Renegotiating Incomplete Contracts: Over and Under Investment of Concessioned Public Infrastructure[¤]

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Abstract

This paper characterizes the equilibria of infrastructure franchising under incomplete contracting and ex-post renegotiation. The parties (government and a …rm) are unable to credibly commit to the contracted investment plan, so that a second step investment (labeled as investments in service quality) is renegotiated by the parties in the revision stage. As expected, the possibility of renegotiation a¤ects initial non-veri…able investments. The main conclusion of this paper is that not only under-investment but also over-investment in infrastructure may arise in equilibrium, compared to the complete contracting level.

KEYWORDS: Infrastructure franchising, incomplete contracts, unveri…able information, renegotiation, under-investment, over-investment.

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1. Introduction

In the 1980's and 90's, both developed and developing countries started a rapid process of privatization of public enterprises. The main reason to justify the privatization process has been to improve efficiency and pursue sustained long-run growth. Privatization and efficiency are more easily achieved in principle in sectors with no natural monopoly characteristics. However, sectors such as utilities (e.g. electricity, telecommunications) and public infrastructure (e.g. highways, ports) show important economies of scale and scope. Thus, at least in such cases, privatization and efficiency are not necessarily directly related. The experience of developing countries shows that important pre-conditions are not satisfied which will ensure the benefit of privatizing firms operating in markets with natural monopoly characteristics. Then, a long process that creates the necessary conditions to privatize firms in these sectors should be previously carried out.[1]

It is interesting to know how important efficiency problems could become when activities with natural monopoly characteristics are privatized. This paper uses contract theory as a way to understand the efficiency consequences of renegotiation in the context of public infrastructure franchising, which is of great importance in developing countries today. There are two theoretical ways to study contracting problems arising when opportunistic parties want to renegotiate their original contract. One way is to assume asymmetric information and the other is to assume symmetric but unverifiable information. In this paper we follow the second approach.[2]

The main contribution of this paper to the literature in incomplete contracting with symmetric but unverifiable information is the possibility of over-investment in public infrastructure; i.e. "white elephants" are therefore not only consequence of public production but also they may arise under private provision of public infrastructure. Over-investment is not new, however, in the new regulatory economics. Tirole (1986) and Besanko and Spulber (1992) have already found that possibility in an asymmetric information setting.

From the public policy perspective, this paper illustrates the importance of

[1] A general discussion about pros and cons of regulation can be obtained in Khan (1988), Tirole (1988), Vickers and Yarrow (1988), and Armstrong, Cowan and Vickers (1994). To see the experience on privatization and the posterior regulation of utilities in developing countries, see Muñoz (1993) and Bitran and Saavedra (1993).

[2] Mandatory references on this topic are Grossman and Hart (1986), Hart and Moore (1988), Bolton (1990), Chung (1991), Aghion, Dewatripont and Rey (1994), and Noldeke and Schmidt (1995).

reducing ambiguities (contract incompleteness) in infrastructure franchising. The more ambiguities in contracts, the more likely to departs from optimal allocations of resources. Furthermore, this paper also highlights the importance of strengthening regulatory and judiciary institutions in order to avoid both excessive ambiguities and opportunistic behavior. The worse prepared institutions in the country, the more likely to end up with either the hold-up problem or with "white elephants" in infrastructure.

A large number of papers has been written during the last two decades regarding the regulation of natural monopolies and procurement[3]. Most of the research uses principal-agent models to discuss the government and the monopolist interaction. This literature, however, has minimized the role of incomplete contracts. Contractual incompleteness is particularly useful when studying developing countries, where the regulatory framework is normally ambiguous and both regulatory and judiciary institutions are technically bad prepared to do their duties. Hence, it is fair to assume either the existence of transaction costs or bounded rationality that impede the government to write a complete (contingent) contract before initial investments are ful…lled[4]. Whatever the explanation to contractual incompleteness is, the parties have to deal with opportunistic behavior arising as a consequence of the non-contracted contingencies.

This paper considers a modi…ed version of the canonical model by Hart and Moore (1988) on symmetric but unveri…able information. They study a procurement relationship between two private parties under incomplete contracting and renegotiation of the price of the good. The main conclusion their paper is the hold-up e¤ect, i.e., the …rm under-invests in infrastructure as compared with the complete contracting situation. Thus, both parties would be better-o¤ if they were able to credibly commit not to renegotiate after sunk investments have been carried out. Other conclusions pertinent to our paper are the following: when the time zero contract is revised, the ex-post surplus is fully appropriated by the party who has more power in the renegotiation game; disputes are not an equilibrium outcome because of the non-veri…ability of the initial investments; the parties are severely constrained when setting the price of the good in the original

[3] Let me only mention La¤ont and Tirole (1993), which contains most of the topics in this area and a large number of references to the relevant literature.

[4] Transactions costs and bounded rationality are the two more widely accepted explanations for the existence of incomplete contracts in practice; see for instance Williamsom (1975) and (1985), Klein, Crawford and Alchian (1977), Hart and Holmström (1987). More formal justi…cations to contractual incompleteness are found in Holmström and Milgrom (1991), Spier (1992), Allen and Gale (1992), Anderlini and Felli (1994), and Bernheim and Whinston (1996).

contract; and when only one party's sunk investment matters, optimal (first best) investment levels can be achieved.

Despite being intuitive, the hold-up effect is not robust to changes on basic assumptions of the model. Two important examples are provided by Aghion, et. al. (1994) and Noldeke and Schmidt (1995)[5]. The main problem with those papers is that they assume courts observe more than they typically observe in practice (at least in developing countries). It is precisely the weakness of the judiciary system in developing countries our main argument to justify why we consider Hart and Moore's setting better suitable to analyze infrastructure franchising in developing countries.

We introduce two main changes to Hart and Moore's: (i) a benevolent government is one of the parties in the relationship; and (ii) the variable to be revised in the renegotiation stage is investment in quality, instead of price. Contrary to standard literature, the main conclusions in this paper are that both under-investment and over-investment are feasible subgame perfect Nash equilibrium outcomes; in the case of renegotiation, the ex-post surplus might be shared by the two parties; legal disputes are irrelevant; the government has total freedom to choose its optimal contract at time zero; and the first best investment levels $(I^*; q^*)$ cannot be achieved even when only the firm's investment matters.

The model is presented in the next section. Section 3 solves the renegotiation game played by the government and the firm after the investment in infrastructure have been sunk and the parties observe the actual state of nature. This section contains one proposition that characterizes the renegotiation process. Section 4 presents a solution for the model. Two propositions respectively characterize the firm's investment decision and the optimal time-zero contract. Finally, section 5 concludes.

[5] The implementation theory tells us that efficient investments are attained if the parties are able to design the revision stage in the original contract (e.g. Aghion, et. al. (1994) and Maskin and Tirole (1997)). One problem with the renegotiation design approach is that contracts should be much more sophisticated than they are in practice. For example, when using the revelation principle to obtain efficient investments, Aghion, et. al. assume courts must observe probabilities of trading when quantity is a discrete variable (let say, trade or not trade). Clearly, this is a strong assumption regarding capabilities of judges in practice. Another approach solving the hold-up effect is to assume courts may observe delivery of the good (Noldeke and Schmidt, (1995)). With this very small departure from Hart and Moore's model, the parties are able to implement the first best writing simple option contracts which give the seller the right to deliver and specify payments contingent on whether delivery takes place. Noldeke and Schmidt assume courts may distinguish whether the seller is stopping delivery on its own rather than by the buyer's pressure.

2. The Model

Let us …rst state what we understand for regulated public infrastructure in this paper. They are ports and airports, highways, tunnels, subways, etc. whose common feature is that all of them have both natural monopoly (high sunk investment levels implying decreasing average costs) and public good characteristics (non-rivalry in consumption). Franchising allow the private sector to participate in …nancing and operating these facilities, but it also gives monopolistic power to the …rm. In the more typical infrastructure franchising design, the government gives to a private …rm the right to build and, later on, operate the facility for a limited number of years. A contract signed when the project is granted regulates both how the facility has to be built (e.g. the level of quality) and how the operation has to be done (e.g. the vector of prices to be charged to users of the facility)[6].

Accordingly, consider an economy where the government decides to privatize the construction of infrastructure. Due to scale economies, it is socially preferred to have only one …rm for each project. A bidding process for each project decides what investor obtains the franchise to build and operate the facility by some speci…ed number of years (T periods). Assume that the economic value of the project and assets are zero for both parties after T. Finally, assume that if the project is aborted in the renegotiation stage, then its economic value is zero for each party and no payments are done as compensation for such a decision.

Since the government behaves strategically, it will commit not to renegotiate veri…able variables (e.g. prices) if and only if such a commitment is credible to the other party. In the real world, the government may set itself high barriers to change contracted prices, especially in public infrastructure franchising and procurement relationships[7]. Accordingly, our model assumes contracted prices are …xed in the original contract and not ex-post revised. Furthermore, let us suppose, without further lost of generality, that the demand for using this facility is deterministic and common knowledge. Therefore, the present value of the

[6] We indistinctly named to this contract as "original contract", "date-zero contract", or "old contract".

[7] As a matter of fact, the Chilean government …xes cap prices in real terms (i.e., indexed by in‡ation) prior to the bidding process. Thus, any renegotiation on those prices is extremely costly because two potential groups of presion –consumers and those …rms that did not get the project– become natural watchdogs of the original contract. Furthermore, our assumption also has theoretical support. Bös and Lülfesmann (1996) show that the …rst best is attained when a benevolent government is the buyer in the Hart and Moore's model.

revenue for the …rm, R, is also deterministic and common knowledge.[8]

This does not mean, however, that the incomplete contracts problem has been solved in the real world. We usually observe …rms and regulators renegotiating other variables that cannot be contracted and that do not have natural watchdogs, such as side payments and the contracted investment plan. When beginning an infrastructure project, …rms confront a big amount of uncertainty in most of the relevant variables, uncertainty which mostly disappear in …nal stages of the construction process. Accordingly, let us assume that the parties do not know the true state of the world, !, at time zero; they learn ! at period 1. Moreover, suppose that to write a complete contingent and enforceable contract is prohibitively costly because the true ! is su¢ciently complex and of high dimension. For simplicity, assume that ! 2 -, a …nite set. The support of - is common knowledge. As we will see soon, ! a¤ects both the consumer surplus and the operational costs of the project.

We assume two steps of investments. An initial investment (I 2 $[I_L; I_H]$), which cannot be contracted upon because it represents unveri…able investment effort decisions[9]. The …rm commits to undertake speci…c investments in infrastructure before period 1, when the uncertainty regarding the true state of the world is still present. This assumption allows the parties to behave opportunistically after those investments are sunk.

We also assume a second step investment (labeled as investment in quality of the service, q) which is undertaken between periods 1 and 2, when all of the uncertainty has disappeared. Consistent with practice, suppose that q is enforceable and, above certain minimum level, non-observable by the users of the facility. Since quality is not directly observable by outsiders, the government cannot commit not to renegotiate this variable in the future[10]. Let us assume that this investment is undertaken in order to produce a workable outcome of the project.

[8] We are implicitly assuming here that, above certain minimum level, the demand is inelastic with respect to the quality of the service. This assumption is consistent with the fact that often the quality of the service is not directly observed by consumers (an example, the quality of the air in a tunnel). Quality of the service above such minimum level becomes a "credence good" to users, which a¤ects surplus but not the demand of the service.

[9] An alternative is to suppose that I cannot be contracted because it is su¢ciently complex, such that no contract may describe it at a reasonable cost. In order to maintain our results, we need to re-de…ne the set of all feasible values of I, such that I 2 £, where £ is a non-empty, compact and bounded set.

[10] The investment in quality, q, might be veri…able by courts only if at least one of the parties is willing to do it in an eventual dispute. This assumption avoids a new renegotiation stage in this game.

The minimum investment in quality required by the government as acceptable is $q_L > 0$; hence, under the current contract, the firm should either invest $q \geq q_L$ or stop the project. Then, assume that $q \in [q_L; q_H]$.

If constructed, the public infrastructure is ready at date 2. From date 2 to T the facility is working and the firm is going to charge a sequence of prices exogenously determined at time zero and operating with a service quality specified by the current contract.

Regarding capabilities of judiciary institutions, we assume that outsiders may only observe whether or not the public infrastructure is built, but neither the probability of its successful construction nor delivery from the firm is observed by courts. This assumption is made in order to avoid the renegotiation design into the original contract.

Assume that the government is a benevolent planner, so its problem is to maximize the expected summation (over ω) of the consumer and firm's surpluses. Define v as the present value of the net consumer surplus. Assume that v depends upon the state of nature (ω) and the quality of the facility (q), in addition to prices and demand level. Let c be the present value of operational costs. It depends upon ω and firm's initial investment (I). Finally, φ is a function that transforms non-monetary costs into monetary costs to the firm. Regarding functions $v; c$ and φ, assume:

Assumption 1 [A.1]. For all ω:

i) $v \in (v_L; v_H)$ if $q \in (q_L; q_H)$

ii) $\{v_q > 0$ and $v_{qq} < 0\}$ if $q \in (q_L; q_H)$

iii) $\lim\limits_{q \to q_L} v_q = \infty; \lim\limits_{q \to q_H} v_q = 0$

Assumption 2 [A.2]. For all ω:

i) $c = 0$ if either the government rejects or the firm aborts the project.

ii) $c_I < 0$ and $c_{II} > 0$

iii) $\lim\limits_{I \to I_L} c_I = \infty; \lim\limits_{I \to I_H} c_I = 0$

Assumption 3 [A.3]. Assume:

i) $\varphi' > 0; \varphi'' > 0$

ii) $\lim\limits_{x \to x_L} \varphi'(x) = 0; \lim\limits_{x \to x_H} \varphi'(x) = \infty$, where $x = (I; q)$

Assumptions 1 and 2 guarantee an interior solution to the planner's problem (first best). Assumption 3 is necessary to obtain unique solutions to the firm and the government's problems, as proved in Propositions 4.1 and 4.2.

The First Best Outcome. The …rst best is our benchmark. It is the solution of an omnipotent and benevolent government. Omnipotence in the benchmark implies the government is able to write a complete contract at date zero. The government is not omniscient, however, because it is unable to see the "true" state of nature when writing the contract. Using assumptions 1 to 3 we can characterize the …rst best, i.e. assuming that speci…c investments can be veri…ed by outsiders, the …rm and the government will invest levels indicated by the time zero contract. Suppose all variables are expressed in monetary units at time zero. The …rst best is the solution to the planner's problem, that is ($I^{¤}$ and $q^{¤}$) solving:

$$\underset{fI;qg}{\mathrm{Max}}\ fE_{!}\left[v(!;q) + R \text{ ¡ } c(!;I) \text{ ¡ } Á(I) \text{ ¡ } Á(q)\right];0g$$
$$\text{subject to } (I;q)2[I_L;I_H] \text{ - } [q_L;q_H]$$

The objective function is jointly strictly concave in $(I;q)$. By assumptions 1 to 3, it is also bounded and continuous in both I and q. If evaluated at $(I^{¤};q^{¤})$, the objective function is greater than zero, then there exists a unique solution to the planner's problem in the interior of the constrained set.

However, there isn't an omnipotent government in practice. In general, when the government and the …rm separately solve their own problems, the …rst best cannot be achieved because of the impossibility of writing a complete contract at time zero. Ex-post opportunistic behavior arises because I is not veri…able by outsiders, neither are realizations of v and c. Thus, incentives to renegotiate arise because property rights on the ex-post …rm's surplus (residual surplus) are not speci…ed by the original contract.

It is important to know when the …rst best investment levels, $I^{¤}$ and $q^{¤}$, may be achieved by the parties. One hypothetical case is when there is no uncertainty, i.e. for any state of nature, $v = v(q)$ and $c = c(I)$. That is true either because (a) both …rm and government want and expect to continue with the project, so the …rm invests optimal levels; or (b) at least one of them wants or expects to abort the project, thus the …rm does not invest at all. In other words, since all uncertainty has disappeared the time zero contract can be complete. A second extreme case when the …rst best may also be attained by the two parties occurs if both parties want to continue the project at $q^{¤}$. Hence, setting $q = q^{¤}$ at time zero contract the …rm will invest the …rst-best level $I^{¤}$. That is because the …rm will continue with the project in any case and it cannot change $q^{¤}$ with its investment decision.

The Second Best Outcome. Conditions to attain the first best in the previous paragraph are, however, rarely satisfied. To explicitly avoid the second case where the first best would be achieved by the two parties, assume:

Assumption 4 [A.4] Non Renegotiation-Proofness.
For any $(I;q) \in [I_L;I_H] \times [q_L;q_H]$:

i) $v(\omega;q) + R - c(\omega;I) - \psi(q) \geq 0 > v(\omega';q) + R - c(\omega';I) - \psi(q)$, for some $(\omega,\omega') \in \Omega$;

ii) $R - c(\omega;I) - \psi(q) \geq 0 > R - c(\omega';I) - \psi(q)$, for some $(\omega,\omega') \in \Omega$

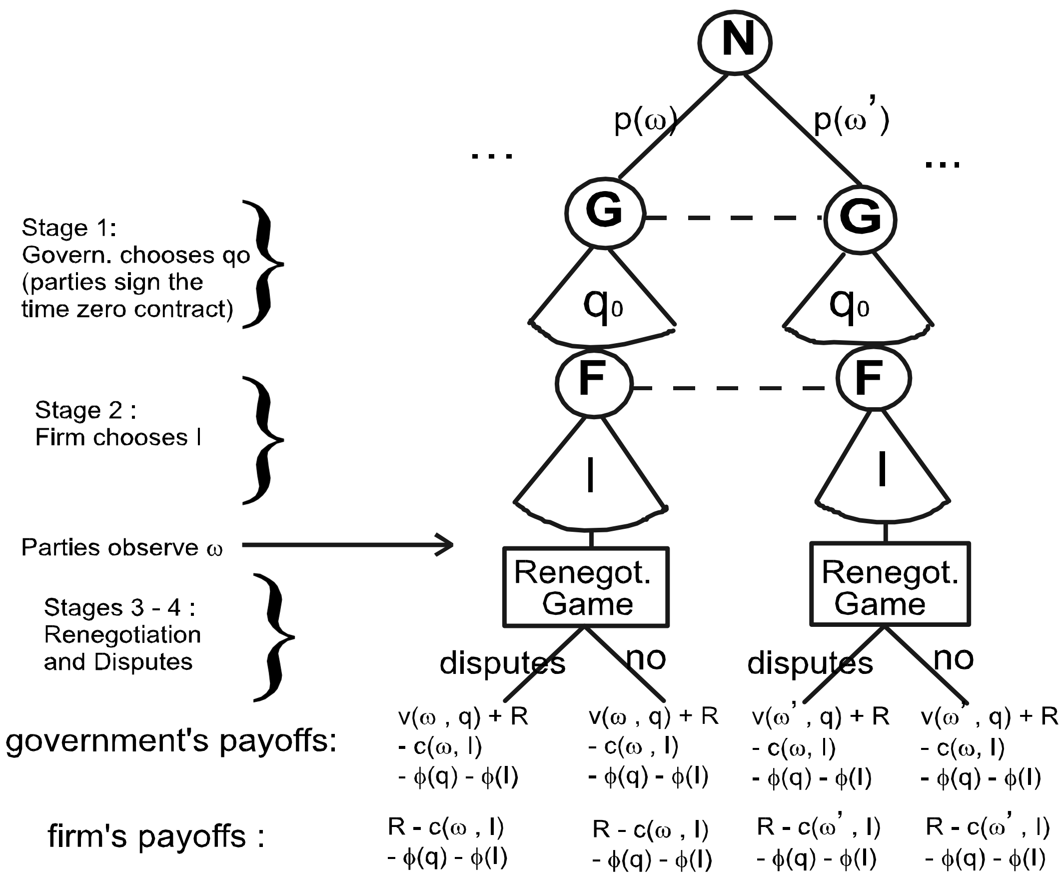
where $v(\omega;q) + R - c(\omega;I) - \psi(q)$ and $R - c(\omega;I) - \psi(q)$ respectively correspond to the government and the firm's ex-post surpluses, for any $(I;q)$ and for a given state of nature ω.

Assumption 4 tells us that both parties have ex-ante positive probabilities of either continuing or aborting the project. Thus, A.4 assumes the contract is not renegotiation-proof for at least one state of nature. This assumption avoids setting q small or big enough to guarantee the firm the complete ex-post bargaining surplus, because there exists some $\omega \in \Omega$ such that the government will be willing to trade. In turn, it implies that even though only the firm investment decision matters, the first best cannot be implemented. This assumption is feasible as a consequence of introducing a benevolent government as a party of the game (buyer) and assuming investment in quality, instead of price, as the only variable in being revised.[11]

The extensive form of the complete game between government and firm after finishing the bidding process is showed below (figure 2.1). As usual, assume that nature moves first. Yet, the true ω is unknown by the two parties until period one. At time zero (first stage) both parties sign the contract that –under non-enforceability on I– implies to set only the investment in quality, q_0 say, to be carried out after the renegotiation step has finished (besides other irrelevant variables to this game, such as prices to be charged to consumers). Let us assume for a moment that the government sets q_0. In the second stage the firm unilaterally

[11] Technically, assumption 4 makes sense because $\{v(q) + R - c - \psi(q)\}$ has a maximum on the interior of $[q_L;q_H]$. That is not the case, however, in a procurement model when price is the variable to be revised because in such a case the buyer's payoff function is always decreasing in the difference between p_1 (default price if trading) and p_0 (default price if not trading). Thus, the first best is attained (when only the firm's investment decision matters) setting $(p_1 - p_0)$ above the $Max_\omega\ v(\omega;\cdot)$ because in such a case the firm becomes the only residual claimant of its own investments.

Figure 2.1: The Complete Game



decides its optimal level of investment, I^S. Both parties realize the true state of nature at period one. Moreover, the government learns the investment level (I) done by the firm. Later on, the renegotiation game takes place. Finally, the dispute game might be the fourth stage, but we know that legal disputes are irrelevant as long as courts cannot verify relevant variables to make a decision ($v; c; I$). The irrelevance of disputes is represented in figure 2.1 as each party obtaining the same payoff on either going or not going into legal disputes.[12]

[12] We left the bidding process out of the game in order to avoid a more cumbersome model (we focus on post-auction problems).

3. The Renegotiation Game

Let us solve the game backwards. Since we have assumed that courts are unable to solve disputes, no party would pursue in a lawsuit in equilibrium (see the first part of Hart and Moore's proof to their Proposition 1). Then, we start solving the subgame called renegotiation game in this section. In this subgame, the government and the firm bargain over a new contract to set the quality investment level, q, to be carried out by the firm before period 2.

At period 1, the true state of nature has already been realized and observed by the parties. Moreover, the investment in infrastructure, I, is already sunk and it is observed by the government. Since the original contract could not specify contingent investment levels, $I(\omega)$ and $q(\omega)$, the resulting $v(\omega; q)$ and $c(\omega; I)$ could not be specified either. Therefore, how the ex-post surplus should be split between the government and the firm was not specified in the original contract, leading to a renegotiation game after date 1. Notice that this is a complete information game because c is known by the parties and v is one-to-one mapping from $[q_L; q_H]$ to $[v_L; v_H]$.

Ex-Post Surplus. Let us define first the type of ex-post surplus we consider. Given the realization of ω and investment I, the government and the firm know the level of the operational costs. Yet, they do not know how large v is, because it depends on q, which will be invested by the firm before period 2. Thus, for any level of q, if the government accepts the project and there is no renegotiation, the firm will obtain ex-post profits equal to $Max\{R - c - \psi(q); 0\}$; profits will be zero only if the firm decides to abort the project. On the other hand, for any level of q, if the firm decides to continue and there is no renegotiation, the government will obtain an ex-post surplus equal to $Max\{v(q) + R - c - \psi(q); 0\}$; this surplus will be zero when the government decides to abort the project[13]. Therefore, government and firm have individual incentives to renegotiate q_0, the specified level of the ex-post investment in quality originally contracted.

If the government wants to continue with the project, its ex-post optimal q is $\hat{q}$, which is the unique solution to maximize $\{v(q) + R - c - \psi(q)\}$ subject to $q \in [q_L; q_H]$. Assumptions A.1 and A.3 ensure that $\hat{q}$ belongs to the interior of $[q_L; q_H]$. Whether or not q_0 is smaller than this optimal level affects the final outcome of the game. On the other hand, it is clear the firm prefers q_L to any

[13] We are implicitly assuming both parties may abort the project and outsiders cannot see what party is inducing this decision.

other q if it continues with the project.

It is interesting to see what happens when no renegotiation takes place. The project will continue if and only if $[v(q_0) + R - c - \psi(q_0) \geq 0]$ and $[R - c - \psi(q_0) \geq 0]$. Otherwise, either the government or the firm would prefer to abort the project. It is implicit in these inequalities the assumption that both parties individually prefer to continue the project (rather than aborting it) when they are going to obtain zero ex-post surplus.

The Bargaining Mechanism. Renegotiation in this context means that the government and the firm exchange a series of messages trying to convince the other party to repudiate the old contract and write a new one. These messages could be read by third parties but they would be a legally valid contract if and only if the government and the firm have signed it.[14]

Also, let us suppose that messages cannot be forged and that there is nothing to stop the government (and the firm, of course) agreeing at any time before period 2 to tear-up the old contract and write a new one. It should be recognized that this is a strong assumption when one party is the government, but it allows the model to be workable.

Finally, let us describe the message technology used in the revision of q. Time between 1 and 2 can be divided into subperiods (say, days). Messages will be exchanged until day D, where D still belongs to period 1 (so q will be invested before 2). A message is a letter containing the signature of the sender and it is sent by a reliable "mail" taking 1 day to arrive. Each party does one collection and one delivery a day. A message delivery the previous day arrives before the collection of the day. Both parties can send several messages in the same day. Messages sent on day D arrive before parties decide either to invest q or to stop the project.

Two useful definitions that we use in the next proposition are $\bar{q}$ and $\underline{q}$. Let $\bar{q}$ be the minimum q such that the firm gets zero ex-post payoffs; i.e. $\bar{q}$ solves $[R - c - \psi(q) = 0]$. Let $\underline{q}$ the minimum q such that government gets zero ex-post surplus; i.e. $\underline{q}$ is the minimum q solving $[v(\underline{q}) + R - c - \psi(\underline{q}) = 0]$.

[14] An alternative stronger assumption is to assume that it is impossible to publicly record a message sent by one party (Hart and Moore's outcome crucially depends on it). Our weaker assumption is enough to analyze the game played in countries with Judiciary System based on Napoleonic Codes, where any contract is valid if and only if it has been signed by both parties (e.g. France, Spain, and some Latin American countries). The stronger assumption is necessary to analyze the game in countries with Judiciary System based on Common Law (USA or Britain, for example).

Proposition 3.1. Consider the model specified in section 2 and suppose assumptions 1 to 4 hold. Let q be the only variable that both parties can credibly renegotiate. Let q_0 be the investment in quality specified by the date-zero contract which will apply if no messages are sent between period 1 and day D. Then, conditional on I^S, q_0 and the realization of ω, the only subgame perfect Nash equilibrium outcome of the renegotiation game, q^S, can take only five possible values:

i) $q^S = q_0$ if $\hat{q} \geq q_0$ and both parties are better off keeping the time zero contract than stopping the project;

ii) $q^S = \hat{q}$ if $\hat{q} < q_0$, both parties get a non-negative ex-post surplus at $\hat{q}$, and no party has all the ex-post power;

iii) $q^S = \tilde{q}$ if the firm is willing to continue with the project at q_L but not at either q_0 or $\hat{q}$; and the government obtains a non-negative ex-post surplus at $\tilde{q}$;

iv) $q^S = \bar{q}$ if the government is willing to continue with the project at either q_0 or $\hat{q}$ but not at q_L; and the firm obtains a non-negative ex-post surplus at $\bar{q}$;

v) $q^S = 0$ if either some party is not willing to continue the project at any $q \geq q_L$ or when one party is willing to continue the project at some $q \geq q_L$, the other party prefers to abort.

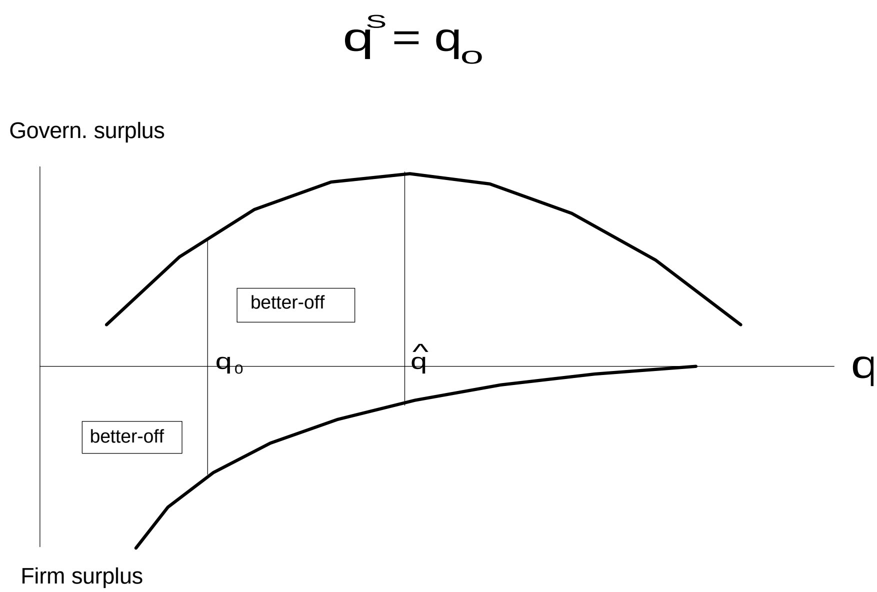
Proof. See Appendix.

Remark 1. This proposition highlights the importance of the relative ex-post power in allocating property rights over the residual asset of the partnership, the ex-post surplus.

The first case, $q^S = q_0$, arises when the parties' ex-post surplus (the outcome of the renegotiation game) run in opposite directions –what is better for the firm (to reduce q) is worse for the government, and viceversa– and both parties would continue the project as specified by the time-zero contract (see figure 3.1, below)[15]. The government would accept a new contract if and only if it specifies a higher investment in quality, but such a contract will never be accepted by the firm. Moreover, since the firm is willing to continue with the partnership at q_0, the government never sends a message asking for replacement of the existing contract

[15] Figure 2 illustrates the government and the firm's ex-post surplus as a function of the investment in quality. Notice that these surpluses are never negative, because both parties have the alternative to abort the project before undertaken q. Therefore, the vertical axis above zero corresponds to the government's ex-post surplus $(v(q) + R - c - \phi(q))$; and the vertical axis below zero corresponds to the firm's ex-post profit $(R - c - \phi(q))$. Finally, the horizontal axis corresponds to quality investment, q.

Figure 3.1: Case i) No Renegotiation Takes Place

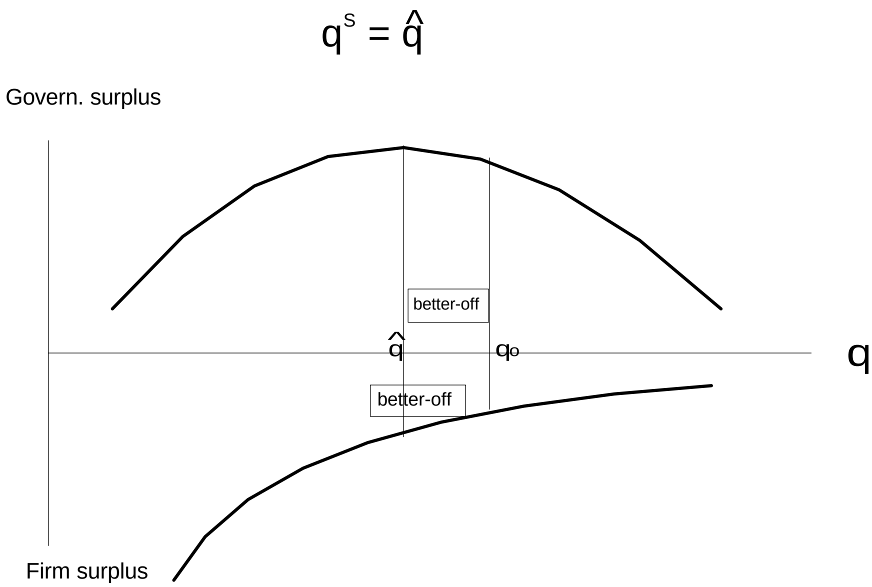


for a new one specifying a lower level of investment in quality. The same argument implies that the firm never tries to reduce the contracted quality investment level. In other words, no party has enough power to change the time zero contract.

In the second case both parties have equal ex-post power, i.e. either both want to continue with the project or both want to stop it. Nonetheless, both parties are ex-post better-off by reducing the investment in quality, so a new contract tiers-up the previous one and fixes q at $\hat{q}$ (see in figure 3.2 the case where both parties want to continue at q_0). Notice that the government will not sign any contract specifying any $q < \hat{q}$ because it knows that the firm will accept a take-it-or-leave-it contract specifying $\hat{q}$ in the last day. Any threat by the firm regarding not signing such a contract is not credible because the firm is better-off at $\hat{q}$ than at q_0.

Case iii) is more interesting. Here the government has all the ex-post power. So, the outcome of the renegotiation game tells us that the government gets all the ex-post surplus. The firm might send a large amount of messages asking for a new contract with $q < \hat{q}$, but the government would always reject them because it knows that in the last day the firm will accept a take-it-or-leave-it message asking

Figure 3.2: Case ii) Renegotiation Makes Both Parties Better-off



for $q = \hat{q}$ (it should be remember that any party prefers to continue when payoffs in such a situation are the same as those when stopping the project). Threat from the firm saying that if the government does not sign a contract asking for some $q < \hat{q}$, the firm will reject the final offer from the government is not credible (see figure 3.3, below).

In case iv) the firm obtains all the ex-post surplus because it has all the ex-post power. That is because the government wants to tier-up the time zero contract and the firm is better-off keeping it more than tiering-up (see figure 3.4, below).

We are implicitly assuming that the net consumer surplus may be negative for some states of nature. No important result of this paper changes ruling out this case.

It is clear that the project is aborted when both parties are better off finishing the relationship than continuing it at any quality investment level (case v)). The same outcome obtains, however, when at least one party is willing to continue the project at some $q \geq q_L$ but it is unable to compensate (in terms of giving up some of its ex-post surplus) the other party to continue with the project. Figure 3.5 shows the case when both government and firm want to continue the project

Figure 3.3: Case iii) The Government has all the Ex-post Power

$q^s = \bar{q}$

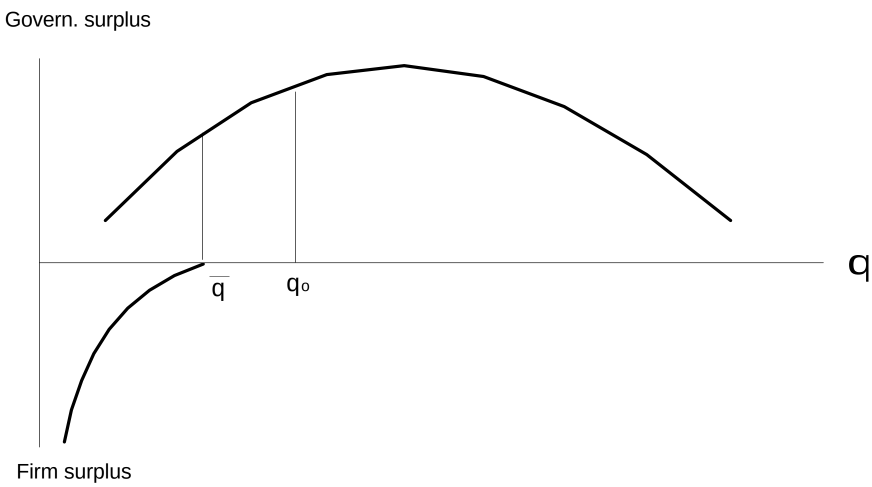


Figure 3.4: Case iv) The Firm has all the Ex-post Power

$q^s = \tilde{q}$

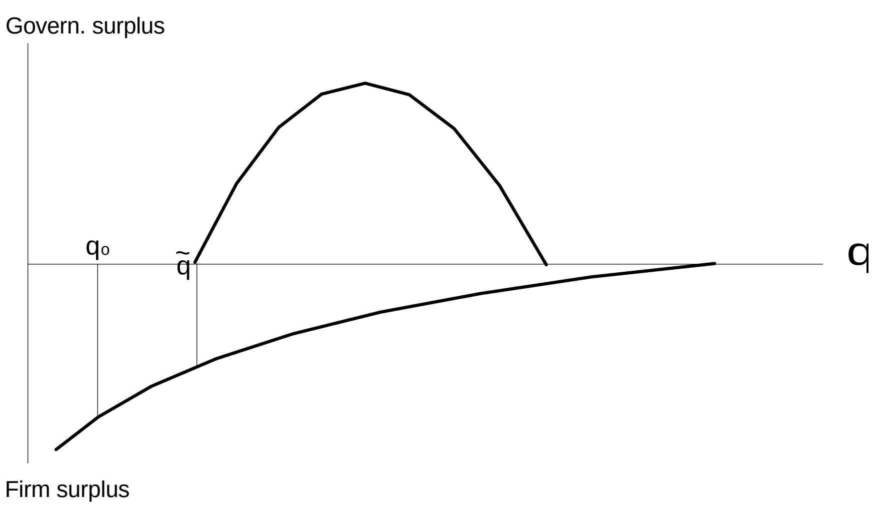
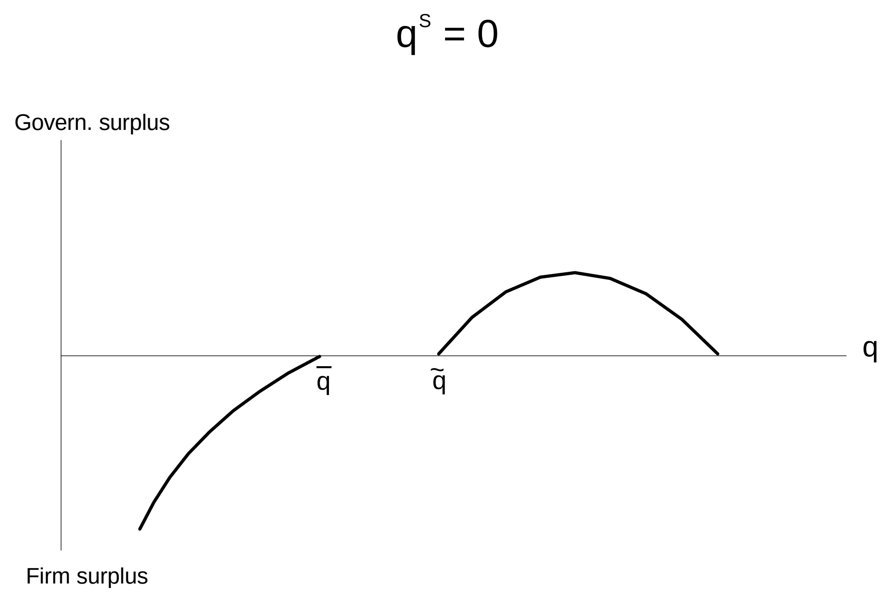


Figure 3.5: Case v) The Project is Aborted



at some q, but in such levels the other party prefers to abort the project.

These results deserve at least one comment. If the time zero contract cannot be renegotiated, only under very strong conditions the project would not be aborted. Such a remark might lead to a wrong conclusion: it is better to allow the parties to renegotiate the contract after sunk investments have been undertaken, because it makes the continuation of the project more likely; moreover, renegotiation allows the achievement of an optimal quality investment level once I is known (ex-post efficient outcome). Such a conclusion is false, however, because the possibility of renegotiation may induce strategic behavior that in turn would lead to either under or over-investment in the first stage, as we will see later. When making the investment decision, the firm is affecting not only its expected ex-post gains but also the government's. This externality most likely impedes the first best from being attained (ex-ante inefficient outcome).

4. The Firm and the Government's Decisions

In this section we solve backward the sequential decisions by the government and the firm in, respectively, choosing the investment in quality to be contracted at time zero, q_0^S, and the investment in infrastructure to be carried out before period 1, I^S. Since the true state of nature is unknown in these stages, both parties make their decisions maximizing expected payoffs.

4.1. Over and Under-Investment Outcome

Assuming that the probability distributions of v and c satisfy the spanning condition described below, the next proposition shows that there exists a unique solution to the firm's problem (I^S, say). The proposition also tells us that I^S is not necessarily below the Pareto optimal level, as expected in a symmetric but unverifiable information model.

Definition [of the Spanning Condition]. There exist two probabilities p_k' and p_k'' such that:

(1) For each $a \in A$ (a non-empty, compact set of actions available to one player),

$$p_k(a) = \lambda(a) \cdot p_k' + [1 - \lambda(a)] \cdot p_k'' \text{ for some } \lambda(a) \in [0;1]$$

(2) $\frac{p_k'}{p_k''}$ is increasing in k, where $k = 1;2;:::;K < \infty$

This condition is an adaptation of that in Grossman and Hart (1983). The first part of the spanning condition ensures that the effect of increasing I in the probability of obtaining a lower cost is independent of the level of I. Similarly, (1) ensures that the effect of increasing q in the probability of obtaining a higher consumer surplus is independent of the of q. The second part of the condition is the (strict) monotone likelihood ratio property, which is a mild assumption. The strict monotone likelihood ratio property implies the higher I, the more likely to obtain a lower cost of operation. Similarly, (2) implies the higher q, the more likely to obtain a higher consumer surplus.

Proposition 4.1. Assume that conditions of Proposition 3.1 are satisfied. Moreover, suppose for each $I \in (0;\infty)$ and $q \in (0;\infty)$ the following conditions hold:

i) for all I and q, the random variables $v(\cdot;q)$ and $c(\cdot;I)$ are statistically independent;

ii) the (non-degenerate) support of $c(\cdot;I)$ is: $\{c_H = c_1 > ... > c_j > ... > c_J = c_L\}$, where $J > 1$;

iii) the probability of c_j, $f_j(I)$, satisfies the spanning condition;

iv) the (non-degenerate) support of $v(\cdot;q)$ is $\{v_L = v_1 < ... < v_i < ... < v_N = v_H\}$, where $N > 1$;

v) the probability of v_i, $h_i(q)$, satisfies the spanning condition.

Then, there is a unique investment level, I^S, consistent with the subgame perfect Nash equilibrium of the subgame that begins in the firm's decision node of the complete game (see figure 2.1, above). Moreover, I^S need not coincide with I^*.

Proof. See Appendix.

4.2. The Time Zero Contract

Since the government knows the firm's best response strategy, the government behaves strategically when choosing the investment in quality, q_0^S, to be contracted at period zero. The government's objective function matches the omnipotent planner's one. Nevertheless, two differences may prevent the achievement of the first best. One is the fact that here the government takes I^S as given; and if $I^S \neq I^*$, then $q_0^S \neq q^*$. The second difference is that in the complete information case q^* cannot be revised, whereas here q_0^S may be revised after I^S is sunk.

Proposition 4.2. Assume that conditions of Proposition 4.1 hold. Then there is a unique q_0^S consistent with the subgame perfect Nash equilibrium of the subgame that begins in the government's decision node of the complete game (see figure 2.1, above). Moreover, q_0^S need not coincide with q^*.

Proof. See Appendix.

Remark 2. Propositions 4.1 and 4.2 dramatically change the main result found in the literature of incomplete contracts with symmetric but unverifiable information. The possibility of revising the original contract allows us to find under-investment and over-investment as feasible outcomes of the game.

The first best investment level can rarely be achieved because of externalities in the firm's decision. Changing I away from I^* reduces both the firm and the government's ex-post surplus, at any q, because it increases expected operational costs. Therefore, for some parametrization it may be valuable to the firm to invest above or below the first best level. This decision may help or hurt the government, but this externality is not considered by the firm.

First of all, consistent with Hart and Moore's paper, it may be worthy to the firm to reduce initial investments below I^*. The firm has incentives to under-invest when the benefit of reducing investment in infrastructure ($\phi(I^*) - \phi(I')$; for some $I' < I^*$) is greater than the sum of expected costs in terms of higher operational costs ($E(c/_{I'}) - E(c/_{I^*})$) and monetary cost in terms of higher investment in quality of the service (when relevant). This is not the case, however, when either no renegotiation takes place –because in such situation the government sets $q_0 = q^*$ and, therefore, the firm invests I^*– or when the original contract is revised, $q^S = \{\bar{q}; 0\}$ in equilibrium[16]. However, under-investment is feasible for any expected situation where this decision is profitable and ex-post renegotiation takes place. This is because both the firm and the government know that the contracted q will change after revised. Therefore, under-investment occurs only if it implies $q^S = \{\hat{q}; \bar{q}\}$.

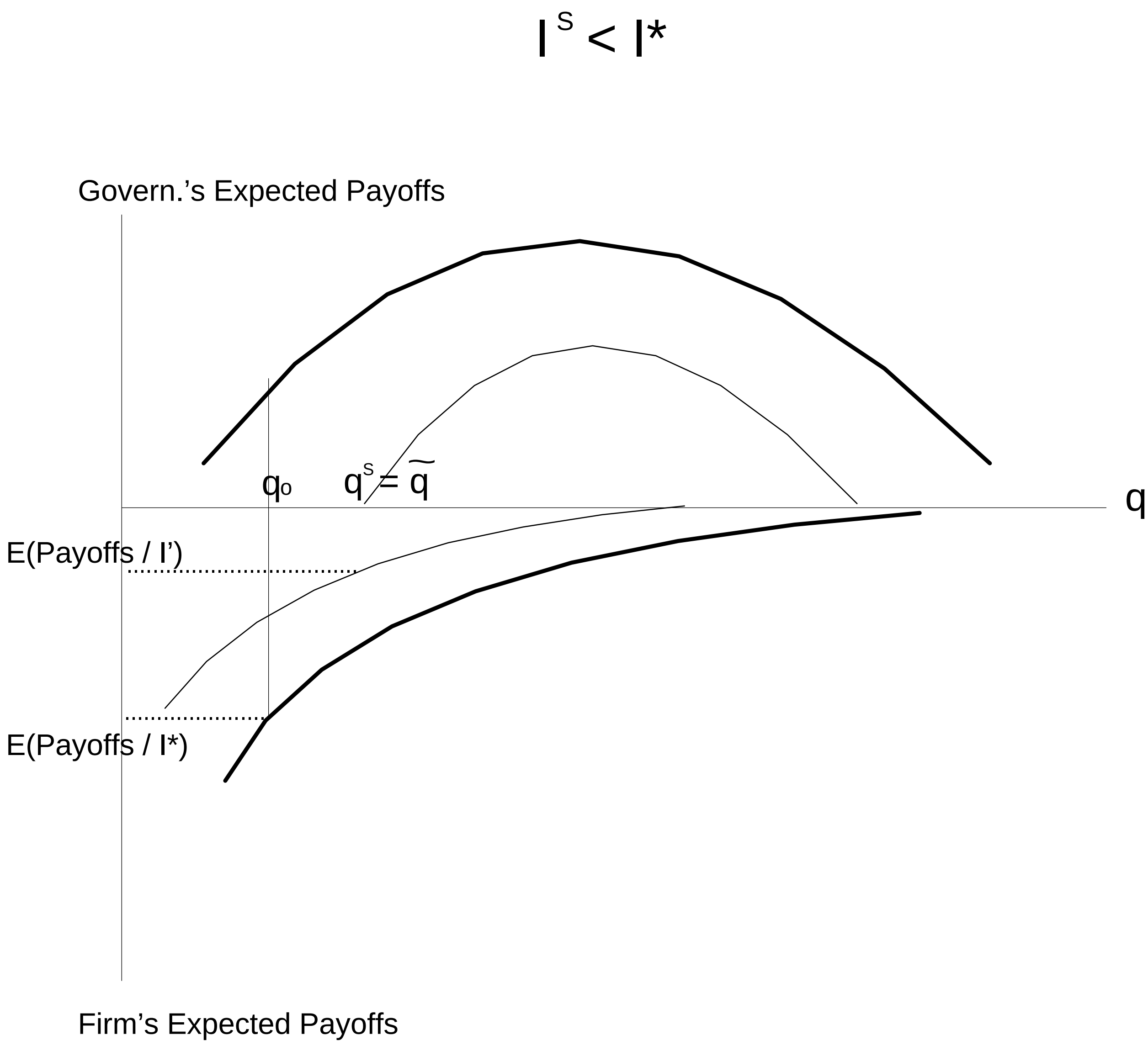
Let us provide an example of under-investment. Assume that the firm under-invests because it expects to drive the government to its reservation utility; i.e. with this decision the government is ex-post in a situation of non-voluntary trade at q_0 (see figure 4.1, below). Then, if the difference $\phi(I^*) - \phi(I')$; for some $I' < I^*$ is greater than $[E(c/_{I'}) - E(c/_{I^*})] + [\phi(\bar{q}) - \phi(q_0)]$, then the firm has incentives to invest less than the first best.

This example illustrates the fact that under-investment is more likely to occur when the firm expects ending up in a situation where it obtains all the ex-post power. Thus, shrinking the ex-post pie (summation of ex-post surpluses) is worthy to the firm because it is compensated by obtaining 100% of the shrunk pie.

Regarding over-investment, it is a direct consequence of the possibility to revise the investment plan. Like under-investment, the firm optimally invests when either no renegotiation is expected or the firm's surplus after renegotiation is equal to zero. Therefore, the firm may over-invest for any ex-post $q^S = \{\hat{q}; \bar{q}\}$.

[16] It is possible to show that it is not an equilibrium strategy to the firm to under-invest when this decision decreases the ex-post investment in quality. This is because the only feasible cases are to end-up at either $q = 0$ or $q = \bar{q}$. At those levels, the firm obtains zero ex-post gain.

Figure 4.1: Underinvestment



Thick lines : Expected payoffs (conditional on I*)
Thin lines : Expected payoffs (for some I'< I*)

Let us illustrate the over-investment outcome with an example. Suppose the parties expects an ex-post situation of $\hat{q} < q^*$. Then, setting $q_0 = q^*$ does not change the fact that $q^S = \hat{q}$. Whenever the firm increases I, not only the same ex-post situation will prevail but also both parties will see a higher ex-post surplus when reducing q to $\hat{q}$. Hence, if the firm's benefit of over-investing ($E(c/_{I^*}) - E(c/_{I'})$; for some $I' > I^*$) is greater than the monetary cost of increasing I above the optimal level ($\phi(I') - \phi(I^*)$), then the firm is better off over-investing (see figure 4.2).

This example shows that over-investment is likely to occur when the firm expects to end up in a situation where both parties are willing to reduce the contracted q. Thus, increasing the pie pays more to the firm in such a situation.

It is important to mention that if we only restrict our attention to non-negative net consumer surpluses ($\hat{q}$ is ruled out as an equilibrium of the renegotiation game), then the only reason to obtain ex-ante inefficiencies in public infrastructure franchising is because both parties expects to share the ex-post surplus after revising the original contract. This result is only feasible because the variable to be revised, q, attains a maximum in the interior of $[q_L, q_H]$. Therefore, it is consistent with the main conclusion by Bös and Lülfesmann (1996). That is, a benevolent government and a firm always attain the first best when revising prices.

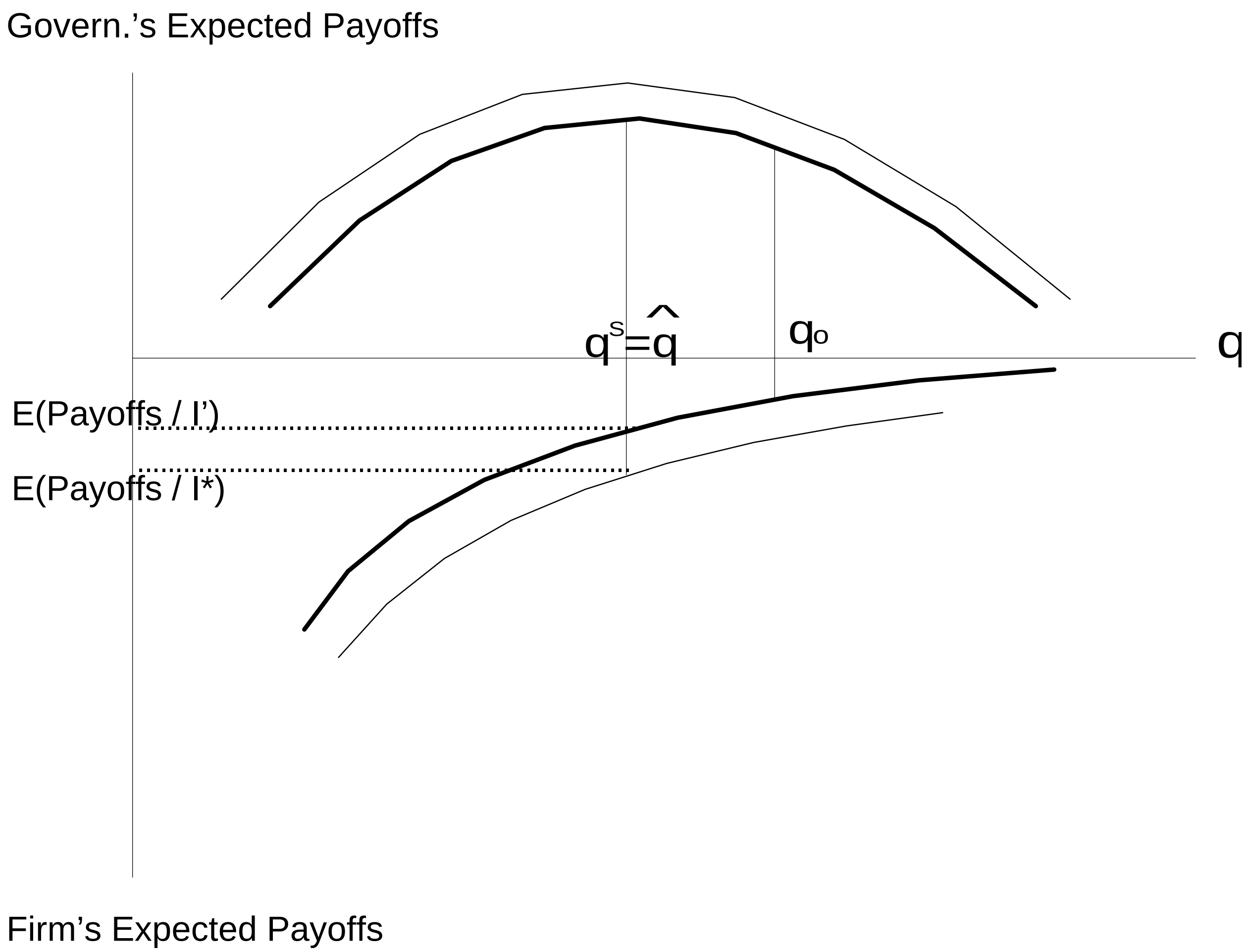
5. Conclusions

This paper presents a model which, departing from Hart and Moore's seminal paper, is capable of dealing with the dynamics of contracts and renegotiation in public infrastructure franchising. One of the main characteristics of this model is that it considers a benevolent government as one of the parties, so the government incorporates the surplus of both consumers and the firm in its objective function. This allows for a better analysis exclusively concentrated on the efficiency consequences of incorporating private capital in the infrastructure sector. The second characteristic is that the variable to be revised during renegotiations is the contracted investment plan (investment in quality), so the ex-post surplus of the government attains a maximum in the interior of its feasible set.

The main contribution of this paper to the literature in incomplete contracting with symmetric but unverifiable information is the possibility of over-investment in public infrastructure franchising; i.e. "white elephants" are therefore not only consequence of public production but also they may arise under private provision

Figure 4.2: Overinvestment

$I^S > I^*$



Thick lines : Expected payoffs (conditional on I*)
Thin lines : Expected payoffs (for some I'< I*)

of public infrastructure. In other words, over-investment may arise in equilibrium even though we assume a rational and benevolent government. Furthermore, as expected, this paper also shows the feasibility of the hold-up effect in this game.

From the public policy perspective, this paper highlights the importance of strengthening regulatory and judiciary institutions in order to avoid both excessive ambiguities in infrastructure franchising and opportunistic behavior by the parties. The more ambiguities in contracts and the worse prepared judiciary and regulatory institutions in the country, the more likely to end up with either the hold-up problem or "white elephants" in infrastructure (i.e. ex-ante inefficiency).

Other interesting conclusions in this paper are:

- In the case of renegotiation, the ex-post surplus might be shared by the two parties. Such a case may happen when the ex-post optimal investment in quality for the government is above the original contracted level and both parties are willing to continue the project at the time zero contract. Furthermore, in the case of renegotiation when the individual objective function of each party are inversely related, the ex-post surplus is fully obtained by the party which has more power in the renegotiation game.

- Disputes are not a subgame perfect outcome because of non-verifiability of the initial investment, I.

- The government has total freedom to choose its optimal contract at time zero, i.e. the contracted investment in quality is some q_0 belonging to the interval $[q_L; q_H]$. The reason is because investment in quality is not directly related with the individual participation constraint of the firm.

- The first best investment levels $(I^*; q^*)$ cannot be achieved even when only the firm's investment matters.

The model may also be used to explain why non-legal disputes might arise in equilibrium, such as political scandals, mutual allegations of corruption and inefficiencies, public pressures from each party in order to induce a more favorable outcome for itself before completing investments, etc. In spite of being of main importance in developing countries, these more political economy applications of the model are not assessed in this paper, however. We leave them for future research.

This paper has shortcomings. First of all, further research must characterize the equilibria of this incomplete contracting situation. Some non-disjoint alternatives are, for instance, to assume speci…c probability distributions on v an c or to restrict the set - to a few states of nature.

Regarding the model, it is not fully realistic. Like Hart and Moore's, our paper implicitly assumes that the government is unable to write more sophisticated contracts. Is the government behaving strategically when not designing the renegotiation procedure in the original contract?. An interesting line of research would be to …nd justi…cations to our assumption, rather than just assuming the existence of transactions costs at time zero. It is also interesting to know whether our conclusions are robust to introduce endogenous renegotiation in the model.[17]

Another point is the assumption that courts are only able to distinguish whether transaction takes place. But, why are courts unable to observe deliveries or intentions of trading?. Why courts cannot randomize?. In this paper we have just assumed that, in developing countries, courts cannot observe more than those in Hart and Moore's paper. Further research ought to assess this point.

Finally, it may be interesting to abandon the benevolent government paradigm. The self-interested paradigm seems to be an adequate assumption to capture the economic problems arising when the government behaves strategically, as it is the case in this paper.[18]

Appendix: Proof of Propositions

Proposition 3.1

Thanks to the message technology originally proposed by Hart and Moore (1988) –that we are using herein with little changes– there are no legal disputes in stage four. This implies that the renegotiation step can be analyzed as a normal form game (see Hart and Moore, pp. 777).

[17] Actually, in a very recent paper, Maskin and Tirole (1997) show that the …rst best may be attained with more sophisticated contracts. They use the same main assumptions of the incomplete contract literature (transaction costs and the possibility that players perform dynamic programming). It turns out that research in theoretical contract theory should strongly work in explaining why agents (including government) prefer simple contracts when complex contracts are at their disposal.

[18] A survey of reasons arguing that claim is Tirole (1994). See further references on this paper.

Case i), $q^S = q_0$ if $\hat{q} \geq q_0$ and both parties are better off keeping the time zero contract than stopping the project. Suppose there are at least one subgame perfect Nash equilibrium outcome which implies quality investment equal to $q^{SPE} \neq q_0$. Since $\hat{q} \geq q_0$, the government (firm) prefers some q above (below) q_0, then changing the quality investment specified in the time zero contract necessarily hurts one of the parties. Assume, without lost of generality, that the government is worse off (i.e. $v(q^{SPE}) - \phi(q^{SPE}) < v(q_0) - \phi(q_0)$). Let us show that government has incentives to unilaterally deviates, so q^{SPE} cannot be a subgame perfect Nash equilibrium. Consider the following strategy for the government: do not send any message before than day D. Suppose that the firm sends messages every day since v and c were known asking for tearing up the old contract and offering q^{SPE} (notice we are analyzing government's deviation from q^{SPE}, not firm's deviation from it). At day D the government sends a message to the firm proposing to tier up all the messages sent by the firm. Later on, the government's strategy is to accept the investment in quality done by the firm and neither to sign nor to reveal any message received from the firm in the case of disputes. If the firm is willing to trade, then $q = q_0$ because any other level would only arise in an eventual dispute (q is verifiable), where the government would sign and reveal some of the messages sent by the firm ($q = q^{SPE}$), and the firm would not sign the unique message sent by the government. But, this is not an equilibrium strategy because the government is better off not signing any message. By the same token, if the firm wants to abort the project, then $q = 0$. But, again this is not an equilibrium strategy because firm's payoff continuing with the project are greater than aborting it ($R - c - \phi(q_0) > 0$). Therefore, it is worthy for the government to deviate from q^{SPE} to q_0, a contradiction.

Case ii), $q^S = \hat{q}$ if $\hat{q} < q_0$, both parties get a non-negative ex-post surplus at $\hat{q}$, and none party has all the ex-post power. Unlike cases where $\hat{q} \geq q_0$, when $\hat{q} < q_0$ both parties have incentives to reduce the quality investment level. The firm prefers to reduce q to q_L and the government only from q to $\hat{q}$. Thus, any strategy involving $q > \hat{q}$ cannot be a subgame perfect Nash equilibrium because it is strictly dominated by $\hat{q}$ for the two players. The same apply for any $q < \hat{q}$ because they are strictly dominated by $\hat{q}$ for the government. To prove it, consider there exist at least one subgame perfect equilibrium evolving $q^{SPE} \neq \hat{q}$. The government deviates from q^{SPE} to $\hat{q}$ using the following strategy: at day D the government sends its unique message to the firm proposing to tier up both the old time zero contract and all the messages sent by the firm, and to fix a new quality investment level equal to $\hat{q}$. Later on, the government accepts the investment in quality done by the firm and do not sign any message received from the firm (if any) to be revealed in the case of disputes. The firm has two choices. If it decides to invest, then $q = \hat{q}$ because any other q is not a subgame perfect Nash equilibrium, otherwise either government or firm would play strictly dominated strategies in the dispute stage as well as we saw in case i). By the same token, the firm is not going to abort the project because its payoff are greater investing $\hat{q}$ than nothing. Therefore, the government has incentives to unilaterally deviates from q^{SPE} to $\hat{q}$, which is a contradiction.

Case iii), $q^S = \underline{q}$ if the firm is willing to continue with the project at q_L but not at either q_0 or $\hat{q}$; and the government obtains a non-negative ex-post surplus at $\underline{q}$. Suppose there is at least one subgame perfect Nash equilibrium outcome which entails quality investment equal to $q^{SPE} \neq \underline{q}$, where $\underline{q} = \phi^{-1}(R - c) \in [q_L; q_H]$ by A.3 (i.e. the firm gets zero ex-post surplus). First of all, $q^{SPE} > \underline{q}$ is not feasible because the firm has unilateral incentives to deviates in order to not lose money following this strategy: do not send any message during 1 and D, do not sign –and do not reveal to a court in the case of disputes– any of the messages received by the government, and abort the project ($q = 0$). Hence, if $q^{SPE} \neq \underline{q}$ then the government must be worse off with q^{SPE} than with $q = \underline{q}$. That is false because the government is better off at $\underline{q}$ using the following strategy: at day D it sends its unique message to the firm proposing to tier-up both the old time zero contract and all the messages sent by the firm, and to fix a new quality investment level equal to $\underline{q}$. Later on, the government accepts the investment in quality done by the firm and do not sign any message received from the firm to be revealed in the case of disputes. The firm has two choices. If it decides to invest, then $q = \underline{q}$ because any other q is not a subgame perfect Nash equilibrium, otherwise either government or firm would play strictly dominated strategies as well as we saw in case i). Therefore, since at $\underline{q}$ the government obtains a non-negative ex-post surplus, it has incentives to unilaterally deviates from q^{SPE} to $\underline{q}$, which is a contradiction.

Case iv), $q^S = \overline{q}$ if the government is willing to continue with the project at either q_0 or $\hat{q}$ but not at q_L; and the firm obtains a non-negative ex-post surplus at $\overline{q}$. The proof here is symmetric to Case iii) except that now the firm has all the power in the revision game because only the government wants to abort the project at q_0. Hence, the firm offers the government's reservation quality investment, i.e. $q = \bar{q}$ which is the unique q solving $v(q) + R - c - \phi(q) = 0$.

Case v), $q^S = 0$ if either some party is not willing to continue the project at any $q \geq q_L$ or when one party is willing to continue the project at some $q \geq q_L$ the other party is better off aborting the project at that level. If some party is worse off at any $q \geq q_L$, then aborting the project ($q = 0$) is the only subgame perfect Nash equilibrium outcome of the renegotiation game (any $q \geq q_L$ is strictly dominated by $q = 0$). Suppose, without loss of generality, that the firm wants to abort the project at any $q \geq q_L$. Regardless what $q \in [q_L; q_H]$ the government offers to the firm in a new contract, the firm is worse off accepting the offer than aborting the project. Thus, the only subgame perfect Nash equilibrium outcome is no messages are sent by any party, the firm doesn't invest at all, i.e. abort the project ($q = 0$), and there are no disputes at the end. The non-verifiability of v, c and I impedes the government to go on legal disputes using the time zero contract. Finally, let us suppose that both parties would continue the partnership at some $q \geq q_L$ but both $\{v(\tilde{q}) + R - c - \phi(\tilde{q})\}$ and $\{R - c - \phi(\bar{q})\}$ are negative at that level; i.e. none party has power to bargain because each one is worse off offering to the other party's reservation q than stopping the project. Therefore, after deleting strictly dominated strategies the only strategy that remains is to stop the project.

This completes the proof ■

Proposition 4.1

Step 1 : Definitions

Let us re-scale the investment such that $I \in (0; 1)$. Let I^a be the actual value of the investment, then define $I = \frac{I^a - I_L}{I_H - I_L}$. The same can be done to re-scale q.

Let rename q^S as q^S_{ij} to highlight that the subgame perfect Nash equilibrium outcome in the renegotiation stage depends on the true state of nature, i.e. $q^S_{ij} = \{q_0; \hat{q}; \bar{q}; \tilde{q}; 0\}$.

Voluntary trade occurs if and only if $[v_i + R - c_j - \phi(q^S_{ij}) \geq 0]$ and $[R - c_j - \phi(q^S_{ij}) \geq 0]$.

Let p'_j and p''_j be the spanning probability distributions over the support of $c(\cdot; I)$. Since $f_j(I)$ satisfies the spanning condition then $f_j(I) = I p'_j + (1 - I) p''_j$, where $\frac{p'_j}{p''_j}$ satisfies the monotone likelihood ratio property. Notice that $\frac{\partial f_j(I)}{\partial I} = p'_j - p''_j$ by definition of $f_j(I)$, so it does not depend on I.

Define $\Delta f_j = \frac{\partial f_j(I)}{\partial I}$. Likewise, define $\Delta h_i = \frac{\partial h_i(q)}{\partial q}$, which does not depend on q.

Define monotonicity by the property $q^S_{i,j+i} \geq q^S_{ij} \geq q^S_{i+1,j}$. This result comes from Proposition 1.

Finally, let $\sum \equiv \sum_{i=1}^{N} \sum_{j=1}^{J}$.

Step 2 : First Order Stochastic Dominance (FOSD)

Let us show that spanning condition (SC) implies FOSD. Since f_j increasing in j implies FOSD, then it is enough to show that SC implies f_j to be increasing.

$\frac{\Delta f_j}{f_j(I)} = \frac{p'_j + p''_j}{I p'_j + (1 - I) p''_j}$ dividing both terms by p''_j :

$$= \frac{\frac{p'_j}{p''_j} - 1}{I \frac{p'_j}{p''_j} + (1 - I) \frac{p'_j}{p''_j}}$$

but $\frac{p'_j}{p''_j}$ is increasing in j (by the monotone likelihood ratio property), so $\frac{\Delta f_j}{f_j(I)}$ is.[19]

Step 3 : Existence and Uniqueness of I^S (given q^S_{ij})

In the complete contracting case (first best), the government's problem is:

$$\underset{I,q}{MAX} \left\{ \sum f_j(I) h_i(q)[v_i + R - c_j - \psi(q)] - \psi(I) \right\}$$

By assumptions A.1 to A.3 this objective function is strictly concave, then FOC are necessary and sufficient for a unique maximum.

$$\frac{\partial\{\cdot\}}{\partial I} = \sum \Delta f_j h_i(q^*)[v_i + R - c_j - \psi(q^*)] - \psi'(I^*) = 0$$
$$\iff A_I(I^*; q^*) = \psi'(I^*)$$

$$\frac{\partial\{\cdot\}}{\partial q} = \sum f_j(I^*) \Delta h_i[v_i + R - c_j - \psi(q^*)] - \psi'(q^*) = 0$$
$$\iff Aq(I^*; q^*) = \psi'(q^*)$$

[19] Notice that the numerator increases on $\frac{p'_j}{p''_j}$ as j changes to $j+1$. On the other hand, the denominator only increases in $I \cdot \frac{p'_j}{p''_j}$ as j changes to $j+1$. Since $I < 1$, then $\frac{\Delta f_j}{f_j(I)}$ is increasing in j.

In the incomplete contracting case (second best), the firm's problem is:

$$\max_I \left\{ \sum f_j(I)h_i(q)[R - c_j - \varphi(q^S_{ij})] - \varphi(I) \right\}.$$

Since $\varphi'' > 0$, this objective function is strictly concave, then FOC is necessary and sufficient for a maximum.

$$\begin{aligned} \frac{\partial\{\cdot\}}{\partial I} &= \sum \Delta f_j h_i(q^S_{ij})[R - c_j - \varphi(q^S_{ij})] - \varphi'(I^S) = 0 \\ &\iff B_I(I^S, q^S_{ij}) = \varphi'(I^S) \end{aligned}$$

Since B_I is bounded (by A.1 and A.2), continuous and strictly increasing in I (by definitions of $f_j(I)$, $\varphi(I)$ and FOSD), and non-negative (voluntary trade); and $\varphi(I)$ satisfies Inada conditions (A.3), then by the intermediate value theorem there exists a unique $I^S \in [I_L, I_H]$ solving $B_I(I^S, q^S_{ij}) = \varphi'(I^S)$

Step 4 : Second Best Investment Decision (I^S)

Two things are important establishing the second best. One is to know how A_I changes as q changes. The other one is to know the difference between $A_I(I^*, q^*)$ and $B_I(I^S, q^S_{ij})$. Let $A_{Iq} \equiv \frac{\partial[A_I]}{\partial q}$. From the complete information's FOC we have:

$$A_{Iq} = \sum \Delta f_j \Delta h_i[v_i + R - c_j - \varphi(q^*)] - \varphi'(q^*) \sum \Delta f_j h_i(q^*) \gtreqless 0$$

because both terms are positive and their magnitude cannot be a priori inferred.

Therefore, under and over-investment are likely to occur no matter the level of q_0. Remember q_0 is determined by the government in a previous stage, so q_0 and q^S_{ij} are known by the firm before deciding I^S.

To prove the second part of Proposition 2 is enough to show one feasible example for each alternative. Therefore, the required conditions for each case are sufficient but not necessary.

(a) under-investment. Assume $q_0 \leq q^*$. A sufficient condition for under-investment is $A_{Iq} > 0$.

Proof: If $q_0 < q^*$, then, $\sum \Delta f_j h_i(q^S_{ij})\varphi(q^S_{ij}) < \sum \Delta f_j h_i(q^*)\varphi(q^*)$ because by Proposition 1 each $q^S_{ij} \leq q_0$.

From Planner's FOC: $\varphi'(I^*) = A_I(I^*, q^*)$

but $A_{Iq} > 0$ implies $\qquad > A_I(I^*, q^S_{ij})$

A_I independent of I $\quad = A_I(I^S; q^S_{ij})$
$\quad > B_I(I^S; q^S_{ij})$
by firm's FOC: $\quad = \psi'(I^S)$
Since $\psi'' > 0$, then $I^S < I^*$.

(b) over-investment. It occurs when $q_0 > q^*$, such that $\sum \Delta f_j h_i(q^S_{ij})\psi(q^S_{ij}) > \sum \Delta f_j h_i(q^*)\psi(q^*)$, $A_{Iq} < 0$. Same proof as above.

This completes the proof ■

Proposition 4.2

Step 1 : Definitions and FOSD

It is possible to re-scale q such that it belongs to $(0, 1)$. Definitions of voluntary trade and monotonicity remain the same.

Define $\Delta h_i = \frac{\partial h_i(q)}{\partial q}$, which does not depend on q by the definition of h_i. Δh_i is (strictly) increasing in i by the (strict) monotone likelihood ratio property. Hence, Δh_i satisfied FOSD.

Step 2 : Existence and Uniqueness of q^S_0 (given I^S and q^S_{ij})

From FOC of the complete contracting case we have:

$$\sum f_j(I^*)\Delta h_i[v_i + R - c_j - \psi(q^*)] = \psi'(q^*)$$
$$\iff \quad A_q(I^*; q^*) = \psi'(q^*)$$

In the incomplete contracting case (second best), the government's problem is:

$$\underset{q_0}{MAX} \left\{ \sum f_j(I^S)h_i(q^S_{ij}(q_0))[v_i + R - c_j - \psi(q^S_{ij}(q_0))] - \psi(I^S) \right\}.$$

Since $\psi'' > 0$, this objective function is strictly concave, then FOC is necessary and sufficient for a maximum.[20]

FOC: $\sum f_j(I^S)\Delta h_i[v_i + R - c_j - \psi(q^S_{ij}(q_0))] = \sum_{\{i,j = q^S_{ij} = q_0\}} f_j(I^S)h_i(q^S_{ij}(q_0))\psi'(q^S_0)$

[20] To clarify concepts only, remember that $q^S_{ij}(q_0)$ is a (KJx1) vector taking only five possible values, i.e. $\{q^S_{11}, ..., q^S_{KJ}\} = \{q_0, ..., q_0, \hat{q}, ..., \hat{q}, \tilde{q}, ..., \tilde{q}, \bar{q}, ..., \bar{q}, 0, ..., 0\}$. Then, changing q_0 will only directly affect those q^S_{ij} where q_0 will be the subgame perfect Nash equilibrium in the renegotiation game. The indirect effect (through $v(\cdot, q)$) is captured by Δh_i.

or, equivalently

$$D_q(I^S; q_{ij}^S) = \sum_{\{i,j = q_{ij}^S = q_0\}} f_j(I^S) h_i(q_{ij}^S(q_0)) \phi'(q_0^S)$$

where D_q is the first term on FOC.

Since both terms are bounded (by assumptions A.1 to A.3), continuous and strictly increasing in q (by definitions of $h_i(q)$, $\phi(q)$ and FOSD), and non-negative (voluntary trade); and $\phi(q)$ satisfies Inada conditions (by assumption A.3), then by the intermediate value theorem there exists a unique $q_0 \in [q_L; q_H]$ solving this problem.

Step 3 : The Second Best Time Zero Contract

Any subgame perfect Nash equilibrium in the first and the second best has to be respectively consistent with:

(#) $A_q(I^*; q^*) = \phi'(q^*)$ and

(##) $D_q(I^S; q_{ij}^S) = \sum_{\{i,j = q_{ij}^S = q_0\}} f_j(I^S) h_i(q_{ij}^S(q_0^S)) \phi'(q_0^S)$

Similarly to proof of Proposition 2, the sign of some feasible inequalities are the main importance to identify sufficient (but not necessary) conditions to either $q_0^S \gtreqless q^*$ as a best strategy for any I^S. The inequalities here are:

$$\sum f_j(I^S) \cdot h_i \phi'(q_{ij}^S(q_0^S)) \gtreqless \sum f_j(I^S) \cdot h_i \phi(q^*)$$

and

$$\sum_{\{i,j = q_{ij}^S = q_0\}} f_j(I^S) h_i(q_{ij}^S(q_0^S)) \phi'(q_0^S) \gtreqless \phi'(q^*)$$

(a) $q_0^S \geq q^*$ or $q_0^S < q^*$ for $I^S < I^*$. The inequality $\sum f_j(I^S) \cdot h_i \phi'(q_{ij}^S(q_0^S)) < \sum f_j(I^S) \cdot h_i \phi(q^*)$ is feasible even though $q_0^S \geq q^*$ because by Proposition 1 $q_{ij}^S \leq q_0^S$ (it is always true for $q_0^S < q^*$). A sufficient condition to the claim is

$$\sum_{\{i,j = q_{ij}^S = q_0\}} f_j(I^S) h_i(q_{ij}^S(q_0^S)) \phi'(q_0^S) > \phi'(q^*)$$

Proof : $A_q(I^S; q^*) < A_q(I^*; q^*)$ by $I^S < I^*$ and FOSD.

by (#): $= \phi'(q^*)$

by suff. condition: $< \sum_{\{i;j=q_{ij}^S=q_0\}} f_j(I^S)h_i(q_{ij}^S(q_0^S))\psi'(q_0^S)$

from (##): $= D_q(I^S; q_{ij}^S)$

which is true if and only if $\sum f_j(I^S)\cdot h_i\psi'(q_{ij}^S(q_0^S)) < \sum f_j(I^S)\cdot h_i\psi(q^*)$:

Since $q_{ij}^S \le q_0^S$; the previous inequality holds even though $q_0^S \ge q^*$.

Therefore, $q_0^S \Rightarrow q^*$ is feasible for any $I^S < I^*$.

(b) $q_0^S > q^*$ for $I^S \ge I^*$. Consider the following weak inequalities $\sum f_j(I^S)\cdot h_i\psi'(q_{ij}^S(q_0^S)) \ge \sum f_j(I^S)\cdot h_i\psi(q^*)$ (which implies $q_0^S > q^*$) and $\sum_{\{i;j=q_{ij}^S=q_0\}} f_j(I^S)h_i(q_{ij}^S(q_0^S))\psi'(q_0^S) \le \psi'(q^*)$. The proof is similar to above.

Therefore, q_0^S need not coincide with q^*. ■

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