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Self-Fulfilling Crises and Central Bank Communication

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Self-Fulfilling Crises and Central Bank Communication*

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Abstract

This paper studies how much information a Central Bank should release to less informed private agents. Agents have dispersed information about the state of the economy, and their actions are strategic complements. Thus, the Central Bank's public disclosure of information can generate an undesirable coordination among agents and self-fulfilling crises. We show that the Central Bank will choose an information structure that sends only two messages. We characterize the optimal information structure and prove that it retains the uniqueness equilibrium property of global games. We also show that, without the ability to commit to an information disclosure rule, the Central Bank could be worse off by releasing public information.

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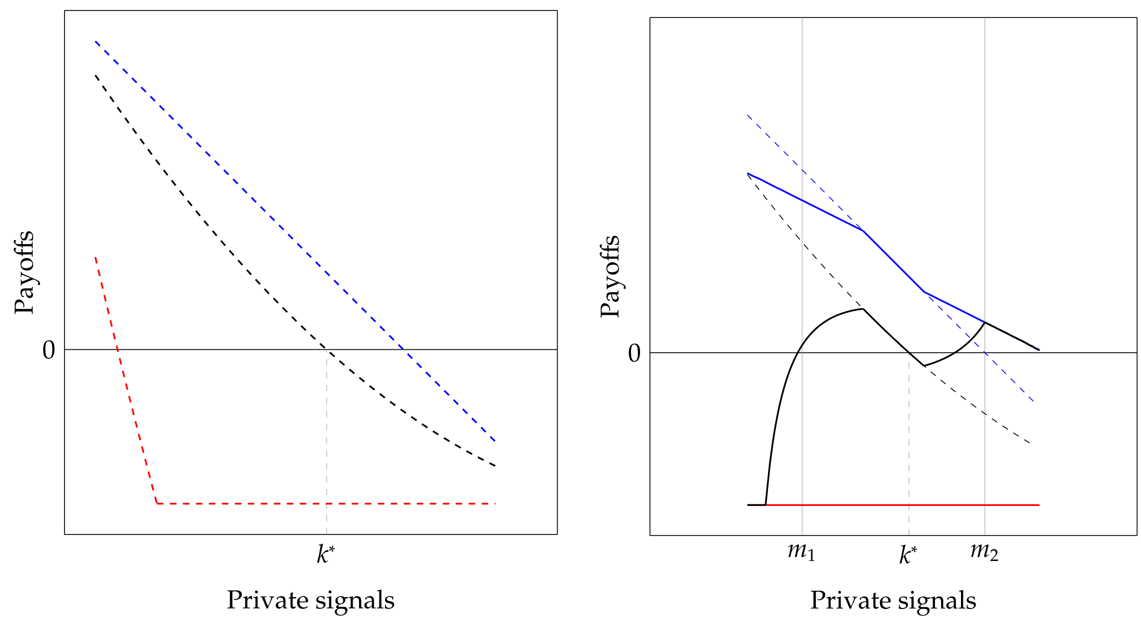
1 Introduction

The effectiveness of an economic policy largely depends on the behavior of private agents. Public disclosure of information about a payoff relevant state can be used to induce them to take actions aligned with the policy. When there is strategic complementarity in the agents' actions, public information has the additional feature of allowing them to coordinate their actions, not necessarily in favor of the policy's success. In this paper, we study how a Central Bank should publicly disclose information in such coordination environments.

We address this question by introducing public communication in the model of speculative currency attacks of Morris and Shin (1998). The exchange rate is initially pegged and the Central Bank sends a credible public signal about the fundamentals of the economy before speculators move. A continuum of privately informed speculators decides whether to attack the currency or not. If sufficiently many speculators attack, the Central Bank is forced to abandon the peg.

The speculators' payoff of a successful attack depends on the unknown state of fundamentals. There are two dominance regions in the space of fundamentals. If fundamentals are weak, the Central Bank abandons the peg regardless of the size of the attack, therefore attacking is a dominant strategy. If fundamentals are strong, a successful attack is not profitable, which makes not attacking the dominant strategy. In between the regions, attacking is only profitable if sufficiently many speculators decide to do so.

The model without public information has a unique equilibrium, in which speculators attack if their private signals are below a cutoff. This result can be explained with the aid of Figure 1 (a). The horizontal line at zero is the payoff to a speculator who refrains from attacking the currency. We depict the expected payoff of attacking as a function of the speculator's private signal, considering different aggregate strategies. In blue, every speculator attacks the currency, regardless of their private signals; in red, no speculator attacks. Neither strategy is possible in equilibrium: for high signals (after the blue curve crosses zero), speculators prefer not to attack even if everyone else attacks; for low signals (before the red blue crosses zero), speculators prefer to attack even if no one else attacks. In black, we depict the expected payoff to a speculator that attacks when the private signal is k



(a) Without a public signal.

(b) With a public signal.

Figure 1: Expected payoff of attacking the currency peg.

and every other speculator follows a strategy with cutoff k. When the black curve crosses zero, we find the unique equilibrium of the model without public signals. In this equilibrium, speculators follow a strategy with cutoff k^*.

In Figure 1 (b), we illustrate how the introduction of a public signal can generate self-fulfilling crises. Suppose that the Central Bank announces that the state lies in the interval (m_1, m_2). The curves in bold depict the expected payoffs conditional on the public information. Since the red curve is always below zero, it is not optimal to attack if no one else is attacking. However, since the blue curve is always above zero, a coordinated attack is now profitable regardless of the private signal. In addition, we have that following a cutoff k^* is still an equilibrium strategy for speculators. Thus, such a public signal leads to multiple equilibria and attacks based on self-fulfilling beliefs that other speculators are also attacking.

In Section 2, we model the public signal structure as a partition of the space of fundamentals. The Central Bank chooses a partition and sends a public signal that reveals the interval where the realized state lies. When a public signal leads to multiple equilibria, as in Figure 1 (b), we assume that the Central Bank only cares about its lowest equilibrium payoff. We also assume that the Central Bank can commit to a disclosure policy before observing the state.

In Section 3, we present our main results. Our first result (Theorem 1) is that, without loss of generality, the Central Bank chooses a two-signal structure. The intuition for this result is as follows. For any partition choice, in the worst equilibrium for the Central Bank, the peg is abandoned if and only if the realized state is below certain threshold. We show that the Central Bank is not worse off by choosing a partition with only two signals, revealing whether the state is below or above that threshold. Such a disclosure rule leads to a unique equilibrium, in which speculators refrain from attacking above the threshold. This is the least costly way for the Central Bank to maintain the peg in that region of fundamentals. A more precise information structure opens up the possibility of multiple equilibria and self-fulfilling crises.

We characterize the optimal partition for the Central Bank in Theorem 2, our second main result. By Theorem 1, we can restrict attention to partitions that divide the space of fundamentals into two regions, low and high. If fundamentals in the high region are strong enough, the currency is not attacked in equilibrium. As the Central Bank adds weaker fundamentals to this region, the possibility of a self-fulfilling crisis eventually arises: not attacking is no longer the unique equilibrium strategy for speculators. Thus, it is optimal for the Central Bank to expand the high region as long as it does not induce attacks on the peg. The optimal partition leads to a unique equilibrium, in which speculators perfectly coordinate their actions based on the public signal, attacking only if the Central Bank announces the low region. Intuitively, with the optimal signal structure the Central Bank exploits the speculators' coordination incentives to prevent attacks as often as possible.

We conclude Section 3 by showing that, if the Central Bank could fully disclose the fundamentals, it would not be optimal to do so. This would lead to a coordinated attack at every state, except when not attacking is a dominant strategy. In contrast, we have that the optimal partition prevents attacks beyond this dominance region.

Section 4 presents a version of the model in which the Central Bank is unable to commit ex-ante (before observing the state) to an information disclosure policy. Without commitment, the Central Bank's ability to change speculators' posteriors about fundamentals is limited by their endogenous beliefs. We show that, in equilibria where speculators have "pessimistic" beliefs, the Central Bank is forced to fully disclose the state.

In Section 5, we present our conclusions. All proofs are presented in the Appendix.

Related Literature This paper is related to the literature on self-fulfilling crises when payoffs are not common knowledge. The idea that *small* deviations from common knowledge can have a large impact on equilibrium outcomes dates back at least to Rubinstein's mail game (Rubinstein (1989)), and has gained great attention since Carlsson and van Damme (1993) and Morris and Shin (1998).

We build on the model of Morris and Shin (1998) to introduce a public signal that generates partial common knowledge. In different settings, the interaction between public and private signals in coordination games has been studied in Morris and Shin (2002), Hellwig (2002), Angeletos and Pavan (2007) and Angeletos et al. (2007).

In particular, our paper contributes to the literature on the role of policy choices in coordination games, as in Angeletos et al. (2006) and Angeletos and Pavan (2009, 2013). Breaking the uniqueness result in Morris and Shin (1998), Angeletos et al. (2006) point out that policy interventions that convey some information about the fundamentals allows for agents to coordinate their actions, which in turn may lead to multiple equilibria. In our model, the Central Bank optimally chooses a disclosure policy that leads to a unique equilibrium, in which agents perfectly coordinate their actions.

This paper also relates to the literature on Bayesian persuasion pioneered by Kamenica and Gentzkow (2011). They study the optimal signal structure from the perspective of a sender who wants to influence a rational Bayesian receiver to take the sender's preferred action. We address this question of information design in the context of a coordination model where the sender faces a continuum of privately informed receivers. In a similar setting, Goldstein and Huang (2016) characterize the optimal policy for a sender who is restricted to announcing a threshold state of fundamentals below which the status quo is abandoned. In our model, this is equivalent to restricting the Central Bank to sending only two signals.

2 Model

2.1 Actions and payoffs

We extend the benchmark model of currency attacks of Morris and Shin (1998) by allowing the Central Bank to release a public signal about the fundamentals of the economy. The state of fundamentals is represented by θ, which is uniformly distributed over $[0, 1]$. The exchange rate is initially pegged at e^*, and its value in the absence of intervention is given by $f(\theta)$. We assume that $f(\cdot)$ is continuous and strictly increasing, with $e^* \geq f(\theta)$ for all θ. A continuum of speculators of measure one has to simultaneously to decide whether to attack the currency peg or not.

A speculator attacks the peg by selling short one unit of the currency at a cost $t > 0$. If the speculator attacks and the peg is abandoned, his payoff is $e^* - f(\theta) - t$, whereas the payoff from attacking when the currency is defended is $-t$. If the speculator does not attack, then his payoff is zero.

The Central Bank derives a value $v > 0$ from maintaining the currency peg. There is a cost $c(\alpha, \theta)$ to defend the peg, where α is the mass of speculators who attack the currency. The cost $c(\cdot, \cdot)$ is continuous, strictly increasing in α and strictly decreasing in θ. Hence, the payoff from defending the peg is $v - c(\alpha, \theta)$, and the payoff from abandoning the peg is zero. The following assumptions are made:

- $c(0, 0) > v$: the Central Bank abandons the peg if fundamentals are sufficiently weak, even if no speculator attacks;
- $c(1, 1) > v$: the Central Bank abandons the peg if every speculator attacks, even if fundamentals are strong;
- $e^* - f(1) - t < 0$: it is not profitable for speculators to attack the currency if fundamentals are strong.

Denote by $\underline{\theta}$ the value of θ that solves $v = c(0, \theta)$. If $\theta \leq \underline{\theta}$, the Central Bank finds it optimal to abandon the peg regardless of the size of the attack. Denote by $\bar{\theta}$ the value of θ such that $e^* - f(\theta) - t = 0$. If $\theta > \bar{\theta}$, attacking is not profitable even if the peg is abandoned.

We assume that $\underline{\theta} < \bar{\theta}$.[1] When the state is common knowledge, we can divide Θ in three intervals, as it has been pointed out in the literature:[2]

- if $\theta \in [0, \underline{\theta}]$, the Central Bank always abandons the peg and attacking is a dominant strategy;
- if $\theta \in (\underline{\theta}, \bar{\theta}]$, attacking is profitable only when many speculators attack;
- if $\theta \in (\bar{\theta}, 1]$, it is never profitable to attack the peg.

2.2 Timing and information

The game has three stages. In the first stage, before observing θ, the Central Bank commits to a disclosure policy, which is announced to the speculators. In the second stage, once θ is realized, a public signal y is sent according to the chosen disclosure policy.[3] Speculators do not observe θ, just the public signal y and a private signal x. Given x and y, speculators simultaneously decide whether to attack the currency or not. In the last stage, the Central Bank observes θ and the size of the attack, and decides whether to defend the currency or to abandon the peg. The structure of the game is assumed to be common knowledge.

The Central Bank can partition the space of fundamentals and announce in which interval the realization of θ lies. We denote a partition of $[0, 1]$ by $P = \{m_n\}_{n=0}^{N}$, where $0 = m_0 < ... < m_n < ... < m_N = 1$, and $N \in \mathbb{N}$.[4] The n-th interval of partition P is denoted by y_n, with

$$y_1 = [0, m_1], y_2 = (m_1, m_2], ..., y_n = (m_{n-1}, m_n], ..., y_N = (m_{N-1}, 1],$$

for $N > 1$. When the public signal $y = y_n$ is sent, it becomes common knowledge that $\theta \in y_n$. When $N = 1$, we let $y_N = [0, 1]$, which means that the public signal is uninformative.

[1] This condition holds for a large v and a small t.

[2] See, for example, Obstfeld (1996).

[3] In line with the Bayesian persuasion literature, an interpretation of the disclosure policy is that an independent and credible Central Bank commits to an information acquisition procedure and to publicly releasing its findings.

[4] In this exposition, we restrict the analysis to partitions with a finite number of intervals. Our results still hold if partitions can have a countable number of intervals.

Given the assumption that the Central Bank commits to a choice of P before learning the true state θ, there is no *strategic learning*, i.e., the choice of P does not change the speculators' beliefs about what the Central Bank knows.[5] In Section 4, we show that commitment is essential for our results.

In addition to the public signal, speculator i observes a private signal x_i, where

$$x_i = \theta + \varepsilon_i.$$

The idiosyncratic noise ε_i is drawn from a distribution with probability density function $g(\cdot)$, and cumulative distribution function $G(\cdot)$. Each ε_i is independent and identically distributed across agents, and independent of θ. We assume that $\text{supp}(\varepsilon_i) = [-\varepsilon, \varepsilon]$, $\varepsilon > 0$, and, as in Morris and Shin (1998), that $2\varepsilon < \min\{\underline{\theta}, 1 - \bar{\theta}\}$.

Since the common prior is uniform, the posterior distribution of θ given a private signal x and a public signal y has probability density function $\phi_y(\theta|x)$, where[6]

$$\phi_{y_n}(\theta|x) = \begin{cases} \frac{g(x-\theta)}{G(x-m_{n-1})-G(x-m_n)}, & \text{if } \theta \in y_n \\ 0, & \text{otherwise} \end{cases}. \tag{1}$$

2.3 Equilibrium

We solve this game by backward induction. In the last stage, given an attack of size α and a state θ, the Central Bank optimally chooses to abandon the peg if and only if $c(\alpha, \theta) \geq v$. In the second stage, speculators observe the public signal and their own private signals. Anticipating the Central Bank's behavior in the last stage, they simultaneously decide whether to attack the currency or not. In the first stage, the Central Bank chooses a partition P. The multiplicity in the second stage of the game poses a selection problem that we solve by assuming that the Central Bank only cares about its lowest equilibrium payoff. An equivalent assumption is that that speculators play the equilibrium strategy that maximizes their own payoff

[5] This is in contrast to Angeletos et al. (2006).

[6] There is a finite number of pairs (x, y) that fully reveal θ: when $y = y_n$ and $x = m_n + \varepsilon$, we have $\mathbb{P}(\theta = m_n | y = y_n, x = m_n + \varepsilon) = 1$; likewise, when $y = y_n$ and $x = m_{n-1} - \varepsilon$, then $\mathbb{P}(\theta = m_{n-1} | y = y_n, x = m_{n-1} - \varepsilon) = 1$. For all other pairs (x, y), the conditional density of θ is given by (1).

(This equivalence follows from Lemmas 1 and 5 below.).

More formally, suppose that the Central Bank chooses a partition $P = \{m_n\}_{n=0}^N$. Let $p_n = \mathbb{P}(\theta \in y_n)$ be the probability that θ lies in the interval y_n of the partition.[7] In addition, consider the subgame that follows the disclosure of $y = y_n$. Denote by V_n the infimum of the Central Bank's equilibrium payoffs when $y = y_n$.[8] We let $V(P) = \sum_{n=1}^N p_n V_n$. The Central Bank's problem is to choose P in order to maximize $V(P)$.

The last stage of the game is straightforward. The Central Bank abandons the peg at θ if the size of the attack is above a threshold $a(\theta)$. For $\theta \leq \underline{\theta}$, $a(\theta) = 0$, and for $\theta > \underline{\theta}$, $a(\theta)$ is the solution to $v = c(a, \theta)$. Note that, given our assumptions on $c(\cdot, \cdot)$, we have that $a(\cdot)$ is continuous in θ and strictly increasing if $\theta > \underline{\theta}$.

We restrict attention to symmetric equilibria. A strategy for a speculator is a function π such that, for each pair of signals (x, y), $\pi(x, y)$ determines the probability of attacking the currency. When all speculators follow $\pi(\cdot, \cdot)$, the size of the attack at θ is given by

$$s(\theta, \pi) = \int_{\theta-\varepsilon}^{\theta+\varepsilon} \pi(x, y(\theta)) g\,(x - \theta)\, dx.$$

where $y(\theta)$ is the public signal sent according to P. Thus, the event in which the peg is abandoned is given by

$$A(\pi) = \{\theta : s(\theta, \pi) \geq a(\theta)\},$$

and the expected payoff from attacking the currency given a pair of signals (x, y_n) is[9]

$$u_{y_n}(x, \pi) = \int_{[x-\varepsilon, x+\varepsilon] \cap A(\pi)} [e^* - f(\theta)] \phi_{y_n}(\theta | x) d\theta - t. \tag{2}$$

In equilibrium, $\pi(x, y) = 1$ if $u_y(x, \pi) > 0$, and $\pi(x, y) = 0$ if $u_y(x, \pi) < 0$.

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[7] Since we assume that θ is uniformly distributed over $[0, 1]$, we have $p_n = m_n - m_{n-1}$.

[8] Such infimum always exists as the Central Bank always has the option to abandon the peg, so the equilibrium payoff is bounded below by 0.

[9] Equation (2) holds for all but a finite number of pairs (x, y), as described in footnote 6. For the sake of brevity, in the remainder of the paper we omit these finite number of cases.

2.4 Equilibrium properties

We now present some auxiliary results. The first result shows that, if speculators are more likely to attack the currency for every private signal x, then the payoff from attacking increases.

Lemma 1. *For a given public signal y, if $\pi(x,y) \geq \pi'(x,y)$ for all x, then $u_y(x,\pi) \geq u_y(x,\pi')$ for all x.*

Proof: See Appendix A.1. □

For $k \in [-\varepsilon, 1+\varepsilon]$, let the indicator function I_k be defined as

$$I_k(x) = \begin{cases} 1, & \text{if } x < k \\ 0, & \text{if } x \geq k \end{cases}.$$

When aggregate short sales are given by I_k (in particular, they do not depend on the public signal y), the proportion of speculators who attack the currency at state θ is given by

$$s(\theta, I_k) = G\left(k - \theta\right). \tag{3}$$

Note that $s(\theta, I_k)$ is strictly decreasing in θ for $\theta \in (k-\varepsilon, k+\varepsilon)$, and constant otherwise.

We denote by θ_k the largest value of θ at which the Central Bank finds it optimal to abandon the peg when short sales are given by I_k. As in Morris and Shin (1998), let $\psi(k) = \min\{\theta_k - k, \varepsilon\}$. Appendix A.2 provides a derivation of θ_k and $\psi(\cdot)$. Threshold θ_k is increasing in k, and the Central Bank finds it optimal to abandon the peg for all $\theta \leq \theta_k$. Function $\psi(\cdot)$ is continuous, $\psi(k) = \varepsilon$ for $k \leq \underline{\theta} - \varepsilon$, $\psi(1+\varepsilon) = -\varepsilon$, and $\psi(\cdot)$ is strictly decreasing for $k \in (\underline{\theta} - \varepsilon, 1+\varepsilon)$.

Let X_y denote the set of possible private signals when the public signal is y, i.e., $X_{y_1} = [-\varepsilon, m_1 + \varepsilon]$ and, for $n > 1$, $X_{y_n} = (m_{n-1} - \varepsilon, m_n + \varepsilon]$. Since the currency peg is abandoned if and only if $\theta \in [0, k + \psi(k)]$, the payoff function $u_{y_n}(k, I_k)$ is given by

$$u_{y_n}(k, I_k) = \int_{k-\varepsilon}^{k+\psi(k)} [e^* - f(\theta)]\phi_{y_n}(\theta|k)d\theta - t, \tag{4}$$

for all $k \in X_{y_n}$.

Lemma 2. *The payoff function $u_{y_n}(k, I_k)$ is continuous in k, for all $k \in X_{y_n}$.*

Proof: See Appendix A.3. □

Let $u(k, I_k)$ be the payoff function when there is no public signal. Then

$$u(k, I_k) = \int_{\max\{k-\varepsilon,0\}}^{k+\psi(k)} [e^* - f(\theta)] \frac{g(x-\theta)}{G(x) - G(x-1)} d\theta - t, \tag{5}$$

Note that the payoff function is continuous in k. The following lemma shows that it is also strictly decreasing in k.

Lemma 3. *For $k \in (\varepsilon, 1-\varepsilon)$, the payoff function $u(k, I_k)$ is strictly decreasing in k.*

Proof: See Appendix A.4. □

3 Optimal signal structure

This section presents the results of the model with commitment when, in case of multiplicity after a partition choice, the Central Bank only cares about its lowest equilibrium payoff.[10] First, we show that there is no loss of generality in considering partitions with at most two intervals (subsection 3.1). We then characterize the optimal partition for the Central Bank (subsection 3.2) and comment on our results (subsection 3.3).

3.1 No loss of generality in two-interval partitions

Cutoff strategies will play an important role in our results. In order to characterize the speculators' payoffs when a cutoff strategy is used, we make the following assumption:

Assumption 1. *Let the public signal be y. For any pair of private signals x_1 and x_2, with $x_1 < x_2$, $\Phi_y(\theta|x_2) \leq \Phi_y(\theta|x_1)$ for all θ, where $\Phi_y(\theta|x)$ is the cumulative distribution function of θ conditional on signals x and y.*

[10] When there is no ambiguity, we say equilibrium when we mean the equilibrium of the subgame that follows the choice of P.

This assumption means that the distribution of θ conditional on y and x_2 first-order stochastically dominates the distribution of θ conditional on y and x_1. In the online Appendix, we show that Assumption 1 is satisfied, for example, if the idiosyncratic noise on $[-\varepsilon, \varepsilon]$ follows a concave or a truncated normal distribution. Assumption 1 leads to the following lemma.

Lemma 4. *Suppose that Assumption 1 is satisfied. When the aggregate strategy is given by I_k, the payoff from attacking the currency, $u_y(x, I_k)$, is decreasing in the private signal x.*

Proof: See Appendix A.5. □

A consequence of Lemma 4 is that if $u_y(k, I_k) = 0$, then following I_k is an equilibrium strategy for speculators that observe the public signal y. As we the next lemma shows, characterizing the Central Bank's payoff will be closely related to the existence of cutoff strategies for each realization of the public signal.

Lemma 5. *For a given public signal y,*

i. if $u_y(k, I_k) < 0$ for all $k \in X_y$, then, in any equilibrium, $\pi(x, y) = 0$ for all $x \in X_y$.

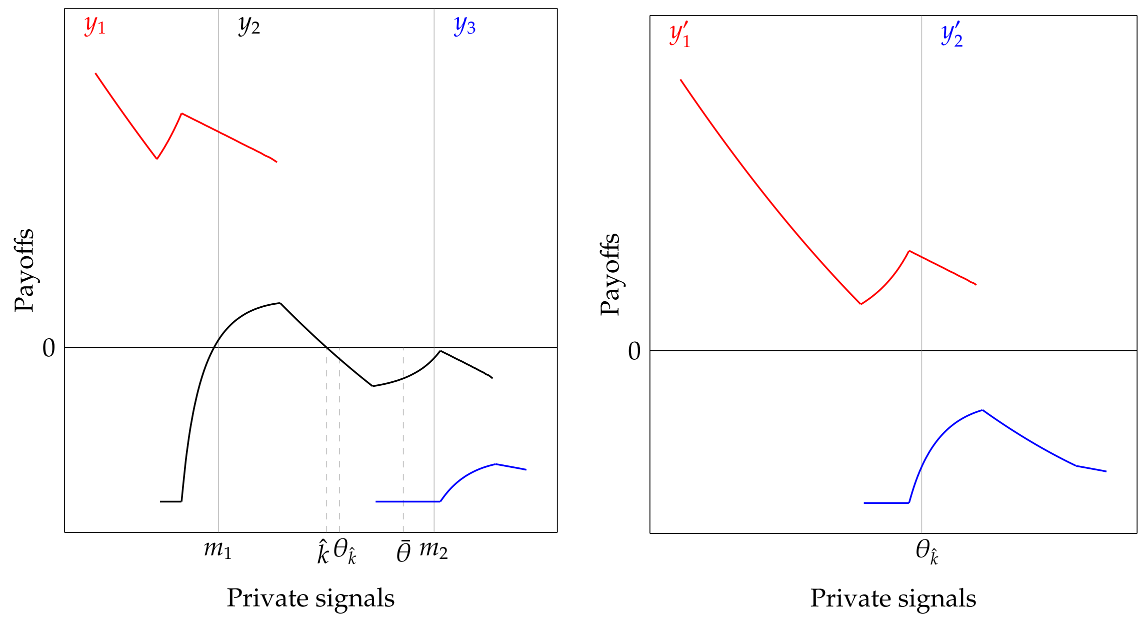
ii. if $u_y(k', I_{k'}) \geq 0$ for some $k' \in X_y$, then, in the worst equilibrium for the Central Bank, speculators use the cutoff rule I_k after observing y, where $k = \sup\{k' \in X_y : u_y(k', I_{k'}) \geq 0\}$.

Proof: See Appendix A.6. □

Recall that $\bar{\theta}$ defines the threshold θ above which investors' payoff from a successful attack is negative. We characterize the equilibrium strategy that minimizes the Central Bank's payoff in the proposition below, which follows directly from Lemma 5.

Proposition 1 (Strategies in the worst equilibrium for the Central Bank). *Consider a partition $P = \{m_n\}_{n=0}^{N}$. The equilibrium strategy that minimizes the Central Bank's payoff is as follows: for all n such that $m_n \leq \bar{\theta}$, speculators always attack the currency if $y = y_n$; likewise, for all n such that $m_{n-1} \geq \bar{\theta}$, speculators never attack the currency if $y = y_n$. Lastly, if n is such that $m_{n-1} < \bar{\theta} < m_n$, speculators never attack if $u_{y_n}(k, I_k) < 0$ for all $k \in X_{y_n}$; otherwise, speculators follow I_{k_n} after observing y_n, where $k_n = \sup\{k' \in X_{y_n} : u_{y_n}(k', I_{k'}) \geq 0\}$.*

Proposition 1 provides the intuition as to why there is no loss of generality in considering only two-interval partitions. If there are several n such that $m_n \leq \bar{\theta}$, then the Central Bank can group all these y_n into a single interval without changing its payoff. Likewise, if there are several n such that $m_{n-1} \geq \bar{\theta}$, the Central Bank can group these y_n together. This implies that we can restrict attention to partitions with at most three intervals.



(a) $P = \{0, m_1, m_2, 1\}$.

(b) $P' = \{0, \theta_{\hat{k}}, 1\}$.

Figure 2: Two-signal structure without loss of generality. The curves depict the payoff function $u_y(k, I_k)$.

Consider a partition P with three intervals, that is, $P = \{0, m_1, m_2, 1\}$, where $m_1 < \bar{\theta} < m_2$. It follows from Proposition 1 that there is a cutoff $\hat{k} = \sup\{k' \in X_{(m_1,m_2]} : u_{(m_1,m_2]}(k', I_{k'}) \geq 0\}$ such that, in the worst equilibrium for the government, speculators attack the currency after observing $y_2 = (m_1, m_2]$ and $x \leq \hat{k}$. This leads to a threshold $\theta_{\hat{k}} \in (m_1, m_2]$ such that the Central Bank abandons the peg if and only if $\theta \leq \theta_{\hat{k}}$. Cutoff $\hat{k}$ and threshold $\theta_{\hat{k}}$ are depicted in Figure 2(a). The curves represent the payoff functions $u_y(k, I_k)$, and we can see that $u_{y_2}(k, I_k)$ crosses zero at the cutoff $\hat{k}$. Now consider the alternative partition $P' = \{0, \theta_{\hat{k}}, 1\}$, that leads to public signals $y_1' = [0, \hat{\theta}_k]$ and $y_2' = (\hat{\theta}_k, 1]$. The payoff functions under partition P' are depicted in Figure 2(b). We can prove that $u_{y_1'}(k, I_k) > 0$ for all k, therefore speculators attack the currency after observing y_1', and the peg is abandoned. However, we can also show that $u_{y_2'}(k, I_k) < 0$ for all k, which implies that speculators refrain from attacking after observing y_2'. Thus, the peg is still abandoned if $\theta \leq \theta_{\hat{k}}$, but the currency is defended at the lowest possible cost for $\theta > \theta_{\hat{k}}$. Since it is cheaper for the Central Bank to maintain the peg at P', this partition is preferred to P.This result is formalized in Theorem 1 below .

In order to prove Theorem 1, we use Lemma 6 (in Appendix A.7), which is an application of the law of total expectation. In that lemma, we show that moving an interval of the partition to the right - that is, increasing the lower bound m_{n-1} or the upper bound m_n - will not increase the payoff $u_{(m_{n-1}, m_n]}(k, I_k)$. This implies that $u_{(\theta', 1]}(k, I_k) < 0$ for all $k \in X_{(\theta', 1]}$. By Proposition 1, there is no attack in $(\theta', 1]$.

Theorem 1 **(No loss of generality in two-interval partitions).** *For any partition $P = \{m_n\}_{n=0}^{N}$ with $N > 2$, there exists $P' = \{m_n'\}_{n=0}^{N'}$ with $N' = 2$, such that $V(P') \geq V(P)$.*

Proof: See Appendix A.8.

Theorem 1 allows us to restrict attention to partitions of two intervals. The Central Bank will divide the state space $[0, 1]$ into two regions, $[0, m]$ and $(m, 1]$, and send two public signals. From now on, we denote the signals by $y_L = [0, m]$ and $y_H = (m, 1]$ and refer to them as the *low* and the *high* public signals, respectively.

3.2 Characterization of the optimal signal structure

When $N = 2$, the Central Bank's problem is equivalent to choosing m, such that speculators will learn whether $\theta \leq m$ or $\theta > m$. Given the choice of m, they observe the public signal $y \in \{y_L, y_H\}$, drawn as follows:

$$y = \begin{cases} y_L, & \text{if } \theta \in [0, m] \\ y_H, & \text{if } \theta \in (m, 1] \end{cases} .$$

For the sake of exposition, let us consider the benchmark environment where there is no public signal. As in Morris and Shin (1998), there is a unique equilibrium where speculators use a cutoff strategy I_{k^*}, leading to a threshold θ^* such that the peg is abandoned if and only if $\theta \leq \theta^*$. Now consider the choice of $m = \theta^*$. When speculators observe the high public signal $y_H = (\theta^*, 1]$, there cannot be an equilibrium where a speculator attacks: Lemma 6 implies $u_{y_H}(k, I_k) \leq u(k, I_k) < 0$ for any $k > k^*$, and, by Lemma 5, the peg is maintained for $\theta > \theta^*$. Therefore, there is no attack after speculators observe y_H, the equilibrium is unique,[11] and the currency peg is abandoned if and only if $\theta \leq \theta^*$, as in the equilibrium of the game without a public signal.

A few observations are in order. First, note that no speculator attacks the currency when $\theta > \theta^*$, whereas without the public signal some speculators would still attack the currency for some $\theta > \theta^*$. Thus, the Central Bank is strictly better off with the introduction of the public signal, since it minimizes the cost of maintaining the peg. Moreover, speculators are also strictly better off now that no one attacks when peg is maintained, and that they only attack when the peg is abandoned.

Note that choosing $m > \theta^*$ is strictly dominated by $m = \theta^*$, since a higher m increases the region where the peg is abandoned, or it makes it more costly to defend the peg (or both). To see this, recall that Lemma 6 implies that $u_{y_L}(k^*, I_{k^*}) \geq u(k^*, I_{k^*}) = 0$. Therefore, speculators will use a cutoff strategy in y_L that will lead to a threshold above θ^*. Any improvement over $m = \theta^*$ must be in the direction of reducing m.

Starting from $m = \theta^*$, as m decreases, the Central Bank is strictly better off as long as the equilibrium is still unique: decreasing m will increase the range of fundamentals where the peg is not attacked. However, reducing m also leads to an increase in $u_{y_H}(k, I_k)$, and eventually it will cross 0 from below for some $k \in X_{y_H}$. When this happens, there is an equilibrium with speculators attacking in y_H, making the Central Bank worse off. Thus, the Central Bank wants to reduce m up to the limit where the equilibrium is still unique. This result is formalized in Theorem 2.

Define M as

$$M = \{m : \text{ in any equilibrium, there is no attack if } \theta \in y_H = (m, 1]\}.$$

[11] We already argued that there is no attack in y_H. We show in Lemma 7 in Appendix A.9 that there is a unique equilibrium if $y = y_L$, where the peg is abandoned.

Note that $M \neq \emptyset$ because $\overline{\theta} \in M$. Define $\underline{m}$ as

$$\underline{m} = \inf M.$$

Theorem 2 (Optimal partition). *$V(P) \leq \overline{V}$ for any partition P, where*

$$\overline{V} = \lim_{m \downarrow \underline{m}} V(P^m),$$

and thus the Central Bank can achieve a payoff arbitrarily close to $\overline{V}$.

Proof: See Appendix A.11. □

As argued above, the Central Bank reduces m as long as there are no attacks in $(m, 1]$. It turns out that, for a sufficiently small $m \in M$, the equilibrium is also unique if $y = y_L$, when every speculator attacks the peg. Therefore, speculators coordinate their actions using the public signal, attacking if and only if they observe y_L.

Note that an equilibrium only exists if $\underline{m} \in M$. However, the Central Bank can achieve a payoff arbitrarily close to $\overline{V}$ and, for m close enough to $\underline{m}$, speculators always coordinate on the public signal. Thus, in the next subsection, we abstract from this existence issue and, when referring to the optimal partition, we mean a partition P^m with m close to $\underline{m}$.

3.3 Discussion

In this section we discuss our two main results, Theorem 1 and Theorem 2. In Theorem 1, we show that, despite having access to a broad message space, the Central Bank cannot improve upon a simple two-signal rule. The key to this result is that committing to such a disclosure rule leads to a unique equilibrium, whereas a more informative communication strategy generates multiple equilibria, some of which have worse outcomes for the Central Bank. In Theorem 2, we characterize the optimal two-interval partition and show that the Central Bank sets m as low as possible, up to the limit where not attacking is the unique equilibrium action for speculators when $y = y_H$.

The first observation we make is that the optimal signal can be interpreted as a recommendation from the Central Bank to speculators about which action they should take. Indeed, y_L could be interpreted as an "attack" recommendation, whereas y_H means "do not attack". Naturally, in equilibrium, those recommendations are followed by speculators. Interestingly, speculators ignore their own private signal.

In order to improve beliefs about the fundamentals when $y = y_H$, the Central Bank commits to acknowledging bad states, i.e., $\theta \leq m$. By disclosing that fundamentals are bad and allowing for a coordinated attack if $y = y_L$, the Central Bank is able to pool intermediate and good states together, minimizing the cost of defending the peg in y_H. The optimal threshold will be the lowest m such that expectations about θ are good enough to prevent an attack if $y = y_H$. Reducing m any further opens up the possibility of self-fulfilling crises, even if the public signal is y_H. Self-fulfilling crises may arise because, in our model, there is a continuum of speculators and their actions are strategic complements. This feature of our model distinguishes this paper from standard Bayesian persuasion models (e.g., Kamenica and Gentzkow (2011)).

Even though it is without loss of generality to assume that $N = 2$, the Central Bank could be arbitrarily precise when fundamentals are bad. It is only when fundamentals are "not too bad" that the Central Bank must be vague, since, as long as y_H remains the same, its payoff does not change. This vagueness is used by the Central Bank to make speculators uncertain about whether the state is intermediate ($\theta \in (m, \bar{\theta})$, where a coordinated attack is profitable) or good ($\theta \geq \bar{\theta}$, where attacking is never profitable), thus preventing them from attacking.

Another characteristic of our equilibrium is that it is Pareto efficient. This happens because the public signal allows for perfect coordination among speculators. In contrast, the game without public information is not Pareto efficient since there are θ at which some speculators attack the currency but the Central Bank defends the peg.

We conclude this section by showing that, even if the Central Bank could fully disclose the state, it would not be optimal to do so. If the state is fully revealed, speculators can coordinate on attacking whenever $\theta < \bar{\theta}$. Since $\underline{m} < \bar{\theta}$ (see Lemma 8, Appendix A.10), there exists $m \in M \cap (\underline{m}, \bar{\theta})$ such that partition $P^m = \{0, m, 1\}$ is strictly preferred to full disclosure. With partition P^m, the Central Bank eliminates currency attacks between $(m, \bar{\theta})$. This proves the following proposition.

Proposition 2 (Full disclosure). *Full disclosure of the state is not an optimal policy for the Central Bank.*

4 No commitment

In this section, we drop the assumption that the Central Bank can commit to a disclosure policy. Without commitment, the Central Bank chooses the public signal after observing the realized state θ. For simplicity, the Central Bank's strategy in the last stage of the game is taken as given.

The game between Central Bank and speculators becomes a signaling game, where θ can be interpreted as the Central Bank's type. A strategy for the Central Bank is a function $y : \Theta \to \Theta^2$ such that when the state is θ, the public signal is $y(\theta) = [\underline{y}(\theta), \overline{y}(\theta)]$.[12] As before, we require that $\underline{y}(\theta) \leq \theta \leq \overline{y}(\theta)$ for all θ.

A strategy for speculators is a function that gives, for every private signal x and every public signal $\hat{y} = [\underline{\hat{y}}, \overline{\hat{y}}]$, the corresponding action to be taken (either to attack or not). As before, let $\pi(x, \hat{y})$ be the aggregate selling strategy. The equilibrium concept in this section is the Perfect Bayesian Equilibrium (PBE) with symmetric strategies for the speculators.

Definition. The strategy profile (y, π) is a PBE if

1. for all $\theta \in [0, 1]$, $y(\theta)$ maximizes the Central Bank's payoff given π;
2. for every pair of signals $(x, \hat{y})$, there exist beliefs $\mu_{x,\hat{y}}$ about θ such that $\pi(x, \hat{y})$ maximizes the speculator's expected payoff given $\mu_{x,\hat{y}}$ and the aggregate strategy π;
3. for each signal $\hat{y}$ such that $\{\theta' : y(\theta') = \hat{y}\} \neq \emptyset$, $\mu_{x,\hat{y}}(\theta)$ is given by Bayes' rule, conditional on x and $y(\theta) = \hat{y}$;

[12] The restriction to closed intervals is made only for simplicity.

4. for each signal $\hat{y}$ such that $\{\theta' : y(\theta') = \hat{y}\} = \emptyset$,

$$\text{support}(\mu_{x,\hat{y}}(\theta)) \subset [x - \varepsilon, x + \varepsilon] \cap \hat{y}.$$

Consider the following profile of (y, π, μ):

$$\begin{aligned} y(\theta) =& \{\theta\}, \quad \forall \theta, \\ \mu_{x,\hat{y}}(\theta) =& \begin{cases} 1, & \text{if } \theta = \max\{x - \varepsilon, \underline{\hat{y}}\} \\ 0, & \text{otherwise} \end{cases} \\ \pi(x, \hat{y}) =& \begin{cases} 1, & \text{if } \max\{x - \varepsilon, \underline{\hat{y}}\} \leq \bar{\theta} \\ 0, & \text{otherwise} \end{cases} \quad . \end{aligned}$$

We claim that this profile is an equilibrium. In this equilibrium, the public signal reveals the true state of the fundamentals, and speculators attack if and only if $\theta \leq \bar{\theta}$.

To see that (y, π) is in fact an equilibrium with beliefs μ, first note that μ satisfies both conditions 3 and 4 of the definition. Now consider speculator i's problem. If $\underline{\hat{y}} = \overline{\hat{y}} = \theta$, given that speculators follow π, it is only profitable for i to attack if $\underline{\hat{y}} \leq \bar{\theta}$, which means that π is optimal on the path of play. Now consider off path signals where $\underline{\hat{y}} < \overline{\hat{y}}$. When $\underline{\hat{y}} > \bar{\theta}$, speculators know that $\theta > \bar{\theta}$ and attacking is indeed not profitable. If $\underline{\hat{y}} \leq \bar{\theta}$ and speculator i receives a private signal $x_i \leq \bar{\theta} + \varepsilon$, he believes that $\theta = \max\{x - \varepsilon, \underline{\hat{y}}\} \leq \bar{\theta}$. The speculator also believes that everyone else received a private signal below $\bar{\theta} + \varepsilon$, and that, following π, they all attack. Hence, attacking is profitable. Finally, if $\underline{\hat{y}} \leq \bar{\theta}$ and $x_i > \bar{\theta} + \varepsilon$, the speculator knows that $\theta > \bar{\theta}$, and it is not profitable to attack. Therefore, π is optimal for i, given μ, y, and that every other speculators follow π.

Now we show that the Central Bank has no profitable deviation from strategy y. Since the peg is not attacked on $(\bar{\theta}, 1]$, there can only be a profitable deviation at θ' if $\theta' \leq \bar{\theta}$. However, according to π, speculators still attack the peg whenever they observe a public signal $\hat{y}$, with $\theta' \in \hat{y}$. This proves that there is no profitable deviation for the Central Bank and that y is optimal.

The PBE above passes the intuitive criterion of Cho and Kreps (1987). As argued above, only types in $[0, \bar{\theta}]$ could benefit from a deviation. However, if the speculators know that $\theta \leq \bar{\theta}$, they can coordinate on attacking the currency peg and, in this case, a deviation is not profitable.

This example stresses the importance of commitment when the public signal allows for agents to coordinate on an action that, from the Central Bank's perspective, is undesirable. When the Central Bank is unable to commit to a disclosure policy, speculators can exploit the fact that types $\theta > \bar{\theta}$ will reveal themselves. In this case, speculators are able to coordinate on attacking the currency peg whenever $\theta \leq \bar{\theta}$. The results are summarized in the following proposition.

Proposition 3 (**No commitment).** *Suppose that the Central Bank cannot commit to a disclosure policy. Then, in the worst equilibrium for the Central Bank speculators coordinate on attacking the currency for all $\theta \leq \bar{\theta}$. With commitment, the Central Bank can avoid attacks on $(\underline{m}, \bar{\theta}]$.*

5 Conclusion

This paper studies how a Central Bank should disclose information when facing agents who can use public signals to coordinate their actions. In this setting, the optimal communication policy must steer coordination in the right direction, inducing agents to take the Central Bank's preferred action as often as possible and reducing the likelihood of self-fulfilling crises. We illustrate this problem with a model of currency attacks, but our framework is suited to analyze a much broader set of coordination games.

Our first finding is that the Central Bank chooses an information structure with only two signals, high and low. In equilibrium, the currency peg is only abandoned after the low signal is sent. The key to this result is that such a vague policy can guarantee the uniqueness of equilibrium, setting a lower bound for the Central Bank's payoff. In contrast, more precise communication leads to multiple equilibria and the possibility of undesirable outcomes.

Intuitively, the low signal is an acknowledgment that fundamentals are too bad for the peg to be maintained, while the high signal indicates that the currency will be defended. We show that the optimal policy is to increase the number of states associated to the high signal while preserving the uniqueness of equilibrium, i.e., as long as agents still find it optimal to follow the recommendation conveyed by the public signal. Thus, under the optimal policy, agents perfectly coordinate their actions. Such a policy leads to Pareto efficiency, since agents only attack the currency when the peg is abandoned.

We also show that the lack of commitment can effectively make the Central Bank hostage to the agents' beliefs. When agents are pessimistic, the Central Bank has to fully disclose the best states of fundamentals in oder to keep the peg. This allows agents to coordinate on attacking the currency for every state in which a coordinated attack is profitable.

A Appendix

A.1 Proof of Lemma 1

Lemma 1. *For a given public signal y, if $\pi(x, y) \geq \pi'(x, y)$ for all x, then $u_y(x, \pi) \geq u_y(x, \pi')$ for all x.*

Proof: Suppose that $\pi(x, y) \geq \pi'(x, y)$ for all x. Then

$$s(\theta, \pi) \geq s(\theta, \pi') \Rightarrow A(\pi) \cap y \supseteq A(\pi') \cap y \Rightarrow u_y(x, \pi) \geq u_y(x, \pi').$$

□

A.2 Derivation of ψ

For $k \in [-\varepsilon, 1 + \varepsilon]$, define θ_k as

$$\theta_k = \sup\{\theta : s(\theta, I_k) \geq a(\theta)\}. \tag{6}$$

θ_k is the largest value of θ at which that the Central Bank finds it optimal to abandon the peg when speculators' aggregate short sales are given by I_k. Since $s(\cdot, I_k)$ is decreasing and $a(\cdot)$ is increasing, the Central Bank abandons the peg if and only if $\theta \leq \theta_k$. Given that $a(\theta) = 0$ for $\theta \leq \underline{\theta}$, the set on the right hand side of (6) is never empty and θ_k is well defined. Moreover, we have that $\theta_k \geq \underline{\theta}$ for all k.

Define $\bar{k}$ as the unique value of k that solves

$$s(1, I_k) = G(k-1) = a(1),$$

that is, $\bar{k} = 1 + G^{-1}(a(1))$. If speculators follow the cutoff rule $I_{\bar{k}}$, the peg is abandoned for every realization of θ. Since $s(\theta, I_k)$ is increasing in k, we then have that $\theta_k = 1$, for all $k \geq \bar{k}$.

Now suppose that speculators follow the cutoff rule I_k, with $k \leq \underline{\theta} - \varepsilon$. In this case, there are no attacks when $\theta > \underline{\theta}$, which implies that $\theta_k = \underline{\theta}$.

Finally, if $k \in (\underline{\theta} - \varepsilon, \bar{k})$, then θ_k is the unique value of θ that solves

$$s(\theta, I_k) = G(k - \theta) = a(\theta). \tag{7}$$

Note that $\theta \leq \underline{\theta}$ cannot be a solution to the equation above, since the left hand side of (7) is strictly positive, while the right hand side equals 0. Thus, $\theta_k > \underline{\theta}$. For $\theta > \underline{\theta}$, we have that $a(\theta)$ is strictly increasing, thus θ_k is strictly increasing in k. In addition, $a(\theta) \in (0, 1)$ implies that $\theta_k \in (k - \varepsilon, k + \varepsilon)$. Note that θ_k is continuous in k for all $k \in [-\varepsilon, 1 + \varepsilon]$.

Define the function ψ as $\psi(k) = \min\{\theta_k - k, \varepsilon\}$, for $k \in [-\varepsilon, 1 + \varepsilon]$. Thus

$$\psi(k) = \begin{cases} \varepsilon, & \text{if } k \leq \underline{\theta} - \varepsilon \\ -G^{-1}(a(\theta_k)) \in (-\varepsilon, \varepsilon), & \text{if } k \in (\underline{\theta} - \varepsilon, \bar{k}) \\ 1 - k \in [-\varepsilon, \varepsilon), & \text{if } k > \bar{k} \end{cases}. \tag{8}$$

From the continuity of θ_k, it follows that $\psi(k)$ is continuous. Since θ_k is strictly increasing for $k \in (\underline{\theta} - \varepsilon, \bar{k})$, then $\psi(k)$ is strictly decreasing for $k > \underline{\theta} - \varepsilon$.

A.3 Proof of Lemma 2

Lemma 2. *The payoff function $u_{y_n}(k, I_k)$ is continuous in k, for all $k \in X_{y_n}$.*

Proof: Using (4), the payoff function when $y = y_n$ is given by

$$u_{y_n}(k, I_k) = \int_{a_{y_n}}^{b_{y_n}} [e^* - f(\theta)]\phi_{y_n}(\theta|k)d\theta - t, \tag{9}$$

where $a_{y_n} = \max\{k - \varepsilon, m_{n-1}\}$, and $b_{y_n} = \max\{\min\{k + \psi(k), m_n\}, m_{n-1}\}$. Since $\phi_{y_n}(\cdot|k)$ and the limits of integration are continuous in k (because $\psi(\cdot)$ is continuous), $u_{y_n}(k, I_k)$ is continuous in k. □

A.4 Proof of Lemma 3

Lemma 3. *For $k \in (\varepsilon, 1-\varepsilon)$, the payoff function $u(k, I_k)$ is strictly decreasing in k.*

Proof: For $k \in (\varepsilon, 1-\varepsilon)$,

$$u(k, I_k) = \int_{k-\varepsilon}^{k+\psi(k)} [e^* - f(\theta)]g(k-\theta)\,d\theta - t = \int_{-\psi(k)}^{\varepsilon} [e^* - f(k-\tilde{\varepsilon})]g(\tilde{\varepsilon})d\tilde{\varepsilon} - t.$$

Since $\psi(\cdot)$ is decreasing and $f(\cdot)$ is strictly decreasing, we have that $u(k, I_k)$ is strictly decreasing in k. □

A.5 Proof of Lemma 4

Lemma 4. *Suppose that Assumption 1 is satisfied. When the aggregate strategy is given by I_k, the payoff from attacking the currency, $u_y(x, I_k)$, is decreasing in the private signal x.*

Proof: Suppose that the aggregate strategy is given by I_k. Let $\mathcal{I}(\theta)$ be an indicator function that equals 1 if the currency peg is abandoned when the state is θ. Since, by assumption, speculators follow a cutoff rule, $\mathcal{I}(\theta)$ is weakly decreasing in θ.[13] Define

$$U(\theta) = [f(\theta) - e^*]\mathcal{I}(\theta),$$

which is negative and increasing. Consider a public signal y and a pair of private

[13] $\mathcal{I}(\theta) = 1$, if $\theta \le \theta_k$; and $\mathcal{I}(\theta) = 0$, if $\theta > \theta_k$.

signals x_1 and x_2, with $x_1 < x_2$. Then

$$\int_0^1 U(\theta)d\Phi_y(\theta|x_2) \geq \int_0^1 U(\theta)d\Phi_y(\theta|x_1),$$

where the inequality comes from Assumption 1 and the fact that U is increasing. Hence

$$\begin{aligned} u_y(x_1, I_k) &= -\int_0^1 U(\theta)d\Phi_y(\theta|x_1) - t \\ &\geq -\int_0^1 U(\theta)d\Phi_y(\theta|x_2) - t \\ &= u_y(x_2, I_k), \end{aligned}$$

which completes the proof. □

A.6 Proof of Lemma 5

Lemma 5. *For a given public signal y,*

i. if $u_y(k, I_k) < 0$ for all $k \in X_y$, then, in any equilibrium, $\pi(x, y) = 0$ for all $x \in X_y$.

ii. if $u_y(k', I_{k'}) \geq 0$ for some $k' \in X_y$, then, in the worst equilibrium for the Central Bank, speculators use the cutoff rule I_k after observing y, where $k = \sup\{k' \in X_y : u_y(k', I_{k'}) \geq 0\}$.

Proof: i. Suppose that $u_y(k, I_k) < 0$ for all $k \in X_y$. Let π be a equilibrium strategy, and suppose by way of contradiction that there is $x \in X_y$ such that $\pi(x, y) > 0$. If this is true, then the set $\{x' \in X_y : \pi(x', y) > 0\}$ is non-empty and we can define $\bar{x}_y$ as

$$\bar{x}_y = \sup\{x' \in X_y : \pi(x', y) > 0\}.$$

Note that $\bar{x}_y \in X_y$ because X_y is right-closed. Also note that, if π is an equilibrium strategy, then for any x' such that $\pi(x', y) > 0$, it has to be true that $u_y(x', \pi) \geq 0$. By the continuity of u_y in the private signal, $u_y(\bar{x}_y, \pi) \geq 0$. From Lemma 1,

$$u_y(\bar{x}_y, I_{\bar{x}_y}) \geq u_y(\bar{x}_y, \pi) \geq 0,$$

which contradicts the assumption that $u_y(k, I_k) < 0$ for all $k \in X_y$.

ii. If $u(k, I_k) > 0$, by continuity (Lemma 2), it has to be true that k is the right bound of the interval X_y and, by the decreasing property of u_y in the private signal (Lemma 4), I_k is an equilibrium strategy. If $u(k, I_k) = 0$, then we know from Lemma 4 that I_k is an equilibrium strategy. Now it is left to show that any equilibrium strategy π features $\pi(x, y) = 0$ for $x > k$. Assume by way of contradiction that there is an equilibrium with $\pi(x, y) > 0$ for some $x > k$. Let $\bar{x}_y = \sup\{x' \in X_y : \pi(x', y) > 0\} \in X_y$. By Lemma 1, $u_y(\bar{x}_y, I_{\bar{x}_y}) \geq u_y(\bar{x}_y, \pi) \geq 0$, which contradicts the assumption that k is the supremum of the set $\{k' \in X_y : u_y(k', I_{k'}) \geq 0\}$.

A.7 Lemma 6

Lemma 6. *Suppose that $y = y_n$ and that speculators follow I_k, for $k \in X_{y_n}$. The payoff function $u_{y_n}(x, I_k)$ is continuous in both m_{n-1} and m_n. Furthermore, it is decreasing in m_{n-1} for $k < m_{n-1} + \varepsilon$, and constant otherwise; it is also decreasing in m_n for $k > m_n - \varepsilon$, and constant otherwise.*

Proof: We want to show that u_{y_n} is decreasing in both m_{n-1} and m_n.

Fix m_{n-1} and consider a change from m_n to $m_n' > m_n$. Recall that, when agents are using a cutoff strategy, $(e^* - f(\theta))\,\mathcal{I}(\theta)$ is a decreasing function of θ, where $\mathcal{I}(\cdot)$ is the indicator function that equals 1 if the peg is abandoned. For $x > m_n - \varepsilon$, we have

$$\begin{aligned}
&u_{[m_{n-1}, m_n']}(x, I_k) + t \\
&= E\left[(e^* - f(\theta))\,\mathcal{I}(\theta) | \theta \in [m_{n-1}, m_n'], x\right] \\
&= E\left[(e^* - f(\theta))\,\mathcal{I}(\theta) | \theta \in [m_{n-1}, m_n], x\right] P(\theta \in [m_{n-1}, m_n] | \theta \in [m_{n-1}, m_n'], x) \\
&+ E\left[(e^* - f(\theta))\,\mathcal{I}(\theta) | \theta \in [m_n, m_n'], x\right] P(\theta \in [m_n, m_n'] | \theta \in [m_{n-1}, m_n'], x) \\
&< E\left[(e^* - f(\theta))\,\mathcal{I}(\theta) | \theta \in [m_{n-1}, m_n], x\right] P(\theta \in [m_{n-1}, m_n] | \theta \in [m_{n-1}, m_n'], x) \\
&+ E\left[(e^* - f(\theta))\,\mathcal{I}(\theta) | \theta \in [m_{n-1}, m_n], x\right] P(\theta \in [m_n, m_n'] | \theta \in [m_{n-1}, m_n'], x) \\
&= E\left[(e^* - f(\theta))\,\mathcal{I}(\theta) | \theta \in [m_{n-1}, m_n], x\right] \\
&= u_{[m_{n-1}, m_n]}(x, I_k) + t,
\end{aligned}$$

that is, $u_{y_n}(x, I_k)$ is decreasing in m_n. For $x \leq m_n - \varepsilon$, we have

$$P(\theta \in [m_n, m_n'] | \theta \in [m_{n-1}, m_n'], x) = 0,$$

therefore $u_{y_n}(x, I_k)$ is constant in m_n. Analogous reasoning shows that $u_{y_n}(x, I_k)$ is decreasing in m_{n-1}, for $x < m_{n-1} + \varepsilon$, and constant otherwise. Regarding continuity, note that the payoff function is given by

$$u_{y_n}(x, I_k) = \int_{\max\{x-\varepsilon, m_{n-1}\}}^{\min\{x+\varepsilon, m_n\}} [e^* - f(\theta)] \, \mathcal{I}(\theta) \frac{g(x-\theta)}{G(x-m_{n-1}) - G(x-m_n)} d\theta - t,$$

which is continuous in both m_{n-1} and m_n. □

A.8 Proof of Theorem 1

Theorem 1. *For any partition $P = \{m_n\}_{n=0}^{N}$ with $N > 2$, there exists $P' = \{m_n'\}_{n=0}^{N'}$ with $N' = 2$, such that $V(P') \geq V(P)$.*

Proof: Given Proposition 1, the only non trivial result left to show is that, for any $P = \{0, m_1, m_2, 1\}$, with $m_1 < \bar{\theta} < m_2$, there is a $P' = \{0, m', 1\}$ such that $V(P') \geq V(P)$.

- *Case 1: the Central Bank maintains the peg for all θ in y_2.* Consider the alternative partition $P' = \{0, m_1, 1\}$. The Central Bank cannot be worse off if $\theta \leq m_1$.

  We know from Proposition 1 that $u_{(m_1,m_2]}(k, I_k) < 0$ for all $k \in X_{(m_1,m_2]}$. Since $m_2 > \bar{\theta}$, we also know that $u_{(m_2,1]}(k, I_k) < 0$ for all $k \in X_{(m_2,1]}$. From Lemma 6 in Appendix A.7,

  $$u_{(m_1,1]}(k, I_k) \leq u_{(m_1,m_2]}(k, I_k) < 0, \quad \text{for all } k \in (m_1 - \varepsilon, m_2 + \varepsilon],$$

  and

  $$u_{(m_1,1]}(k, I_k) = u_{(m_2,1]}(k, I_k) < 0, \quad \text{for all } k \in (m_2 + \varepsilon, 1 + \varepsilon].$$

  These inequalities imply that $u_{(m_1,1]}(k, I_k) < 0$ for $k \in X_{(m_1,1]}$. From Proposition 1, there is no attack if $\theta > m_1$. Thus, $V(P') \geq V(P)$.

- *Case 2: the Central Bank abandons the peg for all θ in y_2.* Consider the partition $P' = \{0, m_2, 1\}$. The Central Bank is not worse off if $\theta \leq m_2$. If $\theta > m_2$, speculators observe the public signal $(m_2, 1]$, and since $m_2 > \bar{\theta}$, no one attacks. Thus, $V(P') \geq V(P)$.

- *Case 3: the Central Bank abandons the peg at some but not all θ in y_2.* From Proposition 1, speculators follow a cutoff rule I_{k_2} after observing y_2, where $k_2 = \sup\{k' \in X_{y_2} : u_{y_2}(k', I_{k'}) = 0\}$. Given the speculators' strategy, there exists $\theta_{k_2} \in (m_1, m_2]$ such that the peg is abandoned if and only if $\theta \leq \theta_{k_2}$. Consider partition $P' = \{0, \theta_{k_2}, 1\}$. From Lemma 6 ,

$$u_{(\theta_{k_2},1]}(k, I_k) \leq u_{(m_1,m_2]}(k, I_k) < 0, \quad \text{for all } k \in (k_2, m_2 + \varepsilon],$$

and

$$u_{(\theta_{k_2},1]}(k, I_k) = u_{(m_2,1]}(k, I_k) < 0, \quad \text{for all } k \in (m_2 + \varepsilon, 1 + \varepsilon].$$

Thus, I_k cannot be an equilibrium strategy if $k > k_2$. By Lemma 5, the Central Bank maintains the peg if $\theta > \theta_{k_2}$. By changing the partition from P to P', the Central Bank no longer has to pay a cost to defend the currency on $(\theta_{k_2}, \theta_{k_2} + \varepsilon)$, therefore $V(P') > V(P)$.

□

A.9 Lemma 7

Lemma 7. *Consider the game following the disclosure of $y = y_L$. If $m \leq \theta^*$, the equilibrium is unique and the peg is always abandoned. If, in addition, $m \leq \bar{\theta}$, then the equilibrium is unique and the speculators coordinate on attacking the currency peg .*

Proof: Using Lemma 3, the proof of existence and uniqueness of equilibrium in the game without a public signal is analogous to the one in Morris and Shin (1998). The speculators follow a cutoff strategy I_{k^*}, such that $u(k^*, I_{k^*}) = 0$, with $k^* \in (\varepsilon, 1-\varepsilon)$, and the peg is abandoned for $\theta \leq \theta_{k^*} = \theta^*$. Since $u(k, I_k) > 0$ for $k \leq \varepsilon$, and $u(k, I_k) < 0$ for $k \geq 1 - \varepsilon$, it follows from Lemma 3 that $u(k, I_k) > 0$ for $k < k^*$, and that $u(k, I_k) < 0$ for $k > k^*$.

Now we turn to the game with a public signal. Consider an equilibrium strategy profile π. Let $\pi(x, y_L)$ denote the probability that a speculator attacks the currency given a private signal x and a public signal $y = y_L$. Define $\underline{x}$ as [14]

$$\underline{x} = \inf\{x \in X_{y_L} : \pi(x, y) < 1\}.$$

Then, from Lemma 1,

$$u_{y_L}(\underline{x}, I_{\underline{x}}) \leq u_{y_L}(\underline{x}, \pi) \leq 0, \tag{10}$$

where the last inequality comes from the fact that $u_y(x, \pi) \leq 0$ if $\pi(x, y) < 1$, and from the continuity of $u_y(x, \pi)$ in x.

From Lemma 6, we have that

$$u_{y_L}(k, I_k) \geq u(k, I_k) > 0, \quad \text{for } k < k^*.$$

Hence, (10) implies that $\underline{x} \geq k^*$, and that $\pi(x, y_L) = 1$ for every $x < k^*$. This means that, in equilibrium, the peg is abandoned for all $\theta \in y_L = [0, m] \subseteq [0, \theta^*]$.

After observing $y = y_L$, speculators know that peg is always abandoned in equilibrium. Thus, a speculator who receives a private signal x attacks the currency if and only if

$$\mathbb{E}[e^* - f(\theta) - t | x, y_L] \geq 0,$$

and it follows that the equilibrium is unique. If $m \leq \bar{\theta}$, attacking is always profitable when $y = y_L$, thus speculators coordinate on attacking the currency peg regardless of their private signals. $\square$

A.10 Lemma 8

Lemma 8. *$\underline{m} < \bar{\theta}$.*

Proof: We need to find $m < \bar{\theta}$ such that $u_{(m,1]}(k, I_k) < 0$ for all k. Consider the partition $P^{\bar{\theta}}$ and let $\bar{k}$ solve $\theta_{\bar{k}} = \bar{\theta}$.[15] First we prove that there is a bound $\delta < 0$ such that $u_{(\bar{\theta},1]}(k, I_k) \leq \delta$ for all $k \in X_{(\bar{\theta},1]}$. Then we use continuity of $u_{(m,1]}(k, I_k)$ in m to show that there is an m below $\bar{\theta}$ that belongs to M.

[14] If $\pi(x, y) = 1$ for all $x \in X_{y_L}$, then define $\underline{x} = \sup X_{y_L}$.

[15] $a(\bar{\theta}) = s(\bar{\theta}, I_{\bar{k}})$, that is, if speculators follow the cutoff rule $I_{\bar{k}}$, the Central Bank is indifferent between defending the currency and abandoning the peg at $\theta = \bar{\theta}$.

Let $\delta = u_{(\bar{\theta},1]}(\bar{k}, I_{1+\varepsilon})$. Note that $\delta < 0$ because of the definition of $\bar{\theta}$. We claim that $u_{(\bar{\theta},1]}(k, I_k) \leq \delta$ for all $k \in (\bar{\theta} - \varepsilon, 1 + \varepsilon]$. To see this, let $k \leq \bar{k}$. If speculators follow I_k, then the threshold for the Central Bank to abandon the peg is $\theta_k \leq \bar{\theta}$, which means that the Central Bank does not abandon the peg on $(\bar{\theta}, 1]$. Hence $u_{(\bar{\theta},1]}(k, I_k) = -t < \delta$ for any $k \leq \bar{k}$. For $k > \bar{k}$

$$u_{(\bar{\theta},1]}(k, I_k) \leq u_{(\bar{\theta},1]}(k, I_{1+\varepsilon}) \leq u_{(\bar{\theta},1]}(\bar{k}, I_{1+\varepsilon}) = \delta,$$

where the first inequality comes from Lemma 1, and the second inequality comes from Lemma 4.Therefore, $u_{(\bar{\theta},1]}(k, I_k) \leq \delta$ for all $k \in X_{(\bar{\theta},1]}$, as claimed.

Define l_m^1 and l_m^2 as

$$l_m^1 = \lim_{k \downarrow \bar{k}} u_{(m,1]}(k, I_{1+\varepsilon}),$$

and

$$l_m^2 = \lim_{k \downarrow \bar{\theta} - \varepsilon} u_{(m,1]}(k, I_{\bar{k}}).$$

Since $u_{(\bar{\theta},1]}(k, I_{1+\varepsilon}) \leq \delta$ for all $k > \bar{k}$, we have that $l_{\bar{\theta}}^1 \leq \delta$. Since $u_{(\bar{\theta},1]}(k, I_{\bar{k}}) \leq \delta$ for $k \in (\bar{\theta} - \varepsilon, \bar{k}]$, we have that $l_{\bar{\theta}}^2 \leq \delta$. From Lemmas 1 and 4, $l_m^1 \geq u_{(m,1]}(k, I_k)$ for $k > \bar{k}$, and $l_m^2 \geq u_{(m,1]}(k, I_k)$ for $k \in (\bar{\theta} - \varepsilon, \bar{k}]$. Then $l_m \equiv \max\{l_m^1, l_m^2\} \geq u_{(m,1]}(k, I_k)$ for $k > \bar{\theta} - \varepsilon$. From Lemma 6, l_m^1 and l_m^2 are continuous in m, and so is l_m. Hence, there exists $m' < \bar{\theta}$ such that $l_{m'} < l_{\bar{\theta}} - \delta/2 \leq \delta/2 < 0$. This implies that $u_{(m',1]}(k, I_k) \leq \delta/2$ for $k > \bar{\theta} - \varepsilon$. In this case, either $u_{(m',1]}(k, I_k) < 0$ for all $k \in (m' - \varepsilon, \bar{\theta} - \varepsilon]$, or there exists $k' = \sup\{k \in (m' - \varepsilon, \bar{\theta} - \varepsilon] : u_{(m',1]}(k, I_k) \geq 0\}$. From Lemma 5, either there is no attack on $(m', 1]$, thus $m' \in M$, or, in the worst equilibrium for the Central Bank, speculators follow $I_{k'}$ after observing $(m', 1]$. In the latter case, the Central Bank abandons the peg for $\theta \leq \theta_{k'} \in (m', \bar{\theta})$. Consider the partition $P^{\theta_{k'}}$. From Lemma 6, $u_{(\theta_{k'},1]}(k, I_k) < 0$ for all $k \in X_{(\theta_{k'},1]}$, and, from Lemma 5, there is no attack on y_H. This means that $\theta_{k'} \in M$. Thus, either $\bar{\theta} > m' \in M$ or $\bar{\theta} > \theta_{k'} \in M$, which implies that $\underline{m} < \bar{\theta}$. $\square$

A.11 Proof of Theorem 2

Theorem 2. *For every partition P, $V(P) \leq \overline{V}$, where*

$$\overline{V} = \lim_{m \downarrow \underline{m}} V(P^m),$$

and thus the Central Bank can achieve a payoff arbitrarily close to $\overline{V}$.

Proof:

Consider a partition P. From Theorem 1, we can assume that $P = P^m = \{0, m, 1\}$. The proof consists of five steps:

i. if $m > \underline{m}$, then $m \in M$;

ii. for all $m' < m$, there exists $m \in M$ such that $V(P^m) > V(P^{m'})$;

iii. if $m > \theta^*$, then $V(P^{\theta^*}) > V(P^m)$;

iv. if $m \in (\underline{m}, \max\{\theta^*, \bar{\theta}\})$, then $V(P^m) = (1 - m)v$;

v. $V(P^{\underline{m}}) \leq (1 - \underline{m})v$.

When all the claims above are true, we have that $\overline{V}$ is well defined, no partition can yield a payoff higher than $\overline{V}$, and the Central Bank can achieve a payoff arbitrarily close to $\overline{V}$ by setting m arbitrarily close to $\underline{m}$. The proofs are presented below.

i. Since $m > \underline{m}$, there exists $m' \in M$ such that $m' < m$. By Lemma 5, $u_{(m',1]}(k, I_k) < 0$ for all $k \in X_{(m',1]}$. By Lemma 6, $u_{(m',1]}(k, I_k) < 0$ for all $k \in X_{(m,1]}$. Using Lemma 5, $m \in M$.

ii. From Lemma 8, we know that $\underline{m} < \bar{\theta}$. Then, in the worst equilibrium for the Central Bank the peg is abandoned when $\theta \in [0, m]$. From Lemma 1, speculators follow a cutoff strategy I_{k_H} after observing y_H, where $k_H = \sup\{k \in X_{y_H} : u_{y_H}(k, I_k) \geq 0\}$. Given the speculators' strategy, there exists $\theta_{k_H} > m$ such that the peg is abandoned if and only if $\theta \leq \theta_{k_H}$. Following the arguments in the proof Theorem 1 (Case 3), $\theta_{k_H} \in M$ and partition $P^{\theta_{k_H}} = \{0, \theta_{k_H}, 1\}$ is preferred to P^m.

iii. See discussion at the beginning of section 3.2).

iv. From Lemma 7 and the fact that $m \in M$, there is no attack on $(m, 1]$ and the peg is abandoned on $[0, m]$. Therefore,

$$V(P^m) = \int_m^1 v \, d\theta = (1 - m)v.$$

and

$$\lim_{m \downarrow \underline{m}} V(P^m) = (1 - \underline{m})v.$$

v. If $\underline{m} \in M$, then, by the same arguments as in part iv, we have that $V(P^{\underline{m}}) = (1 - \underline{m})v$. If $m \notin M$, then Lemma 5 implies that there exists $\theta_k > \underline{m}$ such that the peg is abandoned if and only if $\theta \leq \theta_k$. In this case, $V(P^{\underline{m}}) \leq (1 - \theta_k)v < (1 - \underline{m})v$.

□

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