AUCTIONS, ENTRY DETERRENCE AND DIVISIBILITY OF THE OBJECT FOR SALE*

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Abstract

This paper analyzes entry deterrence strategies at sequential multi-unit English-type repeated auctions, based on entry deterrence observed at a series of yearly auctions of fishing rights occurring since the early 1990s in the *Chilean sea bass* fishery. It analyzes parametric configurations under which incumbent firms could have followed non-cooperative deterrence strategies or else may have colluded for that purpose. A two-stage competition model is developed. In the first stage there occurs sequential auctioning of multiple fishing rights; in the second stage, production rights are used to compete in a homogeneous-good Cournot market. The analysis focuses on the relationship between the number of incumbents, sources of competitive advantage for them, and the number and size of the rights for sale. The core of the analysis lies in answering how the divisibility of the object(s) for sale affects the possibilities of incumbents to deter new rivals' entry.

JEL Classification: D2, D4, Q2

Key words: Collusion; Entry Deterrence; Repeated Auctions; Free Riding.

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1. Introduction

This study analyzes the award results of two of the annual auctions that have taken place since the early 1990s to grant fishing rights in Chile for the industrial fishing of *Chilean sea bass* (*Dissostichus eleginoides*), also known as Patagonian Toothfish or black cod. The two auctions underlying this analysis are the only ones in which (unsuccessful) attempts were made by new rivals to enter the market. It has been argued publicly that these failed entries were the result of collusion in the auctions between the incumbent firms.

To date, there is no published analysis on Chile's experience with auctions for allocating individual fishing quotas; and this, in a global context in which the use of auctions to allocate fishing rights has been, up to now, the exception. A couple of rare exceptions are the cases of geoduck fishing in the state of Washington (US) and fisheries in Estonia and in East Russia (Huppert, 2005; Anferova et al. 2005; Vetemaa et al. 2002). Despite that, in recent years there has been increasing debate in various countries about implementing individual fishing quotas. Peru has recently enacted a regulatory system of this type (since the end of 2008). Another case is the United States, where there is currently debate on the possibility of extending the use of individual fishing quotas. Therefore, our analysis about the Chilean experience with the auctioning of fishing rights should be of international interest.

The lessons that can be drawn from this analysis are also valid for other sectors in which it is necessary to allocate multiple production rights repeatedly and through public tender. There are several examples, ranging from the allocation of licenses to provide air transport services; allocating rights to use the electromagnetic spectrum (Klemperer 2008); the allocation of concession contracts for public access infrastructure investments, such as roads and ports (Porter and Zona 1993); patenting in sequential investment processes of technological innovation (Gilbert and Newbery 1982, Krishna 1993); the allocation of rights to explore and exploit state land suitable for forestry (Baldwin et al. 1997; Athey & Levin 2001); or allocating marine areas with underground reserves of oil and natural gas (Porter 1995; Hendricks & Porter 1988).

In several of the papers cited, collusion arises in repeated auctions with sale of multiple production rights. In recent years, growing theoretical and empirical literature is becoming available on the incentives for competition and the possibility of collusion in such contexts (e.g., Klemperer 2004, 2008; Hendricks and Porter 2007). So far, the predominant focus has been to analyze how different informational assumptions determine the strategies used by competitors (e.g., Hendrick & Porter 1988; Baldwin et al. 1997; Athey & Levin 2001).

In contrast, the focus in this paper is to study how the divisibility of the rights for sale[1] influences (i) incumbents' incentives to deter entry, and (ii) the parametric conditions in which this deterrence may occur in a non-cooperative way, or would require incumbents' collusion. In this regard, we consider the number and size of the rights for sale, the number of incumbents, along with sources of competitive advantage for incumbents such as sunk costs. We model the divisibility of production rights, parameterizing the number of rights while keeping constant the total volume of production to be tendered (given a maximum production quota, defined exogenously). This modeling variant is reasonable and directly applicable to the specific case that prompted this work. This variant, along with explicitly considering the possibility of deterring entry in a non-cooperative form or via collusion, adds to the literature on how the number of rights to be tendered affects incumbents' incentives to deter entry.

Our analysis is related to the overall topic of incumbents' incentives to deter entry of new rivals (e.g., Dixit 1980; Spence 1977, 1979). We develop a theoretical model of incumbents' incentives to deter entry through bidding strategies in sequential auctions of multiple fishing rights. The analysis considers ascending price (oral) auctions, of the English type, and assumes that competitors have perfect information. In this informational context, sealed envelope/second-price auctions produce identical results to ascending price (English) auctions.

In terms of related literature, Gilbert and Newbery (1982; G&N henceforth) analyze the entry deterrence incentives of a monopolist with a transitory patent over one product. The monopolist aims at patenting a second technological innovation (a once-and-for-all decision), which is a close substitute of its former product, before a potential rival does so. In this model, the monopolist always succeeds at patenting first the second product because he chooses a higher R&D investment.[2] The incumbent's maximum willingness to invest in R&D is equal to the difference between (a) the payoff he would obtain from exploiting both patents as a monopoly, and (b) the payoff he would obtain from having just the former license while the second license falls into the entrant's hands, the market thus becoming a duopoly. In contrast, the potential entrant's payoff for patenting the second product will always come from a duopolistic market, as in this model the incumbent has an exogenous first-mover advantage. Therefore, given that the monopolist's payoff will be higher than both duopolists' total payoff (as usually happens

[1] Annex A.1 explains that the Chilean fisheries authority opted to auction the overall quota available in *multiple* lots or fishing rights precisely because it thought that in this way the likelihood of entry of more and smaller firms in this industry would increase.

[2] Although G&N do not model a particular auction scheme, they assume that firms must invest for patenting a new product, this investment being made by the agent most willing to pay for the patent, but paying the second highest valuation (a result that is equivalent to the one we could obtain if the patent had been sold off in a Vickrey-type auction). Thus, G&N model is assimilable to a single-stage auction, the license for sale being of a non-restrictive type (i.e., enabling to produce whatever quantity its owner chooses).

with total industry profits when additional entry occurs in a non-cooperative environment), the monopolist's valuation of the new patent will always be higher than the potential entrant's valuation.

Subsequent works extend the insight developed in G&N (1982) to *multiple* production license auctions. Krishna (1993) argues that with the sequential selling of multiple production licenses (or capacity units), the possibility of entry increases. In this model, a monopolist shares his market with a fringe of small competitors, each of the latter being a price taker. Potential entrants can become part of the fringe if they buy at least one license. The monopolist sets his price knowing that, because of his initial production rights, he will never be capacity constrained.

When the monopolist's valuation of obtaining a new capacity unit is positive and increasing with the level of additional capacity units (which could be of equal or different size), deterring entry at an earlier stage will raise the cost of doing so at future stages, i.e. the prices of later units for sale rise if earlier units are previously bought. Hence, in general the monopolist will be better off by letting earlier capacity units for sale go to the entrant, accepting some dissipation in his price powers.[3] In this model, only the incumbent monopolist holds market power, independently of the auctions' outcome. Thus, entry causes only a limited erosion of the incumbent's profit.

Subsequent works extend the analysis to cases with multiple incumbents (which is more relevant to the case leading to this paper). Rodriguez (2002) analyzes the sequential auctioning of L> 1 licenses of homogeneous quality and permanent validity. Each license enables participation in an oligopolistic market that occurs immediately after completing the sequence of L auctions. In the post-auction phase, entrants also enjoy market power, i.e., the licenses acquired do not imply relevant restrictions of production. Thus, entry can produce a more radical erosion of incumbents' profits (relative to Krishna's (2003) model). This explains why, in some cases, one or more of the incumbents may decide to deter any entry attempt. Rodriguez (2002) only considers the possibility that incumbents use non-cooperative strategies to deter entry.

This model can generate certain entry in each auction by imposing an upper limit on the erosion of the incumbent's profits with the entry of a new rival. Under this assumption, when there are N≥2 incumbents at the beginning of the first auction, the incumbents never manage to deter entry, because spending on deterring entry has characteristics of public good. On the other hand, if there is one initial incumbent, the ability to deter entry depends negatively on the number of licenses auctioned. The latter is related to the overall message in Rodriguez (2002): entry will occur, repeatedly, while there are a sufficiently large number of licenses for sale.

[3] This result assumes that the licenses for sale are of equal or increasing size (as the auctioning sequence proceeds). By contrast, if the licenses for sale decrease in size, at a fast enough rate, all the licenses will go to the monopolist.

In contrast, Hoppe et al. (2006) argue that a greater number of licenses on sale will not necessarily induce more entry. In this model, multiple licenses are auctioned simultaneously. They may have different qualities and multiple incumbents and potential entrants compete for them (non-cooperatively).[4] In this setting, offering a greater number of licenses may even facilitate coordination strategies (not necessarily collusion) between the incumbents to deter entry. However, the relationship between the number of licenses for sale and the number of incumbent plays a key role. For example, when a single license is for sale and there are at least two incumbents, there will be a positive probability of entry. But if the number of licenses equals that of incumbents, no entry occurs. The paper presents a brief analysis on the possibility of collusion between incumbents to deter entry. To do this, it considers the case of a *single* license for sale and also *presume*s the feasibility of perfect collusion between incumbents to achieve full deterrent success.

This latter paper is the closest to our modeling. However, both papers present differences in their set of assumptions. In this regard, the model in this paper seeks to contribute to understanding specific aspects of the case study. It will therefore assume that the auctioned rights are homogeneous and that their size depends on an overall (exogenous) production quota. It will also examine the possibility of collusive deterrence of entry, explicitly considering the case of *multiple* fishing rights for sale.

The outline of the paper is as follows. Section 2 describes relevant facts of the case study underlying the theoretical model. Section 3 presents the model's notation and assumptions. Sections 4 and 5 analyze the case of entry deterrence, first considering the sale of a single lot and then auctions of multiple lots. Section 6 presents conclusions. The Appendices provide additional information.

2. Case study

This section describes relevant facts about barriers to entry into the *Chilean sea bass* industry and a brief summary of the results observed at the series of yearly auctions.

Barriers to Entry

First, the industrial fishing of the *Chilean sea bass* occurs in a fairly limited number of places in the world; mainly in areas of the Southern Pacific (areas in Chile's exclusive economic zone (EEZ) and in surrounding (southern) international waters), in southern South Atlantic and south-Antarctic areas (EEZ

[4] The licenses can differ in quality depending on the degree of demand substitution that they present in connection with the licenses under incumbents' ownership. Furthermore, it is assumed that the incumbents use symmetric strategies and consider only equilibria in mixed strategies. Thus, each incumbent probabilistically defines its bid for each license, resulting generally in a positive probability of entry.

areas of Uruguay, Argentina, Falklands, waters under CCAMLR[5] jurisdiction and in international waters) and in some sectors of the south-western Indian Ocean. In each of these there are restrictive regulations for the granting of licenses and quotas. In the early 2000s the world catch of this species was about 34 thousand tons, with almost a third of these obtained in Chilean waters (Cararsi, 2004).

Second, the ships operating in this fishery are capital intensive (each with a replacement cost of about US$5 million), representing largely a sunk cost. On the one hand, they use species-specific fishing equipment (Bravo, 2001). On the other, since late 1991, most industrial fisheries in Chile are subject to very tight entry regulations and only boats with fishing permits for each of these can operate in them. Finally, vessels operating within Chile's EEZ face similar entry constraints in the rest of the world, if they wish to operate in other areas of similar commercial value.

Note that the restrictive effect of the previous two barriers to entry, on the supply of sea bass worldwide, is enhanced by the different fishing seasons that prevail in the most important fishing grounds of this species worldwide.

A third barrier to entry arises from the minimum efficient scale of operation in this industry. This typically requires, on the one hand, performing fishing operations diversified into several species and, on the other, value-specific investments to create marketing networks to and within export markets. This all demands deployment periods during which the suppliers can gain the confidence of wholesale traders in the end-consumer markets. This need to build trust and reputation is a common requirement in the marketing of seafood. The reason for that lies in the significant informational asymmetries related to the perishability and heterogeneous quality of the final product (whether in terms of its freshness or freezing technique). As a market response to these transaction costs, a few large traders predominate in the international wholesale marketing of fishing products. Thus, expanded scales of operation act as a signal of credible reputation (Doeringer & Terkl 1995; Geirsson & Trondsen 1991; Arnason 2003; Peña & Vespa 2008).

Industry structure

For more than a decade in Chile there have been a small, and stable, number of incumbent firms in the industrial fishing of *Chilean sea bass* (see Table 1). Most of these firms are vertically integrated, or have long-term contracts subject to various vertical restraints, with a small number of international wholesale dealers, each with country- or region-specific operations (Fernandez 2008; Peña & Vespa 2008).

[5] Multi-country organization for fisheries management and research purposes under the authority of the Convention for the Conservation of Marine Living Resources in the Antarctic region, of which Chile is a member country.

Thus, the most important *Chilean sea bass* producers have managed to establish lasting relationships with specific distribution and marketing networks in the major export markets (U.S., Japan and the European Community). A few large wholesale traders dominate in each of these markets, investing heavily in brand equity positioning (Peña-Torres & Vespa 2008).

The global market share of the leading sea bass-fishing companies in Chile is not confined to the one third of the world catch of this species occurring in Chilean waters. A predominant proportion of these firms are part of multinational fishing conglomerates and thus they also fish in the other major fishing grounds of this species in the world. They own fishing licenses and quotas for the areas controlled by the CCAMLR, for the EEZs of Argentina and Uruguay, and they also fish in various zones of international waters.[6] Thus, these companies are part of a small number of firms that operate globally, fishing in different countries and selling their output via integrated distribution and marketing networks, each with value-specific investments for different end-markets.

On the demand side, the production of *Chilean sea bass*, whether fresh or frozen, has no close substitutes in other white-meat fish. In this category of fishing products, the *Chilean sea bass* is by far the species with the highest price. For example, in 2002 a fillet of *Chilean sea bass* in the U.S. cost around US$ 9/pound; while considering the following white-meat fish with the highest commercial value, the fillet of orange roughy was US$ 4.2/pound, haddock US$ 3.5/pound and turbot US$ 3/pound (*El Periódico de Acuicultura*, Pto. Montt, December 2004).

Despite the concentration of world supply and the imperfect demand substitutability, the few large producers/traders of *sea bass* might not be able to have price power, in final product markets, if the catch quota regulations that govern this species prevented them from strategically controlling their sales volumes. However, two sources of information raise doubts about the validity of this hypothesis.

First, in *none* of the years of the period 1993-2002 did the winners of fishing quotas in Chile use 100% of the quotas obtained via auction. The average percentage use in this period, at aggregate level in the fishery tendered, was 72.6% of the total tonnage of fishing rights auctioned (source: Subpesca records). In some of those years, the quota use was about 80-95% of total tonnage awarded via auctions, while in other years even less than 50%. Second, some environmental NGOs, supported by representatives of national fisheries regulatory agencies, have argued that several major companies involved in global *Chilean sea bass* fishing, and which have participated in the auctions analyzed here, exert strategic arbitrage with the fishing quotas acquired in different jurisdictions (quotas which are valid only for

[6] Fishing operations in Chilean waters occur between the months of September through February. In the remaining months, the fleet migrates to fish different grounds, some in international waters and others in sectors controlled by the CCAMLR (Peña & Vespa, 2008).

particular fishing zones; Catarci, 2004). Thus, these companies could use the portfolio of quotas acquired, among other things, to justify, in case of being subject to investigation *in situ,* catches illegally obtained in other parts of the world (ISOFISH, 1999).

We will assume that incumbent firms in Chile, operating in industrial *Chilean sea bass* fishing, are part of a global oligopoly market, in which very few suppliers act, and whose fishing operations are integrated with the distribution and marketing phase in the final consumer markets. It will also be assumed that these integrated operators have pricing power in their sales markets, given the significant barriers to entry into this industry, the imperfect possibilities of substitution on the demand side, and the evidence suggesting that fishing quotas may not necessarily be limiting factors of optimum supply decisions in the world *Chilean sea bass* market.

Auctioning results

Appendix A.0 offers a summary of the auctions' rules. This section summarizes the outcomes observed in the sequence of yearly auctions, focusing on those relevant for the model to follow.

Table 1: ITQ yearly auctions, *Chilean sea bass* industrial fishery (Auctions occurring in December before each fishing season)

Fishing season →	**1993**	**1994**	**1995**	**1996**	**1997**	**1998**	**1999**	**2000**	**2001**	**2002**
Number of annual auction	1	2	3	4	5	6	7	8	9	10
(1) Auctioned rights (tons) (% of TAC)	4500 (90)	450 (9)	585 (9)	675 (9)	540 (9)	540 (9)	405 (9)	405 (9)	378 (9)	378 (9)
(2) Total Revenue (US$, mills.)	7.32	0.88	8.97	0.49	0.39	0.37	0.25	0.24	4.3	0.19
(3) Average selling price (P_{sell}) (US$/ton)[a/]	278	190	1974	115	102	102	76	75	1413	64
(4) P_{sell})/$P_{minimum}$ (number of times)[b/]	4.1	4.9	29.8	1.2	1.2	1.06	1.01	1.05	20.8	1.02
(5) Export Price (FOB) (US$/processed ton.)[c/]	4120	3590	5230	6990	6200	6199	5436	9027	8490	8150
(6)= (3)/(5); as %	6.7	5.3	38.5	1.6	1.6	1.6	1.4	0.8	16.7	0.8
(7) Number of participants (firms without license buying)	14 (3)	10 (6)	10 (6)	8 (2)	8 (0)	9 (3)	7 (0)	8 (2)	6 (1)	5 (0)

ITQ: Individual transferable fishing quotas; TAC: Total allowable (annual) catch
a/: This value is equivalent to a constant annuity, expressed in US$/annual ton of fishing rights bought, paid yearly over a 10-year horizon and whose present value (discounted at 10%) is equal to the present value of 10 identical payments to be made by each buyer of an auctioned fishing license. This calculation considers the TAC-percentage sold at each license and the TAC levels prevailing at each post-auction year (more details at Fernández 2008).
b/: ratio between the average (per ton) selling price of the different licenses auctioned each year and the minimum (per ton) selling price ex-ante defined for each annual auction.
c/: Yearly-average export (fob) price of frozen fillets. The latter is the predominant product format at this fishery (with an average –and fairly constant-- technical conversion rate of about 68%).
Source: Peña-Torres & Vespa (2008)

In the two auctions with attempted entry of a new rival (a different one each time[7]), the average awarding price of the lots auctioned was substantially higher than the average prices observed in the remaining years. In the first auction with attempted entry of a new rival (third annual auction), the average award price was US$ 1974/ton, i.e. 16 times higher than the average award price at auctions without any attempt of entry of new competitors in this industry (see Table 1).[8] In the second auction with an entry attempt (ninth annual auction), again the average award price was clearly higher than that of the other annual auctions.

Row (6) of Table 1 shows the increase in award values in the third and ninth auctions, increases that are far from coinciding with similar changes in export prices of the main processed product in this fishery (most of this fishery production is exported). Also, in neither of the two years with attempt to enter was there any change in regulations, or in any other source of significant cost reduction (e.g., oil price) for the fishing business, which might explain the very significant increase in award prices of the fishing rights auctioned in these two years.

In order to complete the context of discussion, in what follows we provide more information on the results observed in the series of annual auctions that occurred between December 1992 and December 2002. The first annual auction (see Graph 1) involved 14 companies, of which 11 obtained at least one lot. These 11 companies match those that were already fishing for sea bass prior to starting the auction system. The first auction resulted in awarding prices that were on average 4 times higher than the minimum price set ex-ante. This auction was particularly significant because it auctioned 90% of the total annual quota (TAC). Thus, there was particular interest in awarding lots which would enable companies to consolidate their operations in this fishery.

In the second annual auction, the average selling price of fishing licenses was almost a third lower than the previous year's average price (Table 1), but still nearly 5 times higher than the corresponding minimum selling price.

When the first attempt at entry of a new rival did occur (third annual auction), the award prices of the lots rose dramatically: the average selling price (US$/ton) was 10 times the previous year's average and 30 times higher than the corresponding minimum selling price. Graph 2 reports the award prices, along with the respective buying company (identified by Ei, i=1, 2,…), of each of the 10 lots sold in the

[7] Both potential entrants had no experience with fishing or marketing internationally *Chilean sea bass*. Thus, these firms would have had to sink investment, in case of entry success, both in sea-bass specific vessel equipment and international marketing networks.

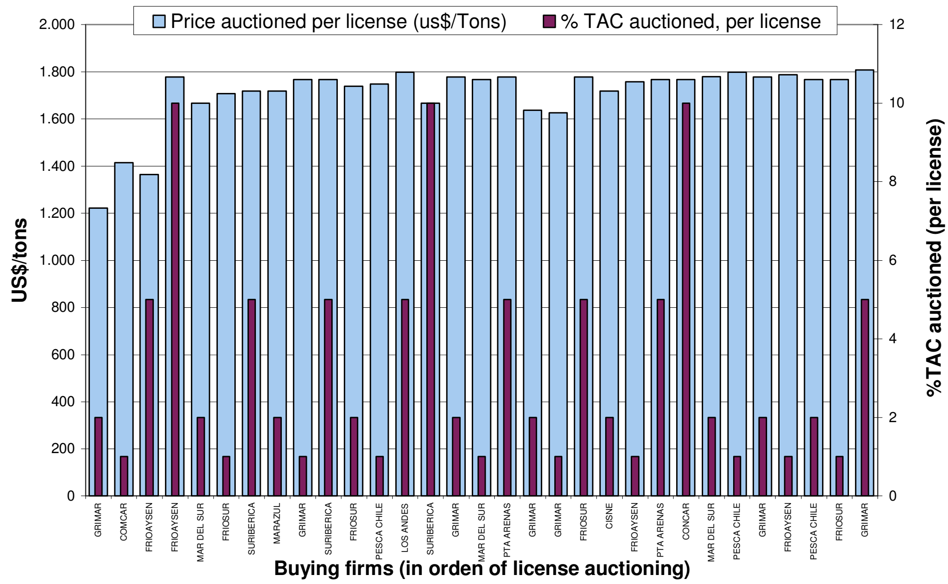
[8] This comparison considers the average award price (US$/ton. of *annual* fishing rights) from the auctions occurring for fishing years 1993-2002, excluding the 1995 and 2001 seasons.

third auction. Note that the total number of firms participating in this auction (i.e., possible independent competitors) was identical to that of the previous year's auction. Thus, the dramatic rise in the average price for the award of the lots sold in the third auction cannot be interpreted simply as the result of increased competitive pressure arising from a greater number of rivals.

At the fourth annual auction, the average award price (US$/ton of fishing rights) clearly decreased, being only 20% higher than the corresponding minimum selling price. As from this auction, and until the eighth, awarding prices were very close to the corresponding minimum selling price (Table 1).

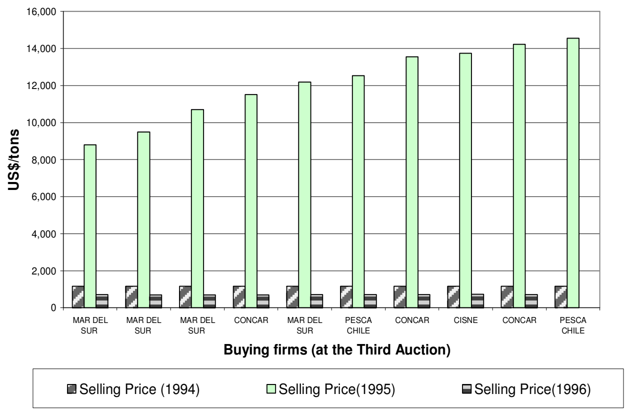
Finally, Graph 3 reports the award prices at the auctions for the 2000 and 2002 fishing seasons, which were representative of the average prices observed in the auctions of 1997-1999. This Graph also shows, in contrast, the significantly higher award prices achieved at the ninth annual auction[9], in which the second attempt of entry by a new rival occurred. Again, this significant increase in award prices cannot be explained as the simple result of an increase in the total number of rivals in the annual auction (Table 1).

Graph 1: First Auction (1993 fishing season)

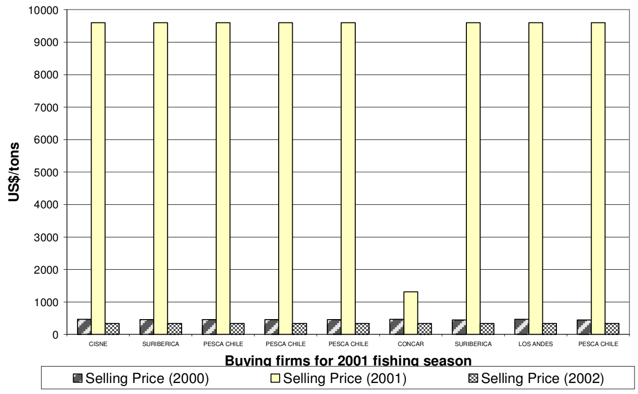


[9] The selling price of the sixth lot sold in this auction is clearly an anomaly. We have no information to give a clear explanation about this.

Graph 2: Third Auction (1995 fishing season)



Graph 3: Ninth Auction (2001 fishing seson)



Source: Own elaboration, based on information provided by Subpesca.
Notes: (1) The selling prices reported in Graphs 1-3 are expressed in US$/(ton of fishing right) and correspond to the present value (discounted at 10% per year) of *the whole series* of annual fishing rights (tons) sold at each license, a series of fishing rights throughout a 10-year period. (2) Graph 1 reports the selling price of each license sold at the first auction, the corresponding buying firm and the TAC-percentage contained in each license.

The arguments presented so far suggest that the only significant change, common to the two auction years with sharp increases in award prices of the rights auctioned, relates to the corresponding entry attempts of a new competitor into this industry. Thus, it is worth analyzing why, associated with these entry attempts, the incumbent firms were willing to pay substantially higher prices (per ton) for the rights acquired in those two years. As an alternative to the hypothesis of a collusive defense of extranormal profits by the incumbents, the following model will analyze parametric conditions for the possibility that the incumbents were willing to pay, in a non-cooperative way, in order to deter the entry of a new rival to the industry.

3. Model: Notation and Assumptions

Assume N≥2 firms involved in a sequence of English auctions. The firms compete to buy L≥1 homogeneous lots that are sold sequentially, each lot representing fishing rights that accrue to firms involved in the final stage in a homogeneous-good Cournot market. All participants have perfect information.[10] Finally, for simplicity, it is assumed that the fishing rights auctioned are not tradable.[11]

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[10] In the industry modeled there appear to be no significant asymmetries of information between rival fishing firms. The imperfect competition that seems to prevail in this industry, at least in the distribution and wholesale production marketing phases, could have other sources of market power (e.g., Anderson 2003; Doeringer & Terkl 1995).

At the start of the auction process, there are I≥1 *incumbent* firms; these firms already have fishing rights and can participate in the final Cournot stage without acquiring any lot; superscript i denotes the incumbent i. There is also a number E of *potential entrants*, who have no fishing rights at the beginning of the auctions and need to purchase lots to participate in the Cournot market. It is assumed that E=1 and superscript e denotes the potential entrant. (The case with E>1 does not provide additional insights on the stylized facts underlying this discussion).

At the end of the auction of L lots, involving N firms ($N = I + 1$), the number of participants $n \leq N$ competing in the post-auction Cournot market is defined.

The auction process begins with bidding on lot 1 and continues sequentially until lot L. For simplicity, the strategies of firms are written as if they are participating in a second-price sealed-bid auction (Vickrey), which, under perfect information, generates the same payments as an ascending price oral auction (Riley & Samuelson 1981, Krishna 2002, Hoppe et al. 2006). Thus, the strategies of the bidders are written as a profile of best responses to the highest bid in each auction (excluding its own bid). The auction is won by the firm that bids highest, committing itself to pay the value of the second highest bid. The rule for resolving ties is: if an incumbent ties with the entrant, the incumbent wins; and if two or more incumbents tie with each other, one of them wins with probability $1/k$, where k is the number of incumbents tied (see Hoppe et al., 2006).

Let s_l^h be the highest bid that firm j notes for lot l (excluding its own). The superscript h indicates whether the highest bid was made by an incumbent (h=i) or incoming (h=e) firm. b_l^j denotes the bid that firm j makes for lot l. In the Cournot market, each firm j has constant (and identical) marginal cost $c \geq 0$, each producing $q_j(n)$. The total (industry) production is $Q = \sum_{j=1}^{n} q_j$ and the inverse demand function is $P(Q) = a - Q$, with $a > c$ and P(Q) being the product price. To compete in the Cournot market, the

[11] Fernandez (2008) develops a related model where firms buy *tradable* fishing rights (tons) in a (single) sequential auction of L lots. The firms sell their catches in a post-auction Cournot market with differentiated goods, whose demand levels are unknown at the auctioning time; being then known (under the assumption of symmetric information for all competitors) only the distribution of a demand-level parameter. After the auction, participants know the true level of demand and correspondingly adjust (by trading) their fishing right endowments. Since the expected demand levels (at the time of the auction) are identical for all the competitors, and all of them being risk neutral, the expected payoffs for them in the sequential auction are then equivalent to the payoffs they get when they participate in a (post auction) deterministic homogeneous-good Cournot market.

entrant must pay a sunk cost F>0, payable only once (when buying its first lot in the auctions). Denote the minimum viable scale of operation by $q^{MIN}(F)$.

The production capacity of each firm depends on the fishing rights they possess. The capacity constraint for incumbents firms is $q_i^R = L_i \cdot q_l + x_i$, and $q_e^R = L_e q_l$ for the entrant. L_j is the number of lots that the firm j acquires in the sequence of auctions, q_l the tons of fish to which each lot l gives a right (all lots are identical) and x_i the fishing rights (tons) which incumbent i holds, prior to opening the auctions. All incumbents are symmetric ($x_i = x$).

At the Cournot stage, the rivals will not necessarily use all the fishing rights they hold. Each firm produces the quantity that maximizes its profit, conditional on its production capacity. Thus, in the Cournot stage there could be two types of firms: *constrained* (with active constraint of production) and *unconstrained*.

At the time of purchasing a new lot, each firm's valuation of the lot will depend on whether its capacity constraint in the Cournot stage is active or not. If a firm has, at the time of bidding for lot *l*, a number of fishing rights less than the optimal Cournot quantity (i.e., has an active capacity constraint at that time), it will value lot *l* according to the gain that its sale can produce in the Cournot market. If its fishing rights at the time of bidding for *l* are greater than the optimal Cournot quantity (*unconstrained* firm), its valuation for lot *l* will not come from its sale in the Cournot stage (sale that would be sub-optimal). Instead, it will depend on another rival acquiring the lot and selling the corresponding catch in the Cournot stage, thus reducing the equilibrium price in that market.

We will assume that: (i) all incumbent firms are always *unconstrained* in the Cournot stage, i.e., even if they do not acquire a lot in the auctions, their x>0 initial tons are sufficient to produce the Cournot optimum.[12] Note that the possible empirical validity of this assumption cannot be dismissed based on the evidence cited in section 2. On the other hand, (ii) the entrant firm will always be *constrained*, even if it acquires all the lots for sale. Thus, incumbents have no incentive to compete among themselves to acquire a lot. Their only reason to buy a lot will be to deter the entry of a new rival, as that entry would lower the market price of each ton of fish. Hence, for the incumbents, the value of the lots auctioned will be zero if a potential entrant does not participate in the auction; otherwise, the value of the lot will be the loss of profits for the incumbent that would be caused by its purchase by the potential entrant.

[12] Under this assumption, a possible rationale for the fishing regulator to define a total fishing quota greater than the incumbents' optimal Cournot production is to understand that policy as a limit facing possible increases in aggregate fishing, given the possibility of entry of new firms.

In this model, the only way in which the incumbents are willing to pay to prevent the entry of a new rival, assuming that the incumbents are independent rivals which do not maintain any collusive agreement between them, is that these firms are *unconstrained* agents. If they were constrained, their Cournot production would be independent of the number of rivals in this market and therefore their valuation to deter entry would be zero. Allowing the incumbent firms to mutate, during the sequence of auctions, from being initially constrained agents to becoming unconstrained firms, would introduce unnecessary complications into the model. Therefore, this possibility is excluded by assumption.

On the other hand, if the incumbents were indeed constrained agents, then the only possible explanation, in this model, for the significant increases observed in the prices of lots sold in the two annual auctions with attempted entry, is that such increases reflect the value (to incumbents) of protecting extra-normal rents, achieved through a collusive agreement to reduce competition between them when acquiring the lots at auction. This deduction assumes that other determinants of the lot valuation remain unchanged in the 2 years with entry attempt, with respect to the rest of the years with auctions. The latter may be reasonably supposed given the evidence presented in section 2.

In relation to the potential entrant, if this firm were allowed, through purchases in the auctions, to become an *unconstrained* Cournot rival, this would generate two effects: First, such entry would generate a greater drop in the Cournot equilibrium price than if the entrant was a constrained rival. Thus, letting the potential entrant enter as an unconstrained rival would generate an even stronger incentive, *ceteris paribus*, for the incumbents to deter its entry (a similar idea is found in Krishna 1993 and Rodriguez 2002). Second, given the entry of the new rival as an unconstrained firm, its behavior in the auctions would be strategically identical to that of the incumbents. Thus, to achieve parsimony in the analysis, we will assume that the potential entrant is always constrained.

The previous assumptions require imposing certain conditions on x and q_l. Let $q^*(I, L_e)$ be the amount of catch as from which the lots purchased at auction become an inactive constraint in the Cournot profit maximization. For all the incumbents to be always unconstrained, let $x \geq q^*(I,0)$. Moreover, to ensure that the potential entrant has a positive valuation for each lot and that they participate in the Cournot stage as a *constrained* agent, let $q^{MIN}(F) < q_l < \dfrac{q^*(I+1,0)}{L}$. That is, (i) the minimum scale of operation will be less than the tons of fish contained per lot and (ii) the fishing rights accumulated in the total of L≥1 lots to be auctioned will be less than the amount needed to become an *unconstrained* agent.

If the firm buying a new lot is incumbent, the other incumbents know that it will not increase the total quantity in the Cournot market, and thus do not care about such a purchase. But if that purchase is made by a new entrant, this will use its entire quota at the Cournot stage. So the incumbents will value that lot at the amount that their profits would fall, given the drop in the Cournot equilibrium price. The valuation of each incumbent for the last lot auctioned will then be[13]:

$$V_L^i = \begin{cases} \Pi^i(Q) - \Pi^i(Q + q_l) > 0 & \textit{If lot L is bought by entrant} \\ \Pi^i(Q) - \Pi^i(Q) = 0 & \textit{If lot L is bought by another incumbent} \end{cases} \tag{1}$$

where $\Pi^i(Q)$ is the Cournot profit, given the volume of supply Q. In turn, the entrant firm will value each lot for the profit that it can obtain in the Cournot stage with the tons acquired. Thus, its valuation for lot l will be:

$$V_l^e = \Pi^e(l \cdot q_l) - \Pi^e((l-1) \cdot q_l) \tag{2}$$

In order to analyze how the incentives of incumbents to collude in auctions change when a new rival can take part, two additional assumptions are introduced which allow this (sequential) auction game to be infinitely repeated, in which the structure of each subgame is identical to that described above:

(A1) The number x>0 of fishing rights that incumbents have at the beginning of each subgame is time invariant. Thus (i) these rights do not lose value over time, (ii) there is no catch inventory transfer from one subgame to another, and (iii) the rights acquired via auction only serve for the immediately following Cournot market.

(A2) At the beginning of each subgame of sequential sales of L≥1 lots, the potential entrant must sink a cost F>0 to participate in this auction process.

With these two assumptions, the subgame described (auction of L lots and the subsequent Cournot market) can be replicated countless times, thus making it possible to analyze the benefits of possible collusion between incumbent firms during their participation in the auction process.

In the next section, the model assumes that a single lot is auctioned in each subgame. From this, three results of interest are derived: (i) The existence of a *free rider* effect, while deterring the entry of a new rival; (ii) the limit imposed by this effect on the ability to collude among incumbents in order to deter

[13] Note that each incumbent knows, at the time of auction of lot $l<L$, the effect its respective bid will have on the outcome of future auctions for additional lots (given the information structure in this model, each player chooses its optimal strategies using backward induction). Therefore, every incumbent will consider the results anticipated in these future auctions when deciding the valuation of lot l<L (since it knows the effect the acquisition of any lot by the entrant would have, on the total number of lots that the entrant might manage to reach when the Cournot stage begins). Thus, the valuation of an *any* l lot does not correspond - strictly speaking - to that expressed in (1). This expression is valid only for the auction of the last lot auctioned.

entry; and (iii) the conditions necessary for a potential entrant to acquire the lot on sale. In Section 5, the model is solved considering the sequential sale of L lots in each subgame. Then, the following will be discussed: (a) how the divisibility of the total volume of production to be auctioned (which is modeled parameterizing the number of lots for sale under the assumption that the total production volume auctioned is held constant) affects the possibility of entry of a new rival in a non-collusive context; and (b) how the divisibility of the total production volume to be auctioned affects the incentives of incumbents to collude in the auctions in order to deter entry.

4. *A* single-*license auction*

This section analyzes the conditions under which, with L=1, the incumbents deter entry when: (i) $V^i \geq V^e$ and hence no kind of coordination is required between the incumbent to deter entry, and (ii) $V^i < V^e$ and therefore the only option for deterring entry is by using some type of coordination among incumbents.

A key aspect in the lot valuation of each incumbent (V^i) is that spending on deterring entry, which any of these firms may make, will have public-good properties: if any incumbent invests in deterring entry, all the others benefit equally with that spending, and incentives therefore exist for these firms to act as free riders. The following shows that, given $V^i \geq V^e$, it will not be a Nash equilibrium in pure strategies for all the incumbents to reach a draw and then divide among themselves the cost of deterring entry. In all Nash equilibria, there will remain the free rider effect.

Proposition 1 (Free Riding Effect; L=1). *If (i) E=1=L, (ii) incumbents act in a non-coordinated way, and (iii)* $V^i \geq V^e$ *, then in pure strategies there will be multiple Nash equilibria in which any one incumbent assumes the entire cost of deterring entry, while the others benefit from the deterrence without incurring any cost.*

Proof. See Appendix A.1

So, the benefit to the incumbents, that at least one of them buys the auctioned lot, has characteristics of public good. This is so because, associated with the spending on deterrence, there is no rival consumption or possible exclusion of the other incumbents from the resulting benefit. So, when the lot valuation of each incumbent is greater than or equal to that of the entrant, only one of them will invest in deterring entry, benefiting all others, who will act as *free riders.*[14]

Note that, keeping conditions (i) and (iii) of Proposition 1, there could never be collusion between the incumbents in order to share the spending to deter the entry of a new rival. In this context there will be no credible punishment for any deviation from the collusion agreement. The threat that, as punishment for whoever deviates from any agreement, some incumbent might allow the entrant to buy the lot under auction, would not be credible. Thus, the presence of a potential entrant, in this specific context, would have no effect whatsoever on the incentives to collude in the auction.

What happens when each incumbent cannot alone deter the entry of the potential new rival, i.e. $V^i<V^e$? In this case, it might be feasible to sustain a collusive agreement to co-finance the cost of deterrence. Thus, Proposition 2 finds the minimum discount factor, when $V^i<V^e$, that allows collusion in an infinite series of auctions, between a set of *n* incumbents (such that for each incumbent participating in this collusive agreement, the benefit of colluding, in each subgame, is greater than the benefit of competing as a Cournot agent), for the purposes of deterring the entry of a new rival. This result makes it possible to analyze later how the sustainability of this collusive agreement changes when the tender for the same total amount of fishing rights (tons) occurs through the sale of multiple lots (L>1).

Proposition 2 (Collusion between incumbents; each subgame with L=1). *If (i) there is an infinite sequence of identical and mutually independent subgames, where each subgame corresponds to the auction of a single lot (L=1), this being followed immediately by an instance of Cournot competition in which firms use their production rights, (ii)* $V^i < V^e$ *, (iii)* $n{\cdot}V^i > V^e$ *, and (iv)* $1 < n \leq I$ *incumbents establish a collusive arrangement by which they share equally the total cost of deterrence in each auction, under the threat of punishing the occurrence of at least one deviation from that agreement by reverting forever to strategies that allow the entry of a new rival, then the minimum discount factor that sustains collusion between the n incumbents will be:* $\underline{\delta} = \dfrac{V^e}{n{\cdot}V^i}$*, where* δ= (1/1+r)<1, r being the discount rate.

Proof. *See Appendix A.2*

Proposition 2 presents a mechanism to sustain a collusive agreement that aims to reduce the cost each incumbent must cover to succeed in deterring entry. Using this logical device, a parametric condition then follows (δ(n; L=1)) denoting the sustainability of the agreement.

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[14] As all incumbents are equal, there will be as many Nash equilibria as number of incumbents participating in the auction. If mixed strategies were allowed, each of the incumbents would deter the entry of a new rival with a given probability, and entry would therefore occur with the complementary probability (see Hoppe 2006).

The minimum discount factor calculated in Proposition 2 shows that the incentives of incumbents to collude increase, *ceteris paribus,* to the extent that the entrant's valuation of the lot auctioned is smaller and approaches the incumbents' valuation, until eventually $V^i \geq V^e$, in which case the *free rider* effect makes it impossible for the incumbents to collude.

The conditions necessary in the post-auction competition so that $V^i < V^e$, when L=1, are identified below. If any of the necessary conditions for $V^i < V^e$ is not satisfied, then $V^i \geq V^e$, in which case it is possible to achieve the deterrence to entry without collusion among the incumbents. Identifying these conditions makes it possible to analyze the effect, on the feasibility of entry deterrence strategies, of: (a) the size of the rights for sale (the total tons of fish to which it gives rights), (b) the level of fixed (sunk) costs of entry, and (c) the number of *rival* incumbents in the post-auction market.

Proposition 3 (Non-coordinated entry deterrence; L=1): *If there is an infinite sequence of identical and mutually independent subgames, where each subgame corresponds to an auction of a single lot (L=1), this being followed immediately by a Cournot competition stage, in which firms use their production rights, then $V^e > V^i$ if and only if the following three conditions are jointly met:*

(i) $I > 1$

(ii) $\frac{F}{(a-c)^2} < \frac{I^2 - 2}{[(1+I)\cdot(2+I)]^2}$

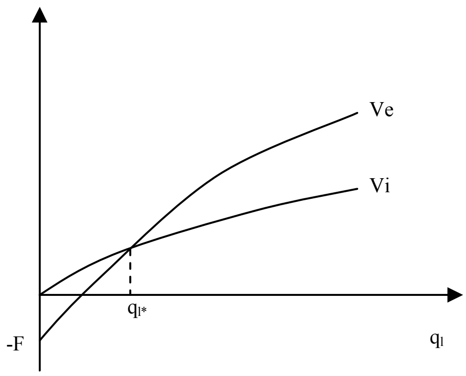
(iii) $q_l > \frac{(a-c)(I-1) - \sqrt{(a-c)^2 (I-1)^2 - 4 \cdot I \cdot F (I+1)}}{2I}$

Proof. *See Appendix A.3*

Figure 1 below shows the valuations of a representative incumbent and of the potential entrant for the lot auctioned in terms of tonnage auctioned, for given values of cost *F (*which affects the vertical position of the function V^e) and the number *I* of *rival* incumbents in the post-auction market (the value of I affects the slope of both value functions). Thus, while meeting conditions (i) and (ii) of Proposition 3, *ceteris paribus*, necessarily $V^e > V^i$ to the extent that the amount auctioned is greater than q_l*.

In relation to the auctions for fishing years 1995 and 2001, Propositions 1 and 3, despite being valid for the simplification L=1, help to motivate the possibility that the deterrence of entry that was observed in both years might have occurred as a result of non-cooperative strategies between the incumbents, and not necessarily because of collusion between them.

Figure 1



Thus, in the context discussed so far, the occurrence of entry deterrence without incumbents' collusion would have required a breach of at least one of the three conditions outlined in Proposition 3. For example: (a) the breach of condition (i). This could happen if the incumbents were perfectly colluding in the post-auction market, so *de facto* I = 1, or if the industry was directly a natural monopoly, in which case the cost advantage in favor of the incumbent, given F>0, would always enable the monopolist to prevent a new rival's entry. Or (b) a breach of condition (ii), in such a way that the entry cost F was sufficiently high in relation to the expected profitability of the oligopolistic market, the latter approximated by an indirect proxy of the unit margin of Cournot profit (a-c>0), thus increasing the advantage of competition in favor of the incumbents. Or (c) a breach of condition (iii), in such a way that the size of the lot auctioned was small enough. In this regard, note that a greater value of q_l will be associated with higher profits for the entrant and therefore a higher V^e (as long as p-c>0). At the same time, the loss of Cournot profits for *each* incumbent (and thus the value of V^i) will be lower if I is higher, for a given a value of q_l and the resultant fall in the price of Cournot equilibrium with a new rival's entry. In this case, compliance with condition (i) of Proposition 3 will be favored, and thus the possibility of entry of the new rival.

Another interesting deduction from the conditions identified in Proposition 3 is that when the number of incumbents is equal to or less than two (see conditions (i) and (ii)), and even if F = 0, the incumbent(s) will always deter the new rival's entry. This is due to the first-mover advantage in favor of the incumbent, which arises from the possession of an initial amount x>0 of fishing rights, such that the incumbent is always an unrestricted competitor in the post-auction market. This limits the profits of the potential entrant, if it manages to get in, at most to the earnings of a duopolist, while the incumbent will remain monopolist (or duopolist, if I=2) if it deters entry. Thus, the loss of benefits of the incumbent, if there is entry to the Cournot market, will be greater or at most equal to the entrant's potential gains, in which case entry will never occur. This reproduces the result obtained in the one shot entry game of G&N (1982), where an incumbent monopolist always prevents the entry of a new rival.

5. Sequential auction of L>1 lots

This section discusses how the incumbents' ability to deter entry will be affected by the fact that a given total quantity of fishing rights (tons) is auctioned in a greater or lesser number of fishing licenses. To solve the maximization problem of the incumbents and the entrant in a series of sequential auctions of L>1 lots, firms' strategies are defined in terms of different states. These states, or histories, are the number of lots that the entrant has acquired prior to the current auction of a new lot. Thus, if, at the time that bidding starts for lot l, the entrant already owns 3 lots, the history in l will be $h_l = 3$. The set of possible histories is $H_l = (0,1,2,...,l-1)$, indicating all the possible histories that may face a player at the time of starting the auction of lot l. The expected payment for the incumbent j when auction l takes place depends on $E^j[h_{L+1}|h_l]$, i.e. the expected number (in *l*) of lots that the entrant may acquire by the end of the auction of the L lots. Thus, in the competition stage L+1 (Cournot market), the number of lots acquired by the entrant will be h_{L+1}, which is estimated by each incumbent, at the time of the the auction of the l-th lot, conditional on the amount of lots that the entrant has already accumulated up to that point in time.

A strategy for incumbent j will be a vector $(b_1^j, b_2^j, b_3^j, ...b_L^j, q_j(h_{L+1}))$, where $q_j(.)$ denotes the Cournot production of incumbent j and $b = (b_1^j, b_2^j, b_3^j, ...b_L^j)$ is a function of the expected value (in the *l*-th stage) $E^j[h_{L+1}|h_l]$. The notion of equilibrium that we use enables the number of subgames to be summarized in such a way that auction participants choose their strategy according to the state they face (h_l), no matter in what order the entrant acquired the different lots it has. The transition function between h_t and h_{t+1} will be $\Lambda_{t+1} = f(h_t, b_t, s_t^h)$. Finally, the law of conditional probability that ensures independence of the strategies with respect to time will be as follows:

$$E^j[h_l = \theta | h_{l-1} = \omega] = E^j[h_{l+s} = \theta | h_{l+s-1} = \omega].$$

The strategies thus defined correspond to Markovian strategies and the concept of equilibrium used is called *Markovian recursively undominated equilibrium* (sophistication of a perfect Markovian equilibrium; see Rodríguez 2002). The benefit obtained by incumbent j for purchasing the l-th lot will be:

$$w_l^j = \begin{cases} E\left[\left(\Pi^j(h_{L+1}|h_{l+1}=h_l) - \sum\limits_{b_l^j > S^h} s^h(h_{L+1}|h_{l+1}=h_l)\right)\Big|h_l\right] - s_l^h & \text{if } b_l^j > s_l^{i\neq j} \text{ or } b_l^j \geq s_l^e \\ E\left[\left(\Pi^j(h_{L+1}|h_{l+1}=h_l) - \sum\limits_{b_l^j > S^h} s^h(h_{L+1}|h_{l+1}=h_l)\right)\Big|h_l\right] - \frac{1}{k}\cdot\left[s_l^h\right] & \text{if } b_l^1 = b_l^2 = \ldots = b_l^j = \ldots = b_l^{k-1} = b_l^k = s_l^h \\ E\left[\left(\Pi^j(h_{L+1}|h_{l+1}=h_l) - \sum\limits_{b_l^j > S_j} s^h(h_{L+1}|h_{l+1}=h_l)\right)\Big|h_l\right] & \text{if } s_l^j > b_l^j \\ E\left[\left(\Pi^j(h_{L+1}|h_{l+1}=h_l+1) - \sum\limits_{b_l^j > S_j} s^h(h_{L+1}|h_{l+1}=h_l+1)\right)\Big|h_l\right] & \text{if } s_l^e > b_l^j \end{cases}$$

where k denotes the number of incumbents who tied for highest bid, while $E\left[\Pi^j(h_{L+1}|h_{l+1}=\theta)|h_l\right]$ denotes the expected benefit for the incumbent firm j at the time of auction l, given that the entrant at the time of that auction has h_l lots and after the auction will have θ lots, with $\theta = \{h_l, h_l+1\}$. For its part, $E\left[\left(\sum\limits_{b_l^j > S_j} s_j^h(h_{L+1}|h_{l+1}=\theta)\right)\Big|h_l\right]$ denotes the expected payments to be made by the incumbent firm j if it wants the history h_{L+1} to be performed in the final stage, given that at stage l the current history is h_l and that the history in stage $l+1$ would be θ.

It is shown below that bidding for a given total quantity Q_l (tons) of fishing rights in multiple (L>1) lots can make it easier, in relation to auctioning the same amount Q in a single lot, for the incumbents to share the total spending required to deter entry without collusion. And this, even in contexts of relative values (V^i/V^e) in which it is impossible to deter entry collusively when the same tonnage Q_l is auctioned in a single lot.

To explain this idea, the following Proposition 4 considers a parametric context such that if L=1 then $V^i \geq V^e$, it thus being impossible for the incumbents to collude to share the total expenditure required to deter entry. In this context it is shown that, for the case where multiple lots are sold such that I≥L>1, and imposing an upper bound on the number of times each incumbent is willing to deter entry of a new rival (by imposing a ceiling on F), in all the *Markovian recursively undominated equilibria* (MRUE), L incumbents will share symmetrically and non-cooperatively the total cost of deterrence, each taking turns to buy a lot (with as many equilibria of this type existing as there are possible forms of taking turns).[15]

[15] A note of caution on the interpretation of the following result: While it will demonstrate the existence of non-cooperative Nash equilibria as described in Proposition 4, it is hard to imagine as likely, in practice, that symmetrical incumbents such that I>L, and which decide simultaneously, use assumptions such as those required by the equilibria described in Proposition 4, without them having some kind of *explicit* coordination (i.e. collusive) mechanism. (In this regard, see Hoppe et al. 2006, p. 378).

Proposition 4 (Entry Deterrence via Turns; I≥L>1). *Let* $q_l = \dfrac{Q_l}{L}$*, where* q_l *is the amount of fishing rights in each lot,* Q_l *the total (constant) amount auctioned, and L>1 the number of lots. Assume further that: (i) the incumbents (I≥2) and the entrant (E=1) use pure strategies, (ii) if the total tonnage* Q_l *is tendered in a single lot* $(L = 1) \Rightarrow V^i \geq V^e$*, and (iii) each incumbent cannot deter more than once an entrant which wants to acquire its first lot, as long as* $F < \dfrac{q_l}{(I+1)^2} \cdot \left(I \cdot (a - c - q_l) - q_l / 2\right)$*. Then, if* $I \geq L \Rightarrow$ *the potential entrant will not acquire any lot since there will always be a different incumbent to deter its entry.*

Proof: *See Appendix A.4*

Equilibria in which entry deterrence occurs via turns among incumbents are particularly interesting from the point of view of the stylized facts. In the two years of auction with entry deterrence, alternating patterns were evident among some of the incumbent firms when buying the various lots sold in each year (see Figures 2 and 3). However, further analysis on this last point goes beyond the reaches of this paper.

To the extent that the ceiling imposed on the cost F, established in Proposition 4, is relaxed, the number of non-cooperative equilibria in which incumbent firms may deter entry of a new rival, through not necessarily symmetrical turns, will increase. Thus, and maintaining the assumption of pure strategies, some incumbents may deter entry more than once, while others do so only once. In all these non-cooperative equilibria entry will never occur, as long as I≥L.

Thus, and given the sale of multiple lots such that $I \geq L > 1$, by increasing the number of lots for sale, with Q_l constant, non-collusive entry deterrence becomes viable for a wider range of parameter values. Indeed, and as an example: while L>1, the result of non-collusive entry deterrence will also be feasible for cases where if L=1 then $V^e > V^i$, but also $n \cdot V^i \geq V^e$ is met, with n≤I and where *L=I>1*. This insight is formalized in the following Corollary.

Corollary 4.1 (L=I>1). *Let* $q_l = \dfrac{Q_l}{L}$*, where* q_l *is the amount of fishing rights in each lot,* Q_l *the total (constant) amount auctioned, and L>1 the number of lots. Assume that (i) if* $L = 1 \Rightarrow V^e > V^i$ *and (ii)* $\underline{n} \cdot V^i = V^e$*, with* $1 < \underline{n} \leq I$ *and (iii)* $Q_l \leq (a - c) \cdot \left(\dfrac{I \cdot (I - 2 \cdot \underline{n}) + 1}{I \cdot (I + 1 - \underline{n}) - 1} \right) > 0$*. Then, if the* Q_l *tons are auctioned in L=I>1 lots, there will be at least one Markovian recursively undominated equilibrium (MRUE) in which each incumbent acquires one lot and therefore no entry takes place.*

Proof: *See Appendix A.5*

The intuition behind Corollary 4.1 is the following: Considering, first, that (a) auctioning Q_l tons of fish in multiple (L>1) lots allows the incumbents to share the *Total Cost of Deterring Entry* (*TCDE*) and, secondly, (b) focusing on the particular case in which the aggregate valuation of $\underline{n}\leq I$ incumbents (relative to the amount Q_l auctioned) equals the corresponding valuation of the entrant, then dividing Q_l in a number L of lots which is equal to or greater than $\underline{n}>1$ will allow the cost to be borne by each incumbent, for each of them to purchase a single lot, to be less than its respective V^i, thereby avoiding, in a non-collusive manner, the entry of a new rival.

The result in Corollary 4.1 imposes a ceiling condition on Q_l. This is because the *TCDE* does not remain constant for every number L>1 of lots into which Q_l is divided for its sale. To get more insight into this, the following lemma identifies parametric conditions such that: (i) the value of *TCDE* will be growing, constant or decreasing as the number of lots for sale increases, and (ii) the value of *TCDE,* for the case with L>1, will be greater, equal to or less than the value of *TCDE* with L=1.

Lemma 4.2 (TCDE with L>1 lots). *Let* $q_l = \dfrac{Q_l}{L}$*, where* q_l *is the amount of fishing rights in each lot,* Q_l *the total (constant) amount auctioned, and L>1 the number of lots. The Total Cost of Deterring Entry (TCDE) for incumbents, i.e. the cost of acquiring the* Q_l *tons for sale (*$L \cdot \Pi^e(Q+q_l)$*), will be greater (less or equal) when* Q_l *is divided into L>1 lots, relative to the TCDE when the same* Q_l *tons are auctioned with L=1, as long as:*

$$(i) \quad L < (\geq) \frac{Q_l^2}{F \cdot (I+1)} \cdot$$

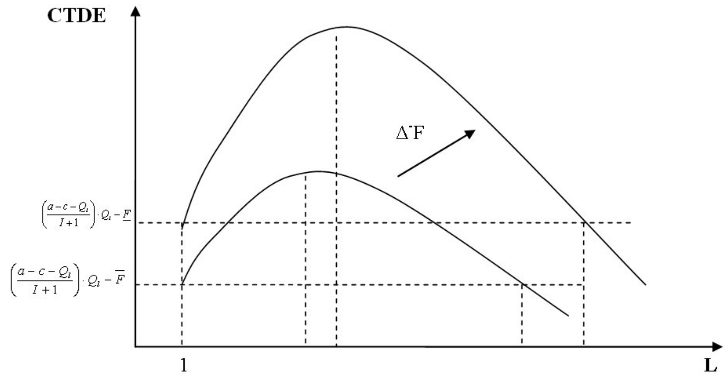
Likewise, the TCDE that the necessary set of incumbents must pay, to ensure success in deterring entry, will be increasing (decreasing or constant) with respect to increases in L, as long as:

$$(ii) \quad L < (\geq) \frac{Q_l}{\sqrt{F \cdot (I+1)}} \cdot$$

Proof: *See Appendix A.6*

The above lemma shows that for a range of values of L> 1, the *TCDE* will increase as the spending on entry deterrence becomes more divisible, although this increase occurs at a decreasing rate. Indeed, there is a value of L as from which the *TCDE* decreases while L keeps increasing. Figure 2 plots the relationship between *TCDE* and L, for two levels of cost F. As F falls, *TCDE* increases, *ceteris paribus*, and there will also be: (a) a larger range of values L>1 for which the *TCDE* is increasing, as well as (b) a larger range of values L>1 in which the corresponding *TCDE* will overtake the *TCDE* with L=1.

Figure 2



In terms of the per-incumbent cost of deterrence, i.e. the *TCDE* divided by the number of incumbents necessary to succeed in deterring entry, and considering the case I≥L, dividing Q_l in L>1 lots will always imply a lower per-capita spending on deterrence (which is equals to $\Pi^e(Q+q_l)$), relative to the per-capita deterrence cost when L=1. This is because $\Pi^e(Q+q_l)$ will always be less as each lot for sale is smaller, which in this model will necessarily occur whenever Q_l is divided into a greater L. Thus, for I≥L, increases in L should make more likely, *ceteris paribus*, a result of entry deterrence without collusion.

What will happen with the per-incumbent cost of deterrence as L increases, when I<L? In this case, the per-capita cost of deterrence is $(L/I)\cdot\Pi^e(Q+q_l)$. Thus, the per-capita cost will behave in a manner equivalent to how the value of TCDE ($L\cdot\Pi^e(Q+q_l)$) varies as L changes (which is described in part (ii) of Lemma 4.2). Note that the value of the function 'per-capita cost of deterrence' corresponds to that plotted in Figure 2 but now divided by I (with the range of L values starting at L=I). Thus, the function 'per-capita cost of deterrence' may or may not have, depending on the value of I, a range of increasing values as L increases.

The following Proposition 5 analyzes how the divisibility of Q_l affects the sustainability of a collusive agreement between incumbents, the purpose of which is to deter a new rival's entry. As in the case of Proposition 2, we rule out by assumption the existence of coordination costs for maintaining a collusive agreement between the participating incumbents.

Proposition 5 (Collusive Agreement, with L>1 in each subgame). *Let* $q_l = \dfrac{Q_l}{L}$*, where* q_l *is the amount of fishing rights in each lot,* Q_l *the total (constant) amount auctioned, and L>1 the number of lots. Assume further that: (i) the incumbents (I≥2) and the entrant (E=1) use pure strategies; (ii) if the* Q_l *tons were auctioned in a single lot, then* $V^e > V^i$*; (iii)* $Q_l > (a-c) \cdot \left(\dfrac{I \cdot (I - 2 \cdot \underline{n}) + 1}{I \cdot (I + 1 - \underline{n}) - 1} \right)$*, where* $\underline{n}$ *is such that* $\underline{n} \cdot V^i = V^e$*, and (iv)* $1 < n \leq I$ *incumbents collude to deter entry, with* $n > (L \cdot \Pi^e(Q + q_l) / V^i)$*,*[16] *by the following mechanism: Any incumbent acquires all the lots and then each incumbent belonging to the agreement pays proportionately its share of the total TCDE spending. If any incumbent, of those needed to sustain the agreement, deviates at least once from the agreement (i.e. pays less than agreed), then the other incumbent members of the agreement revert to Cournot strategies, and therefore entry occurs in all the subsequent subgames. Under these conditions, the minimum discount factor that ensures sustainability of the collusive agreement described is:* $\underline{\delta} = \dfrac{L \cdot \Pi^e(Q + q_l)}{n \cdot V^i}$

Proof: See Appendix A.7.

Note that in Proposition 5, since L>1, the *TCDE* for the colluding incumbents is $L \cdot \Pi^e(Q + q_l)$, while the corresponding TCDE in Proposition 2 (when L=1) is $\Pi^e(Q + Q_l)$. Moreover, Lemma 4.2 has shown that there is a range of values L>1 such that the corresponding TCDE will be strictly greater than in the case L=1. Thus, given the values of the minimum discount factors derived in Propositions 2 and 5, and while condition (i) of Lemma 4.2 is met with the inequality sign <*(*>), then auctioning a given Q_l in L>1 lots will *hamper (facilitate*) the feasibility of a collusive agreement between incumbents, relative to the case when Q_l is auctioned with L=1. A greater 'difficulty' for colluding means that, with L>1, the minimum discount factor that makes feasible the collusive agreement will be higher than that corresponding to the case with L=1 (for a given number of incumbents involved in the collusive agreement). Or, for a given r>0, when L>1 the number of incumbents needed to make the collusive agreement viable is higher than that required when L=1.

[16] This inequality is required for the corresponding minimum discount rate, that makes collusion feasible, to be positive.

6. Conclusions

Spending on entry deterrence can have public-good characteristics for incumbent firms. Thus, the possibility of colluding to deter a new rival's entry will be unfeasible as long as the loss of benefits for the representative incumbent, as result of that entry, is greater than the potential entrant's valuation of entering into the market.

The analysis has pinpointed conditionings for the entry-triggered loss of profits (for the representative incumbent) to be greater, or less, than the benefit of entering for the potential new rival. Firstly, the competition intensity achievable in the post-auction market, proxied in this model by the prevailing industrial concentration in that market. Secondly, the sunk costs of entry (proxy of the incumbents' advantage of competition). And thirdly, the overall quota auctioned and the degree to which this is split for its sale (which directly affects the value of each production right on sale).

With respect to the divisibility of the rights auctioned, it has been found that auctioning a certain production capacity in *multiple* licenses can make it easier, firstly, for the incumbents to use a coordination mechanism so as to share the total cost of entry deterrence. In our model, not all the possible mechanisms for sharing the total deterrence-cost are collusive in nature, i.e., that require a "real and conscious meeting of minds to reach the result of coordination" (according to Richard Posner's definition). However, we have also argued that in a number of the non-cooperative equilibria that would enable a group of incumbents to share the total expenditure on deterrence, it is unlikely --from a practical point of view-- that the incumbents, given the form in which these have been modeled, agree to use precisely the strategic assumptions necessary to sustain these equilibria.

Secondly, it has also been shown that increases in the number of lots for sale can increase the total cost of deterring a new rival's entry (Lemma 4.2). Thus, the effect it may have to sell a given level of production capacity in a greater or lesser number of licenses, on the possibility of new entry, will ultimately depend on how the resulting *trade-off* of incentives for the incumbent firms is resolved; having, on the one hand, greater ease for finding ways to share the total cost of of deterring entry and, on the other, to deal with increases in the latter cost.

In relation to how an increase in the number of lots for sale might affect incumbents' incentives to collude and thus deter entry, in our model the effect depends on how such increase changes the total deterrence cost, as analyzed in Proposition 5.

Note that in all the derived equilibria in which entry deterrence occurs, be it in a non-cooperative form or via collusion, the incumbents spend resources on preventing a new rival's entry; and this, in order to defend the appropriation of extra-normal rents in the final oligopoly market. In our model, such an expenditure produces no positive effect whatsoever on social welfare. Thus, the allocative efficiency implications of entry deterrence in this paper do not differ qualitatively from the equivalent implications of other oligopoly equilibria analyzed in classic works on the subject (Dixit 1980, Spence 1977).

Therefore, the reasons explaining the occurrence of entry deterrence in the model analyzed always involve, whenever deterrence occurs, illicit competition strategies by the incumbents. This is inasmuch as its unequivocal purpose is to prevent artificially, i.e. without any positive effect on social welfare, the entry of a new rival, at the expense of perpetuating the occurrence of allocative losses in the final competition market.

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Appendices

A.0 Auctioning Rules

In the case of industrial fisheries *in State of Recovery* or in *Infant Development* status[17], the Chilean Fisheries Law allows Subpesca, the Chilean fisheries regulator, to sell Individual Transferable Quotas (ITQs) through public auctioning. According with Chilean fisheries law, industrial vessels are those with more than 18 meters of length and weighting more than 50 tons of registered gross tonnage. The *Chilean sea bass* industrial fishery in Chile was at *Infant Development* status at the start of the ITQs auctioning process.

The ITQs for sale are 10-year valid fishing rights corresponding to percentual rights over the global annual quota (TAC), jointly fixed each year by Subpesca and the National Fisheries Council; the latter being a multi-party body representing different interest groups (Peña-Torres 2002a,b).

A Subpesca Decree (from year 1991) defined the auctioning procedure and the number of licenses for sale at each auction, as follows. At the first annual auction, 100% or 90%18 of the corresponding TAC will be for sale. The ITQs for sale correspond to fishing rights that lose yearly 10% of their value. When the first auction involves 90% of the TAC, as in the fishery under analysis, 31 licenses will be for sale, sequentially and in a single act. Three of these licenses provide (each one) fishing rights for 10% of the TAC. Eight of them (each one) fishing rights for 5% of the TAC. Other 10 licenses will provide (each one) fishing rights for 2% of the TAC. The remaining 10 licenses each provide fishing rights for 1% of the annual TAC. Subpesca's motivation for so defining the licenses for sale was to promote the participation of small firms at these auctions.

From the second year onwards, ITQs will continue to be for sale yearly. These ITQs also have a 10-year validity and are percentual rights over the the current TAC. However, they will no longer lose their value gradually but once and for all, at the end of its tenth year. When at the first auction 90% of the TAC has been sold off, fishing rights equivalent in total to 9% of the TAC will be sold at each following annual auction; this total corresponding to the fraction annually liberated from the ITQs sold at the first auction. At each of these following yearly auctions, 9 fishing licenses will be for sale, each corresponding to 1% of the TAC. Thus, from the second auction onwards, equal-sized licenses will be for sale at each auction. Notice that both entry attempts, by a new rival, occurred at this type of auctions. And this will be the setting that we consider at our modeling.

Other relevant features of the auctions under analysis are: (1) The same participant cannot buy, directly or indirectly, more than 50% of the total fishing rights sold at each annual auction. However, there is no legal maximum limit upon the TAC's total percentage that a same participant can buy and accumulate along the whole series of yearly auctions. (2) A minimum selling price is ex-ante defined by Subpesca at each yearly auction. Up to the December 2002 auction, those interested in participating at each yearly auction knew, with at least two-month anticipation, the corresponding minimum selling price. (3) The auctioning process occurs in a single act, the sequential auctioning of the different licenses following an ex-ante defined ordering, and the bidding prices being orally expresed. (4) Each license is granted to the highest bidding price that is above the minimum selling price. (5) The total value of each

[17] Fisheries in *State of Recovery* are those re-opened after a full fishing ban lasting for at least three consecutive years. Fisheries *in Infant Development* are those whose initial landings, at the time of declaring them in that status, are equal or below 10% of the estimated global annual quota.

[18] At *Infant-Development* fisheries that have official landing records for dates previous to the first auction, Subpesca can assign, based on historical records, up to 10% of the *TAC*, and the rest vía public auction. If previous landing records exist, Subpesca then assigns three-year valid ITQs among the firms having those records. Afterwards, the ITQs are renewed for another 10 years, according to the TAC-percentage that those firms caught during the first three years of ITQ operation.

winning bid is paid to Subpesca in ten identical annual payments. (6) The auctioned ITQs can afterward be sold or leased.19

Starting from the December 2003 auction, Subpesca modified the auctioning rules for selling Chilean sea bass fishing rights. Since then, Dutch-type (descending price) oral auctions started to be used and the minimum selling price became unknown to the participants at each auction. The analysis of the auctioning results under these new rules goes beyond the reaches of this paper.

A.1 Proof Proposition 1

Since each subgame is independent of the rest, in each one the payment function of the incumbents is identical. A representative incumbent j, at the start of the auction of the single lot tendered, will have in each subgame the following payments function:

$$w^j = \begin{cases} \Pi^j(Q) - s^h & \text{if } b^j > s^{i\neq j} \text{ or } b^j \geq s^e \\ \Pi^j(Q) - \dfrac{1}{k}\left[s^i\right] & \text{if } b^1 = b^2 = b^3 = \ldots b^j = \ldots = s^i \\ \Pi^j(Q) & \text{if } b^j < s^{i\neq j} \text{ and } s^{i\neq j} \geq s^e \\ \Pi^i(Q + q_l) & \text{if } s^i < s^e \end{cases}$$

where k is the number of incumbents tied for the highest bid s^i. The payment function for the potential entrant will be:

$$w^e = \begin{cases} \Pi^e(Q+q_l) - s^i & \text{if } b^e > s^i \\ 0 & \text{if } b^e \leq s^i \end{cases}$$

Thus, it will not be equilibrium when more than one incumbent spends to deter entry; this is because, for any incumbent, its payments are greater if another deters the entry. On the other hand, there will be equilibrium if any incumbent buys the the single lot auctioned, given that the payments for this incumbent to deter entry (on its own) are greater than or equal to the payment for permitting entry ($\Pi^j(Q) - b^e \geq \Pi^j(Q+q_l)$). This is because the equilibrium strategy of the entrant is to bid exactly its valuation, $\Pi^e(Q+q_l)$, and given that $V^i \geq V^e \Leftrightarrow \Pi^i(Q) - \Pi^i(Q+q_l) \geq \Pi^e(Q+q_l)$ ●

A.2 Proof Proposition 2

Proposition 2 assumes that the entrant's valuation for a lot is greater than the maximum valuation of each incumbent:

$$\Pi^i(Q) - \Pi^i(Q+q_l) < \Pi^e(Q+q_l) \tag{2.1}$$

[19] There is no public register of the transfer prices at these post-auction markets.

Additionally, it assumes an infinite sequence of identical and mutually independent subgames. An optimal strategy for the potential entrant is to offer, in each sub-game, a bid that is greater than the representative incumbent's valuation and less than or equal to its own valuation ($\Pi^i(Q)-\Pi^i(Q+q_l)<b^e\leq\Pi^e(Q+q_l)$). This strategy ensures the entrant pays, as maximum, its own valuation. The best non-cooperative response of the incumbents will be to offer a necessarily lower value than the entrant bid. Thus, for each subgame, the only perfect Nash 'trembling hand' equilibrium (Selten 1975)[20] will be that the incumbents bid exactly their valuation, while the entrant offers one unit above that. In this case, the entrant buys the lot at auction, and the benefit to each incumbent, in each subgame, is $w_i^{NE}=\Pi^i(Q+q_l)$.

Suppose now that a group of 1<*n*≤I incumbents, such that $n\cdot V^i>V^e$, collude to jointly pay the cost of deterring entry, in the infinite series of auctions, by the following mechanism: the *n* colluded incumbents offer exactly the entrant's valuation, therefore tie in each auction and must each pay the *n-th* part of the bidding offer, preventing in this way a new rival's entry. If any of the colluding incumbents deviates at least once from the agreement, they return forever to the non-collusive Nash equilibrium of each subgame, in which each offers its valuation and the potential entrant manages to acquire the lot auctioned in each auction. This threat is credible because, given the deviation, no incumbent has incentives to unilaterally carry out a strategy different to that proposed in the punishment phase, because the punishment strategy is a trembling hand Nash equilibrium in every subgame.

For each incumbent belonging to the collusive agreement, the benefit in each sub-game from staying in the agreement is: $w^{col}=\Pi^i(Q)-\frac{\Pi^e(Q+q_l)}{n}$, while the benefit of the *(I-n)* remaining incumbents (*free riders)* will be $\Pi^i(Q)$. The benefit of an incumbent that deviates from the collusive agreement would be, in the deviation period, equivalent to that of the free riders (denote it by $w^{dev}=\Pi^i(Q)$). After the deviation occurs, the benefit to any future incumbent in each future auction would be $w^{NE}=\Pi^i(Q+q_l)$

If the n incumbents play a trigger strategy as described above, collusion is feasible as long as the expected payments from colluding (RHS of equation 2.2) are greater than or equal to the expected payments from deviating (LHS of equation 2.2):

$$w^{desv}+\delta\cdot w^{NE}+\delta^2\cdot w^{NE}+...\leq w^{col}+\delta\cdot w^{col}+\delta^2\cdot w^{col}+...$$

$$\Rightarrow w^{desv}+\frac{\delta}{1-\delta}w^{NE}\leq w^{col}\cdot\frac{1}{1-\delta}\Rightarrow\delta\geq\frac{w^{desv}-w^{col}}{w^{desv}-w^{NE}} \tag{2.2}$$

where δ= (1/1+r)<1, and r is the discount rate.

$$\Rightarrow\delta\geq\frac{V^e}{n\cdot V^i} \tag{2.3}$$

[20] Perfect Nash equilibrium 'trembling hand' eliminates the use of weakly dominated strategies. Consistently, when analyzing sequential auctions with L>1 lots, a Markovian Equilibrium sophistication is used that also excludes the use of weakly dominated strategies.

A.3 Proof Proposition 3

This proof consists of four parts, A to D. Given the assumption of an infinite sequence of identical and mutually independent subgames, here we shall focus on analysing a single sub-game (with L=1).

(A) Definition of V^e and V^i

The valuation of a representative incumbent, if the potential entrant is to acquire the lot auctioned, will be the loss of profit in the Cournot stage caused by the entrant's purchase, i.e.:

$$V^i = \Pi^i(Q) - \Pi^i(Q + q_l)$$

Solving the Cournot game with I (unconstrained) identical firms and no entrant at the final stage, the profit of each incumbent will be:

$$\Pi^i(Q) = \left(\frac{a-c}{I+1}\right)^2$$

The profit of each incumbent in the final stage, when the entrant acquires the lot auctioned and enters as a constrained agent, will be:

$$\Pi^i(Q + q_l) = \left(\frac{a-c-q_l}{I+1}\right)^2$$

Substituting in the valuation of the representative incumbent, we obtain:

$$V^i = \frac{2(a-c)\cdot q_l - q_l^2}{(1+I)^2} \tag{3.1}$$

The entrant's valuation (which equals the profits earned in the final stage from marketing the tons purchased in the auction), will be:

$$V^e = \frac{(a-c)\cdot q_l - q_l^2}{(1+I)} - F \tag{3.2}$$

(B) $V^e > V^i \Rightarrow I > 1$

Let check, for *I=1*, whether there is any value q_l such that $V^e > V^i$. Replacing (3.1) and (3.2) gives $V^e > V^i$ if:

$$\frac{(a-c)q_l - q_l^2}{2} - F > \frac{2(a-c)q_l - q_l^2}{4}$$

Rearranging, we obtain $q_l^2 < -4\cdot F$, which is a contradiction, so $V^e > V^i \Rightarrow (i) \quad I > 1$ •

(C) If $I > 1$ and $V^e > V^i \Rightarrow \dfrac{F}{(a-c)^2} < \dfrac{I^2 - 2}{[(1+I)\cdot(2+I)]^2}$

Using (3.1) and (3.2), $V^e > V^i$ if:

$$F < q_l \frac{[(a-c)(I-1) - I \cdot q_l]}{(1+I)^2} \tag{3.3}$$

As $\left(\frac{(I^2-2)(a-c)^2}{[(1+I)\cdot(2+I)]^2}\right) \geq \left(q_l \cdot \frac{[(a-c)(I-1) - I \cdot q_l]}{(1+I)^2}\right)$ holds for any $q_l \leq \frac{a-c}{2+I}$ (range of q_l values in which the entrant is always a constrained agent in the Cournot stage), then:

If $I > 1$ and $V^e > V^i \Rightarrow$ $\quad (ii) \quad \frac{F}{(a-c)^2} < \frac{I^2-2}{[(1+I)\cdot(2+I)]^2}$ •

(D) We have to prove that satisfying conditions (i) and (ii) in Proposition 3 => if $q_l > q_l^*$, where

$$q_l{*} = \frac{(a-c)(I-1) - \sqrt{(a-c)^2(I-1)^2 - 4 \cdot I \cdot F(I+1)}}{2I}$$

Then, necessarily, $V^e > V^i$.

The value of this critical amount q_l^*, as from which the valuation of the potential entrant is greater than the loss it causes to every incumbent, it is deduced from $V^e = V^i$. A value $q_l{*}$ is sought such that the potential entrant, should it enter the industry, it does so as a constrained agent ($q_l \leq \frac{a-c}{2+I}$**).** Replacing then the expressions for V^i and V^e:

$$\frac{2(a-c)\cdot q_l - q_l^2}{(1+I)^2} = \frac{(a-c)\cdot q_l - q_l^2}{(1+I)} - F$$
$$\Rightarrow q_l{*} = \frac{(a-c)(I-1) - \sqrt{(a-c)^2(I-1)^2 - 4 \cdot I \cdot F(I+1)}}{2I}$$

Note that $q_l{*} = 0$ when F=0, and that $q_l{*}$ increases with increasing F>0. Therefore, for the entrant to be able to purchase a lot, the three conditions outlined in Proposition 3must be jointly met in •

A.4 Proof Proposition 4

Proposition 4 supposes that (ii) $V^i \geq V^e$ with L=1, which implies the following condition:

$$V^i \geq V^e \Rightarrow \frac{2(a-c)\cdot q_l - q_l^2}{(1+I)^2} \geq \frac{(a-c-q_l)\cdot q_l}{1+I} - F \Rightarrow F \geq q_l \frac{[(a-c)(I-1) - I \cdot q_l]}{(1+I)^2}$$

Likewise, with L>1, an incumbent cannot deter more than 2 times, i.e.:

$$2 \cdot V_l^e(h_l = 0) > V_l^i(h_l = 0) \Rightarrow 2 \cdot \left(\frac{a - c - q_l}{I + 1} \right) \cdot q_l - 2 \cdot F > \left(\frac{2 \cdot (a - c) - q_l}{(I + 1)^2} \right) \cdot (q_l)$$

$$\Rightarrow F < \frac{q_l}{(I + 1)^2} \cdot \left(I \cdot (a - c - q_l) - q_l / 2 \right) \qquad \text{(iii)}$$

Thus, conditions (ii) and (iii) mean that Proposition 4 is valid for a range of F values. This range is not empty since $\left((a - c) \cdot (I - 1) - I \cdot q_l\right) < \left(I \cdot (a - c - q_l) - q_l / 2\right)$, for all $q_l < 2 \cdot (a - c)$, which is satisfied since one of our basic assumptions is that the total amount auctioned ($L \cdot q_l$) is less than the optimal Cournot quantity for the representative incumbent, if entry occurs, i.e.:

$$q_l < \left(\frac{a - c}{I + 2} \right) \cdot \frac{1}{L} \qquad (4.1)$$

For purposes of this demonstration, the I≥2 incumbents are ordered according to the (sequential) number of the lot that each intends to acquire: i=1 intends to acquire lot 1 and so on, up to lot L, which i=L intends to acquire. If there are more incumbents than the number of lots, there will be incumbents who will benefit through *free riding* of the costs borne by the other incumbents to deter entry.

Let I≥L: Solving by backward induction, starts by analyzing the set of possible histories when auctioning the last lot L, i.e. $H_L = (0,1,2,...,L-1)$. Thus, if $h_L = L - 1$, i.e. if the entrant has acquired the previous L-1 lots, the entrant's valuation for lot L will be:

$$V_L^e(h_L = L - 1) = \Pi^e(Q + L \cdot q_l) - \Pi^e(Q + (L - 1) \cdot q_l) \Rightarrow V_L^e(h_L = L - 1) = \frac{(a - c) \cdot q_l - q_l^2 \cdot (2 \cdot L - 1)}{1 + I} \qquad (4.2)$$

While the valuation of incumbent i=L for lot L will be:

$$V_L^{i=L}(h_L = L - 1) = \frac{q_l}{(1 + I)^2} \cdot (2(a - c) - (2L - 1) q_l) \qquad (4.3)$$

So that the entrant does not acquire lot L, it must be that:

$$\frac{q_l}{(1 + I)^2} \cdot (2(a - c) - (2 \cdot L - 1) \cdot q_l) \geq \frac{(a - c) \cdot q_l - q_l^2 \cdot (2 \cdot L - 1)}{1 + I}$$

$$\Rightarrow q_l \leq \left(\frac{I - 1}{I} \right) \cdot \left(\frac{a - c}{2 \cdot L - 1} \right) \qquad (4.4)$$

On the other hand, given condition (4.1) and that it can also be shown that $\left(\frac{a-c}{I+2}\right)\cdot\frac{1}{L} < \left(\frac{I-1}{I}\right)\cdot\left(\frac{a-c}{2\cdot L-1}\right)$ is satisfied ∀I≥2, then it will necessarily be that i=L purchases the L-th lot if the entrant has already acquired the others, paying for this lot $V_L^e(h_L = L-1)$.

If the current history when lot L is auctioned is $h_L = L-2$, the condition for the valuation of the incumbent L to be greater than or equal to the valuation of the entrant (for lot L) will be:

$$q_l \leq \left(\frac{I-1}{I}\right)\left(\frac{a-c}{2L-3}\right) \tag{4.5}$$

Note that condition (4.5) is less demanding than condition (4.4), i.e.:

$$\left(\frac{I-1}{I}\right)\cdot\left(\frac{a-c}{2\cdot L-1}\right) < \left(\frac{I-1}{I}\right)\cdot\left(\frac{a-c}{2\cdot L-3}\right)$$

So, the fewer lots the entrant has accumulated at the time of auction L, the more likely it will be that the incumbent firm may acquire the last lot to be awarded. In effect, the necessary condition for incumbent L to be awarded the L-th lot on sale will be less restrictive to the extent that the entrant has fewer lots accumulated. This implies that the L-th incumbent will always acquire lot L. For incumbent L-1 and all those preceding, the same reasoning applies. Thus, by backward induction it follows that the only possible history is $h_{L+1} = 0$. Therefore, in all stages of auction a different incumbent deters entry of the potential new rival, in each case paying $V_1^e = \Pi^e(Q+q_l)$. Additionally, given condition (iii), each incumbent cannot deter the entry of the new rival more than once. Thus the only possible equilibrium will be one in which every incumbent necessary, to totally prevent the entry of the new rival, will pay only once the cost of acquiring a lot in each auction subgame with L lots.•

A.5 Proof Corollary 4.1

Condition (ii) implies:

$$\underline{n}V^i = V^e \Rightarrow F = Q_l\,\frac{\left[(a-c)(I+1-2\cdot\underline{n}) - Q_l\cdot(I+1-\underline{n})\right]}{(1+I)^2} \geq 0$$

As in the proof of Proposition 4, the I>1 incumbents are ordered according to the (sequential) number of the lot that each intends to acquire. Solving by backward induction, begins by analyzing the case in which incumbent i=L observes, at the auction of the *L-th* lot, $h_L = 0$. In this case, the entrant's valuation of lot L will be:

$$V_L^e(h_L = 0) = \frac{(a-c)\cdot q_l - q_l^{\,2}}{1+I} - F \tag{5.1}$$

While the valuation of incumbent i=L for lot L will be:

$$V_L^{i=L}(h_L=0)=\frac{q_l}{(1+I)^2}\cdot(2(a-c)-q_l) \qquad (5.2)$$

So that the entrant does not acquire lot L, it must be that (5.2) is greater than or equal to (5.1):

$$\Rightarrow F \geq q_l\,\frac{[(a-c)(I-1)-I\cdot q_l]}{(1+I)^2} \qquad (5.3)$$

Given the assumption L=I>1, then $q_l = Q_l/I$. In this case, condition (5.3) will be:

$$F \geq \frac{Q_l}{I}\cdot\frac{[(a-c)(I-1)-Q_l]}{(1+I)^2} \qquad (5.4)$$

Then, combining (5.4) with condition (ii) of the Corollary, we get:

$$Q_l\,\frac{[(a-c)(I+1-2\cdot\underline{n})-Q_l\cdot(I+1-\underline{n})]}{(1+I)^2}\geq\frac{Q_l}{I}\,\frac{[(a-c)(I-1)-Q_l]}{(1+I)^2}$$

$$\Rightarrow Q_l \leq (a-c)\cdot\left(\frac{I\cdot(I-2\cdot\underline{n})+1}{I\cdot(I+1-\underline{n})-1}\right)>0$$

Therefore, condition (iii) of the Corollary is enough for the incumbent i=L to acquire the lot number L. The incumbent i=L-1, looking at history h_{L-1}=0, knows that if the incumbent i=L purchases the lot L, then i=L-1 will also acquire the lot number L-1. Therefore, given h_{L-1}=0, condition (iii) is also enough for the incumbent i=L-1 to acquire its corresponding lot at auction. The same reasoning applies to all the incumbents, and thus there is at least one MRUE-equilibrium in which each incumbent acquires a lot and the entrant does not enter. •

A.6 Proof Lemma 4.2

When auctioning L>1 lots, the *TCDE* will be equivalent to L times the entrant's valuation for the first lot that it is possible to purchase:

$$L\cdot\Pi^e(q_l)=L\cdot\left(\frac{a-c-(Q_l/L)}{I+1}\right)\cdot\frac{Q_l}{L}-L\cdot F \qquad (6.1)$$

This demonstration is divided into two parts (A and B). Part (A) shows that the *TCDE* will be greater (less or equal) when L>1, in relation to the case when L=1, provided that condition (i) of the Lemma is met. Part (B) calculates the slope of the TCD with respect to L and assesses in which parts of the function it is increasing, decreasing or constant (condition ii).

(A) $L \cdot \Pi^e(q_l) >(\leq) \Pi^e(Q_l) \Rightarrow (i)\ L <(\geq) \frac{Q_l^2}{F \cdot (I+1)}$

In effect:

$$\left(\frac{a-c-Q_l/L}{I+1}\right) \cdot Q_l - L \cdot F >(\leq) \left(\frac{a-c-Q_l}{I+1}\right) \cdot Q_l - F \Rightarrow L <(\geq) \frac{Q_l^2}{(I+1) \cdot F}$$

(B) $\frac{\partial(L \cdot \Pi^e(q_l))}{\partial L} >(\leq) 0$, depending on the following condition: $(ii) L <(\geq) \frac{Q_l}{\sqrt{F \cdot (I+1)}}$

The partial derivative with respect to L of the expression (6.1) will be:

$$\frac{\partial L \cdot \Pi^e(q_l)}{\partial L} = \frac{L}{I+1} \cdot \left(2 \cdot \frac{Q_l^2}{L^3} - \frac{Q_l}{L^2} \cdot (a-c)\right) + \left(\frac{a-c-Q_l/L}{I+1}\right) \cdot \frac{Q_l}{L} - F \tag{6.2}$$

Expression (6.2) will be positive (negative or zero) whenever the following condition is met:

$(ii) L <(\geq) \frac{Q_l}{\sqrt{F \cdot (I+1)}}$ •

A.7 Proof Proposition 5

Condition (iii) of Proposition 5 ensures that, in each subgame, the MRUE-equilibrium concept defines strategies that allow the entrant to buy all the lots auctioned. This is because this condition is exactly the opposite of Condition (iii) of Corollary 4.1, which ensured that the incumbent i=L, given $h_L=0$, acquires the last lot auctioned. So condition (iii) of Proposition 5 ensures that the last incumbent is not interested in buying the *L-th* lot auctioned, even if it is the only one that the entrant would be able to acquire.

On the other hand, Proposition 4 showed that the fewer lots the entrant has accrued, the greater the incentives for any incumbent to deter entry. Thus, if incumbent i=L cannot deter entry when $h_L=0$, it will not deter entry for any possible history $h_L>0$.

Now, the non-collusive (Cournot) equilibrium described in Proposition 5 gives incumbents, in each subgame, the following benefit:

$$w^{NE} = \Pi^i(Q + L \cdot q_l) \tag{7.1}$$

In contrast, the benefit of a sustainable collusive agreement as the one defined in Proposition 5 is, in each sub-game, the profit gained in the Cournot stage if no entry occurs, less the TCDE divided by the number of incumbents colluding:

$$w^{col} = \Pi^i(Q) - \frac{L \cdot \Pi^e(Q+q_l)}{n} \tag{7.2}$$

For the incumbents to be interested in making a collusive agreement, it must be that $w^{col} > w^{NE}$, which implies that $n > (L \cdot \Pi^e(Q+q_l)/V^i)$. Finally, the optimal deviation for an incumbent who belongs to the collusive agreement is not to pay its share of the TCDE and to take advantage during the period of deviation of the benefit of being *free riders* ($w^{desv} = \Pi^i(Q)$). The penalty that the members of the agreement apply in case of deviation is to return permanently to the non-collusive (Cournot) equilibrium (thus allowing full entry). Then, for collusion to be possible, it must be that:

$$w^{desv} + \frac{\delta}{1-\delta} w^{NE} \leq w^{col} \cdot \frac{1}{1-\delta} \Rightarrow \delta \geq \frac{w^{desv} - w^{col}}{w^{desv} - w^{NE}} \tag{7.3}$$

Replacing in (7.3):

$$\delta \geq \frac{L \cdot \Pi^e(Q+q_l)}{n \cdot \left(\Pi^i(Q) - \Pi^i(Q + L \cdot q_l)\right)} = \frac{L \cdot \Pi^e(Q+q_l)}{n \cdot V^i} \tag{7.4}$$

Then, the minimum discount factor that makes collusion feasible will be: $\underline{\delta} = \dfrac{L \cdot \Pi^e(Q+q_l)}{n \cdot V^i}$ •