumers.* In a smoother environment (AC versus BB), Ricardians have less incentive to save on a precautionary basis. This implication is not necessarily bad for Ricandian consumers. Extra saving could also be costly in terms of welfare.

Implication (5): *For these reasons, it is expected that non-Ricardians will prefer AC over BB while (based on the first two implications) Ricardians might prefer BB over AC.*

The results of this analysis are shown in Figures 5 and 6. To begin, Figure 5 shows the net welfare gain (measured as a percent of steady state consumption) implied in comparing acyclical versus balanced budget, against the variance of commodity prices.

Figure 6. Welfare Gain (WG) and Variance of Copper Prices (σ_{copper})*



WG: welfare under acyclical minus welfare under balanced budget regime (measured in percent of steady state consumption).

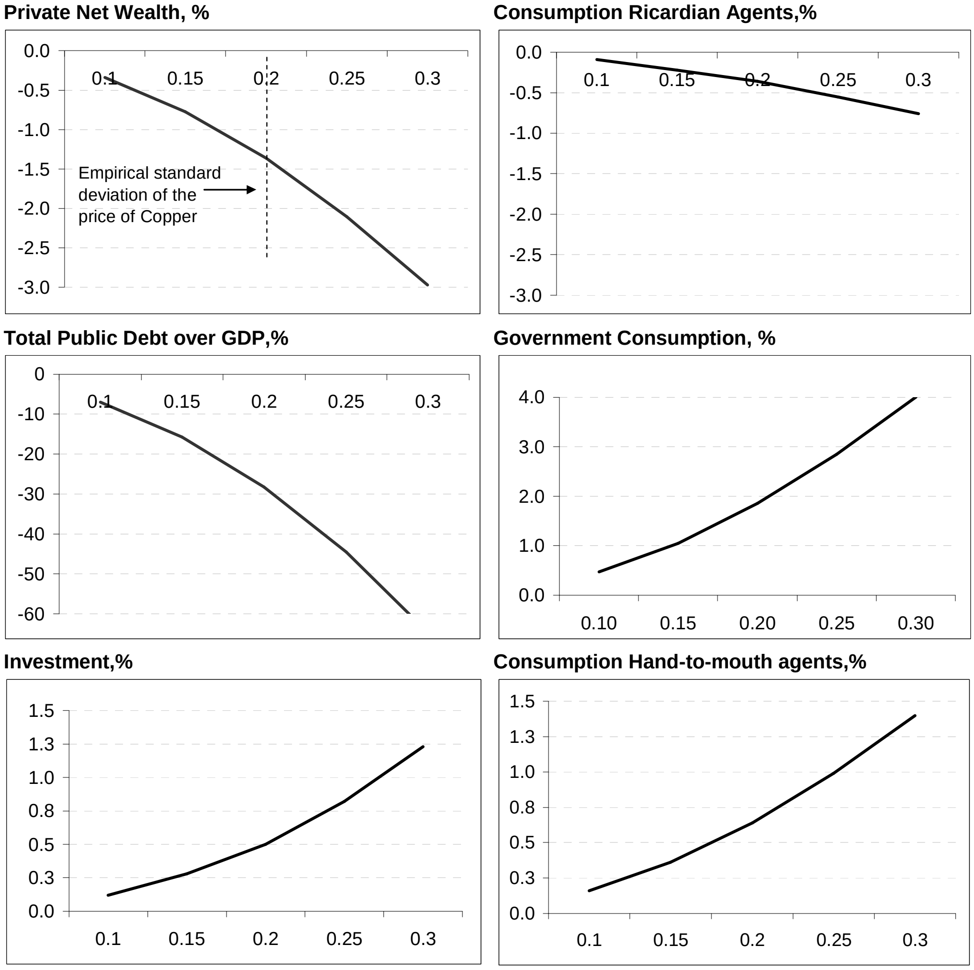
As Figure 5.a has already foreshadowed, "hand-to-mouth" consumers benefit from the acyclical rule, not only because their consumption stream is smoother, but also because it is slightly higher. On the other hand, Ricardian consumers loose with this rule. However, the loss computed with the conditional expectation formula is smaller because, as said, it takes into account the consumption foregone whenever the agent is involved in precautionary saving. The right panel in Figure 6 shows that average welfare goes up with the acyclical rule if one weighs both consumers equally.

Figure 6 suggests that the larger is the variance of the shock σ_{copper},, the higher is the "hand to mouth" welfare. Figure 7 shows why: as σ^{copper} grows, so does C^r under the acyclical regime, both absolutely and relative to the balanced budget regime. An analogous result holds for government expenditure. Hence, "hand-to-mouth" agents benefit from the (Keynesian) demand stimulus, which results from the acylical spending rule.

By contrast, Ricardian agents suffer somewhat under the acyclical rule relative to the balanced budget rule. Their consumption is slightly less volatile under the acyclical regime (Figure 4). However, in the more volatile balanced-budget environment, Ricardian households build their own precautionary assets, that include capital stock—from which they are able to later consume. Figure 6 supports these results. It shows that as σ_{copper} grows, the Ricardian agents' consumption C^o under the acyclical rule decreases. Once again, this reflects their lower asset levels that they do not build up in a more certain environment (a lower precautionary savings motive). As a result, their earnings and average consumption decreases over time.

Figure 7. Difference between regimes

(acyclical minus balanced-budget rule for different commodity price volatilities)



*Each line is the average of the series that resulted from the simulations with the acyclical spending rule minus those obtained with the balanced-budget rule.

In other words, even though the government has a mechanical acyclical rule it acts as if itself were an agent with a precautionary savings motive: it builds up a prudential asset stock that cushions spending today against shocks while also permitting it to spend more in the future. In turn, this provides a beneficial externality for "hand to mouth consumers:" the government is providing a substitute for the precautionary savings that they themselves cannot do. Figure 7 illustrates, under the acyclical rule that the government does what Ricardian consumers would otherwise do under the balanced-budget regime. It accumulates a large amount of assets and ends up with larger revenues and spending. The stock of assets can amount to a large share of GDP if uncertainty increases steadily. Nevertheless, and as we explained in Section 3.2, the mechanical acyclical rule causes an accumulation of assets that is far from being optimal from the perspective of the Ricardian consumers and hence the welfare of this kind of agents is much lower with the acyclical rule.[19]

These results also have important implications for the design of a general fiscal rule (see equation 54). Recall that under a completely acyclical regime, α_r=0 and under a balanced budget regime, α_r=1. By contrast, μ_x is a debt targeting parameter: an increase in the value of μ_x implies that the government is targeting more closely the debt stock (the target is the initial value of zero), but with more volatility in government expenditures.[20]

Table 2 shows the average welfare gains obtained using the conditional expectations for a given volatility of the commodity price shocks, as measured in consumption units for both consumers over a grid of values for μ_x and α_r.

Table 2. Average welfare gains

	α_x	**0**	**0.25**	**0.5**	**0.75**	**1**
	0.010	0.1038	0.0981	0.0792	0.0447	0.00
	0.033	0.1076	0.1008	0.0810	0.0456	0.00
μ_x	**0.055**	**0.1091**	0.1016	0.0828	0.0480	0.00
	0.078	0.1085	0.1032	0.0831	0.0486	0.00
	0.100	0.1075	0.1047	0.0861	0.0495	0.00

As one would expect, raising either μ_x or α_r helps Ricardians but hurts hand-to-mouth units. However, raising μ_x is not identical to raising α_r. Table 2 suggests that, if we maintain the acyclical element (keep $\alpha_\tau = 0$) but increase somewhat the debt targeting parameter μ_x, we can benefit Ricardian agents at a very small cost to the hand to mouth agents. But, no such result can be obtained by raising α_r: while Ricardian households always gain, the loss suffered by "hand-to-mouth" households is even greater. Intuitively, α_r is a larger and blunter instrument than μ_x. If μ_x rises, the stock of debt (or assets) must return to zero more quickly than otherwise. If α_r rises, more volatility is introduced directly— through the commodity price channel. By contrast, if μ_x rises, more volatility in introduced but less directly—through the spending channel.

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[19] Note that the composition of assets is not invariant to the fiscal rule. The acyclical rule encourages more domestic investment in physical capital than the balanced budget. Lower volatility encourages more plant and equipment to be built within the country. By contrast, higher volatility under the balanced budget regime encourages Ricardian consumers to invest abroad due to the precautionary saving motive and the absence of domestic bonds.

[20] In a more general context, when target debt b^* is, for example, 50 percent of GDP, a term like $\mu_x (b_t - b^*)$ would be necessary in the rule.

Indeed, we find that, conditional on $\alpha_r = 0$, there is a value of μ_x that maximizes average welfare gains. Therefore, we say that it is the best degree of government spending stabilization. So long as the gain to Ricardian consumers from increasing μ_x exceeds the loss suffered by "hand to mouth" agents, the former can compensate the latter. Note however, that such an optimum will only coincide with one that a social planner would choose for a special case, namely where the social planner's weights on the utility of "hand-to-mouth" and Ricardians coincide *exactly* with the values of λ and (1-λ), as defined above.

VI. SUMMARY AND CONCLUSIONS

We assess the welfare implications of reducing the volatility and procyclicality of government expenditures in countries that specialize in a primary resource-based commodity export, facing strong fluctuations of their fiscal income due to commodity price volatility. Public spending does contribute to aggregate demand (a Keynesian channel) and hence output. Importantly, government expenditure was assumed to be useless. This is so because our focus is to understand under this extreme assumption if there is some role for government spending in stabilizing external shocks and the business cycle.

Our policy, an acyclical spending rule, was geared to helping the most vulnerable "hand-to-mouth" consumers. We found that the policy was effective: it provided a substitute financial cushion, hence reducing the volatility of their consumption and increasing welfare. This policy boosted their mean consumption through (Keynesian) aggregate demand channels. However, others in society did not fare so well. "Ricardian" households that were able to optimize over time suffered. Initially, they saved less, a response that we would expect from agents with a precautionary saving motive. Their consumption was slightly less volatile, but over time their average consumption was also less. This negative wealth effect has an intuitive interpretation: effectively, under the acyclical spending rule, government accumulates assets and limits what was the role of Ricardian households, namely to smooth consumption administering their assets optimally.

An obvious alternative way of increasing consumers' welfare is relaxing one of our modeling assumptions and allowing public spending to be useful. This is equivalent to giving back part of government spending to the consumers reducing our lump-sum taxes. However, we do not consider this case here since we are tracing specifically the effect on welfare of stabilizing government spending.

We found the best degree of spending stabilization by moving slightly away from the perfectly acyclical rule, what increases the welfare of the Ricardian households at a low cost to the hand-to-mouth households. If the asset position of the government is limited through a somewhat more aggressive debt (asset) targeting stance (in our model a slightly higher $\mu_{x)}$; Ricardians saved more initially than before, building up more assets, hence boosting consumption and welfare. In other words, with full capital mobility and financial market participants having access to a wide range of financial instruments, it may be better (welfare reaches a higher level) also to include a debt / asset target—even at the expense of extra volatility in expenditures.[21]

In conclusion, if policy makers wish to cushion society's most vulnerable agents—those without access to capital markets and who have presumably the lowest wealth, our results show that fiscal policy should de-link public expenditures from current revenues. We conclude that the acyclical rule, in this context, has a positive effect on the welfare of society as a whole depending on how restricted, financially, the consumers truly are.[22]

[21] This could be implemented empirically through infrequent revisions of permanent income and spending.

[22] This paper also touches upon some more general issues in optimal fiscal policy. For example, our optimal debt level is, essentially, a net credit position (in average). This is similar to a conclusion found in Aiyagari et al (2002). In this aspect, our work recognizes that one goal of a government is to provide a financial cushion for "hand-to-mouth" households that are unable to do so for themselves (Tanner and Carey, 2005).

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Appendix 1: **A**symmetries and the risk premium

Given that our artificial economy does not have an analytical solution, we use an example to illustrate how the wedge (the risk premium) between the costs of external funds and the opportunity cost of the country produces an asymmetry in the model.

We assume in this simpler case that the premium depends on public debt. Thus, the complete dynamics of the public debt can be defined by two equations: the fiscal rule and the budget constraint.

Fiscal Rule:

$$P_1^G G_1 = \overline{IT} - \left(\mu_x + \frac{R_1 - 1}{R_1} \right) B_1 \tag{A1}$$

Budget constraint:

$$P_1^G G_1 = \left(\overline{IT} + \varepsilon_1 \right) - B_1 + \frac{B_2}{R_1} \tag{A2}$$

Substituting (2) into (1), we get an equation for explaining the dynamics of the public debt.

$$B_2 = \left(1 - \mu_x R_1\right) B_1 - \varepsilon_1 R_1 \tag{A3}$$

We assume that the risk premium is linear in B_2

$$R_1 = R^* \left(1 + \omega B_2\right) \tag{A4}$$

Substituting (A4) into (A3) and expressing the new equation in terms of B_2, we have:

$$B_2 = \frac{\left(1 - \mu_x R^*\right) B_1}{\left(1 + \omega R^* \left(\mu_x B_1 + \varepsilon_1\right)\right)} - \frac{\varepsilon_1 R^*}{\left(1 + \omega R^* \left(\mu_x B_1 + \varepsilon_1\right)\right)} \tag{A5}$$

Assuming that shocks are distributed in such a way that $\forall t$ there is the same probability of getting a positive or negative shock of equal size ε,

$$\varepsilon_t = \begin{cases} \varepsilon \quad with \quad probability \quad 0.5 \\ -\varepsilon \quad with \quad probability \quad 0.5 \end{cases}$$

It is possible to compute the expected value of the debt level in the second period B_2:

$$E(B_2)=\frac{1}{2}\left(\frac{(1-\mu_x R^*)B_1}{(1+\omega R^*(\mu_x B_1+\varepsilon))}-\frac{\varepsilon R^*}{(1+\omega R^*(\mu_x B_1+\varepsilon))}\right)$$
$$+\frac{1}{2}\left(\frac{(1-\mu_x R^*)B_1}{(1+\omega R^*(\mu_x B_1-\varepsilon))}+\frac{\varepsilon R^*}{(1+\omega R^*(\mu_x B_1-\varepsilon))}\right) \quad \text{(A6)}$$

We can study equation (A6) to comprehend the role of parameters μ_x and ω, and the size of ε in the dynamics of the public debt

Case A1. If $\mu_x > 0$ and $\omega = 0$, the dynamics towards the steady state is determined completely by the parameter μ_x.

$$E(B_2)=B_2=(1-\mu_x R^*)B_1 \quad \text{(A6')}$$

Nevertheless, there are no asymmetries because the wedge between the costs of external and domestic funds has no effect on the foreign interest rate that is relevant for the domestic economy.

Case A2. If $B_1 = 0$ (to eliminate μ_x in all equations) and $\omega > 0$, the expected value for B_2 equals

$$E(B_2)=\underbrace{-\frac{1}{2}\left(\frac{\varepsilon R^*}{1+\varepsilon\omega R^*}\right)}_{\textit{positive shock}}+\underbrace{\frac{1}{2}\left(\frac{\varepsilon R^*}{1-\varepsilon\omega R^*}\right)}_{\textit{negative shock}} \quad \text{(A6'')}$$

This expression is rearranged as follows:

$$E(B_2)=-\frac{1}{2}\left(\frac{1}{\frac{1}{\varepsilon R^*}+\omega}\right)+\frac{1}{2}\left(\frac{1}{\frac{1}{\varepsilon R^*}-\omega}\right)>0. \quad \text{(A7)}$$

We can gain further intuition of this equation examining the case of $\varepsilon \to \infty$, i.e., the variance of the shock tends to infinity. Only with very large values of ε, the expected value of B_2 is negative and depends on ω.

$$E(B_2)=-\frac{1}{2}\left(\frac{1}{\omega}\right)+\frac{1}{2}\left(\frac{1}{-\omega}\right)=-\frac{1}{\omega}<0 \quad \text{(A8)}$$

Thus, the parameter that governs the bias of a net credit position of the government is ω. We observe that this expression can tend to zero if $\varepsilon \to 0$, indicating that this bias also depends positively on the degree of uncertainty (variance). Finally, these parameters interact jointly with the parameter μ_x to define the final size of the asymmetry.

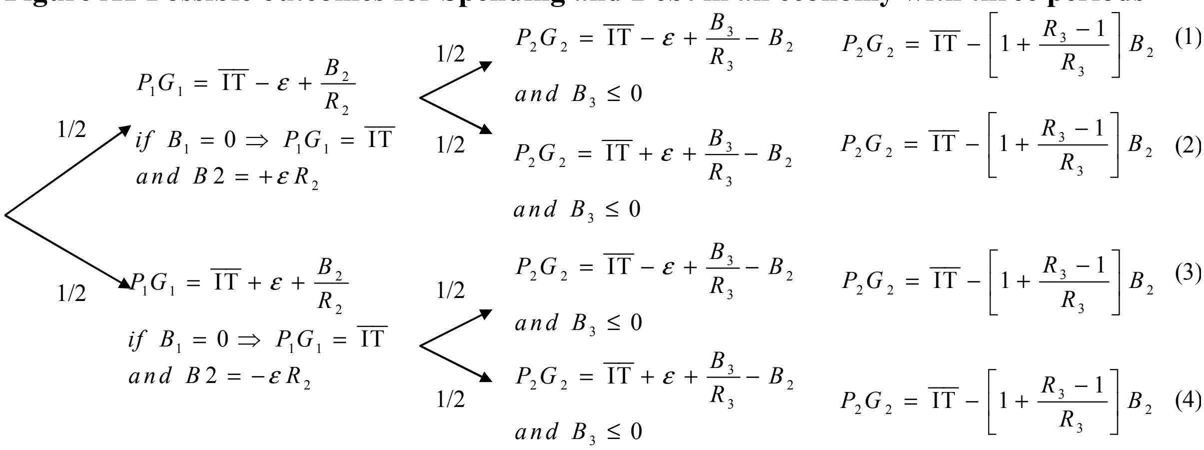
$$E_1(\Delta B_2) = -E_1(R_2)\mu_x B_1$$

In the presence of debt, bad times, R^{bad} is large and so is μR^{bad} while whenever B is negative, the good times, R^{good} is small, thus μR^{good} is also small. Therefore, in the bad times the government tightens more than what it increases spending in the good times.

Why the government accumulates assets in our model?

In a simple case of three periods we show that equation A7 will force the government to accumulate assets as a cushion that allows it to fulfill at the same time the fiscal rule (spending smoothing) and the transversality condition ie avoiding to be at the end of the world with a positive debt.

Figure A1 Possible outcomes for Spending and Debt in an economy with three periods



In figure A1 we show all the possibilities for the evolution of debt/assets in a world with three periods, whenever shocks are distributed as before with size ε or $-\varepsilon$. Given that the world ends in period 3, the transversality condition imposes that B_3 should be smaller or equal to zero, ie., the government could not have outstanding debt at the end of the world. However, given that in equation A7 the expected value of debt in period 2 is positive, in some of the four cases in the figure this condition does not hold. In particular, it does not hold in case (1), the top branch of the figure, in which the shocks were both negative (periods two and three). This is also true in the second branch in which the first shock was negative and the second one was positive. Given, that the government must fulfill the transversality

condition in all cases, it accumulates assets based on the worst scenario (1) in order to be covered in all four possible results.
Using the equality in row 1 of the figure:

$$P_2 G_2 = \overline{\text{IT}} - \varepsilon + \frac{B_3}{R_3} - B_2 = \overline{\text{IT}} - \left[\mu + \frac{R_3 - 1}{R_3} \right] B_2 ,$$

And the fact that $B_2 = \varepsilon R_2$, it is possible to find B_3/R_3

$$\frac{B_3}{R_3} = \varepsilon + B_2 - \left(\mu + \frac{R_3 - 1}{R_3} \right) B_2 \leq 0$$

Which should be smaller or equal to zero for the transversality condition to hold: $B_3 \leq 0$.

In that case, the minimum asset accumulation in period two is:

$$B_2 \leq \frac{-\varepsilon}{\left[1 - \left(\mu + \frac{R_3 - 1}{R_3} \right) \right]}$$

In conclusion, due to uncertainty, the government accumulates assets to be able to follow the fiscal rule, and at the same time meet the transversality condition in all possible cases. The larger the variance of the shocks (ε), the more it saves.